Black holes in massive gravity

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OUTLINE

- Introduction
- Black holes in massive gravity
- Perturbations and (in)stability
- Conclusions

INTRODUCTION

Motivations:

why modify gravity

- Modifying gravity - explain Dark energy, cosmological constant problem, to cure non-renormalizability problem, theoretical curiosity etc.

- There are many ways to modify gravity: f(R), scalar-tensor theories, Galileons, Horndeski theory, KGB, Fab-four, higher-dimensions, DGP, massive gravity...

massive gravity

- Naively, cancellation of the cosmological constant, because of the Yukawa decay;
- Small cosmological constant due to small m

Massive gravity:

problems

Physical ghost

When modifying gravity, extra degrees of freedom appear, which alter gravitational interaction between bodies

} USUAL PROBLEMS of massive gravity

Fierz-Pauli massive gravity

Expand the Einstein-Hilbert action:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_{GR} = M_P^2 \int d^4x \sqrt{-g} R = \int d^4x \left(-\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} \right) + \mathcal{O}(h^3)$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\partial_\mu\partial_\nu h - \frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}\partial_\rho\partial_\mu h^\rho_\nu + \frac{1}{2}\partial_\rho\partial_\nu h^\rho_\mu - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho\partial^\sigma h_{\rho\sigma} - \Box h)$$

2 propagating spin: 2 massless gravitons, spin-2

$$x^\alpha \rightarrow x^\alpha + \xi^\alpha, \;\; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}$$

Fierz-Pauli massive gravity

Fierz-Pauli action *(Fierz&Pauli'39)*:

$$S_{PF} = M_P^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

Linearized Einstein-Hilbert term

mass term

$$x^\alpha \to x^\alpha + \xi^\alpha, \; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}$$

5 healthy degrees of freedom (because of a particular choice of the potential, h=0)

for a generic mass term 6 d.o.f., one is necessary Ostrogradski ghost

Non-linear completion ?

Non-linear massive gravity

potential for metric

Need to construct a mass term -> introduce an extra metric

$g_{\mu\nu}$:physical metric, matter couples to it

$f_{\mu\nu}$:an extra metric (may be dynamical or fixed)

Construct a potential, following the rules:
- general covariance under diffeomorphisms (common to the two metrics)
- has flat spacetime as solution for physical metric (or not, for cosmology)
- when expanding around flat metric the potential takes a specific form, the Pauli-Fierz form

Non-linear massive gravity

potential for metric

building block: $\mathbf{g^{-1}f}$

$$S_{int}^{(2)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-f} \, H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right)$$

(Boulware & Deser'72)

$$S_{int}^{(3)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-g} \, H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$

(Arkani-Hamed et al'03)

where $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

Non-linear massive gravity

Extra degrees of freedom

New degrees of freedom due to the broken diff invariance =>

Change of predictions, i.e. Solar system tests

Non-linear effects restore General Relativity in some region
due to the non-linear effects

Vainshtein'72
EB,Deffayet,Ziour'09'10

Non-linear massive gravity

Boulware-Deser ghost

Generically there are two propagating scalars: one is a ghost !

(Boulware & Deser'72)

dRGT

the model

Massive gravity without Boulware-Deser ghost *(de Rham-Gabadabze-Tolley'10)*

g is physical metric;
f is fixed (flat) or extra dynamical metric.

$$\mathcal{K} = \mathbb{I} - \sqrt{\mathbf{g^{-1}f}}, \;\; \mathcal{U} = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4$$

$$\mathcal{U}_2\left(\mathcal{K}\right) = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$

$$\mathcal{U}_3\left(\mathcal{K}\right) = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)$$

$$\mathcal{U}_4\left(\mathcal{K}\right) = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right)$$

$$S = M_P^2 \int d^4x \sqrt{-g}\left(\frac{R[g]}{2} + m^2 \mathcal{U}[g,f] - m^2 \Lambda_g\right) + \frac{\kappa M_P^2}{2} \int d^4x \sqrt{-f}\left(\mathcal{R}[f] - m^2 \Lambda_f\right)$$

Equations of motion

$$G^{\mu}{}_{\nu} = m^2 \left(T^{\mu}{}_{\nu} + \Lambda_g \delta^{\mu}_{\nu} \right)$$

$G_{\mu\nu}$ is the Einstein tensor for metric $g_{\mu\nu}$

$$\mathcal{G}^{\mu}{}_{\nu} = m^2 \left(\frac{\sqrt{-g}}{\sqrt{-f}} \frac{\mathcal{T}^{\mu}{}_{\nu}}{\kappa} + \Lambda_f \delta^{\mu}_{\nu} \right)$$

$\mathcal{G}_{\mu\nu}$ is the Einstein tensor for metric $f_{\mu\nu}$

No matter Lagrangian.
The energy-momentum tensor from interaction term:

$$T_{\mu\nu} = \mathcal{U} g_{\mu\nu} - 2 \frac{\delta \mathcal{U}}{\delta g^{\mu\nu}} = [\text{function of } \mathcal{K}]$$

$$\mathcal{T}_{\mu\nu} = -2 \frac{\delta \mathcal{U}}{\delta f^{\mu\nu}} = [\text{function of } \mathcal{K}]$$

BLACK HOLES

Black holes

Schwarzschild metric

Ansatz (bi-Eddington-Finkelstein form):

$$ds_g^2 = -\left(1 - \frac{r_g}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right]$$

Non-diagonal term from interaction:

$$T^r{}_v = -\mathcal{T}^r{}_v = \frac{C\left(\beta(C-1)^2 - 2\alpha(C-1) + 1\right)(r_f - r_g)}{2r}$$

$$\alpha \equiv 1 + \alpha_3, \;\; \beta \equiv \alpha_3 + \alpha_4$$

These must vanish due to the Einstein equations and the ansatz for the metrics

Black holes

Schwarzschild metric

Two choices: $\begin{cases} r_g = r_f & \text{bi-diagonal} \\ \beta(C-1)^2 - 2\alpha(C-1) + 1 = 0 & \text{non-bidiagonal} \end{cases}$

Also we need to tune the cosmological constants to balance the diagonal part:

$$\Lambda_g = -(C-1)(\beta(C-1)^2 - 3\alpha(C-1) + 3),$$
$$\Lambda_f = \frac{1}{\kappa C^3}\left(C^3(1-\alpha+\beta) - 3C^2\beta + 3C(\alpha+\beta) - 2\alpha - \beta - 1\right)$$

Black holes

Charged black holes

Ansatz (bi-Eddington-Finkelstein form):

$$ds_g^2 = -\left(1 - \frac{r_g}{r} + \frac{r_Q^2}{r^2} - \frac{r^2}{l_g^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r} - \frac{r^2}{l_f^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right].$$

$$A_\mu = \left\{\frac{Q}{r}, 0, 0, 0\right\}$$

$$T^\mu{}_\nu = \begin{pmatrix} \Lambda_m^{(g)} & 0 & 0 & 0 \\ T^r{}_v & \Lambda_m^{(g)} & 0 & 0 \\ 0 & 0 & \Lambda_m^{(g)} & 0 \\ 0 & 0 & 0 & \Lambda_m^{(g)} \end{pmatrix}$$

$$\Lambda_m^{(g)} = -(C-1)\left(\left(\beta(C-1)^2 - 3\alpha(C-1) + 3\right)\right)$$

$$T^r{}_v = -\frac{C}{2}\left(\beta(C-1)^2 - 2\alpha(C-1) + 1\right)\left(\frac{r_g - r_f}{r} - \frac{r_Q^2}{r^2} + \frac{r^2}{l_g^2} - \frac{r^2}{l_f^2}\right)$$

Black holes

Charged black holes

Both sets of the Einstein equations are satisfied by,

$$
\begin{aligned}
\beta(C-1)^2 - 2\alpha(C-1) + 1 &= 0, \\
\sqrt{2} M_P r_Q &= Q, \\
(C-1)\left(\left(\beta(C-1)^2 - 3\alpha(C-1) + 3\right)\right) + \Lambda_g &= \frac{3}{m^2 l_g^2}, \\
-\frac{1}{\kappa C^3}\left(C^3(1-\alpha+\beta) - 3C^2\beta + 3C(\alpha+\beta) - 2\alpha - \beta - 1\right) + \Lambda_f &= \frac{3}{C^2 m^2 l_f^2}.
\end{aligned}
$$

Black holes

Charged black holes - special choice of potential

$$\beta = \alpha^2$$

$$ds_g^2 = -g_{vv}dv^2 + 2g_{vr}dvdr + g_{rr}dr^2 + r^2 d\Omega^2,$$
$$ds_f^2 = C^2\left[-f_{vv}dv^2 + 2f_{vr}dvdr + f_{rr}dr^2 + r^2 d\Omega^2\right]$$

$$C = 1 + \frac{1}{\alpha}$$

The Einstein equations are satisfied.

A change of coordinates is allowed in each metric

$v \to v(v,r),\ r \to r(v,r).$

Black holes

Rotating solutions

Original Kerr metric

$$ds_g^2 = -\left(1 - \frac{r_g r}{\rho^2}\right)\left(dv + a\sin^2\theta d\phi\right)^2$$
$$+ 2\left(dv + a\sin^2\theta d\phi\right)\left(dr + a\sin^2\theta d\phi\right) + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

f is fixed, but unusual form

$$ds_f^2 = C^2\left[-dv^2 + 2dvdr + 2a\sin^2\theta drd\phi + \rho^2 d\theta^2 + \left(r^2 + a^2\right)\sin^2\theta d\phi^2\right]$$

Obtained from $ds_M^2 = -dt^2 + dx^2 + dy^2 + dz^2$

by:

$$t = v - r,\ x + iy = (r - ia)e^{i\phi}\sin\theta,\ z = r\cos\theta$$

$$r \to Cr,\ v \to Cv,\ a \to Ca$$

Black holes

Rotating solutions

Only non-diagonal terms of $\mathcal{K}^{\mu}{}_{\nu}$ are $\mathcal{K}^{r}{}_{t}$, $\mathcal{K}^{r}{}_{\phi}$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \lambda_g & 0 & 0 & 0 \\ T^{r}{}_{v} & \lambda_g & 0 & T^{r}{}_{\phi} \\ 0 & 0 & \lambda_g & 0 \\ 0 & 0 & 0 & \lambda_g \end{pmatrix}$$

$$T^{r}{}_{v} = -\left(\beta(C-1)^2 - 2\alpha(C-1) + 1\right)\frac{C r_g r}{2\rho^2},$$

$$T^{r}{}_{\phi} = -\left(\beta(C-1)^2 - 2\alpha(C-1) + 1\right)\frac{C a r_g r \sin^2\theta}{2\rho^2}$$

$$\lambda_g = -(C-1)\left(\beta(C-1)^2 - 3\alpha(C-1) + 3\right)$$

Condition

$$\beta(C-1)^2 - 2\alpha(C-1) + 1 = 0$$

PERTURBATIONS

Perturbations

spherically symmetric ansatz for perturbations

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h^{(g)}_{\mu\nu}, \; f_{\mu\nu} = f^{(0)}_{\mu\nu} + h^{(f)}_{\mu\nu}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa}\delta\left(\frac{\sqrt{-g}}{\sqrt{-f}}\mathcal{T}^{\mu}{}_{\nu}\right) .$$

$$h^{\mu\nu}_{(g)} = e^{\Omega v}\begin{pmatrix} h^{vv}_{(g)}(r) & h^{vr}_{(g)}(r) & 0 & 0 \\ h^{vr}_{(g)}(r) & h^{rr}_{(g)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2\sin^2\theta} \end{pmatrix} \qquad (\Omega > 0)$$

$h^{\mu\nu}_{(-)} \equiv h^{\mu\nu}_{(g)} - C^2 h^{\mu\nu}_{(f)}$ i.e. $h^{vv}_{(-)}(r) = h^{vv}_{(g)}(r) - h^{vv}_{(f)}(r)$

Perturbations

spherically symmetric ansatz for perturbations

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h^{(g)}_{\mu\nu}, \; f_{\mu\nu} = f^{(0)}_{\mu\nu} + h^{(f)}_{\mu\nu}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) .$$

$$h^{\mu\nu}_{(f)} = \frac{e^{\Omega v}}{C^2} \begin{pmatrix} h^{vv}_{(f)}(r) & h^{vr}_{(f)}(r) & 0 & 0 \\ h^{vr}_{(f)}(r) & h^{rr}_{(f)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2 \sin^2\theta} \end{pmatrix} \qquad (\Omega > 0)$$

$h^{\mu\nu}_{(-)} \equiv h^{\mu\nu}_{(g)} - C^2 h^{\mu\nu}_{(f)}$ i.e. $h^{vv}_{(-)}(r) = h^{vv}_{(g)}(r) - h^{vv}_{(f)}(r)$

Perturbations

spherically symmetric ansatz for perturbations

The advanced time v is regular at the future horizon, we require $h^{\mu\nu}_{(g,f)}(r)$ to be regular at the horizons

At infinity, instead, it is more suitable to use the Schwarzschild time $t \sim v - r$ to separate between temporal and spatial components.
In the asymptotic region $h^{\mu\nu}_{(g,f)}(r)$ must behave as $o\left(e^{-\Omega r}\right)$ to be physically acceptable.

Non-bidiagonal case

perturbed stress tensor and constraints

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) .$$

$$\beta(C-1)^2 - 2\alpha(C-1) + 1 = 0$$

$$\delta T^{\mu}{}_{\nu} = \frac{\mathcal{A}\left(r_S - r_f\right)}{4r} e^{\Omega v} \begin{pmatrix} 0 & 0 & 0 & 0 \\ h^{\theta\theta}_{(-)} & 0 & 0 & 0 \\ 0 & 0 & \frac{h^{vv}_{(-)}}{2} & 0 \\ 0 & 0 & 0 & \frac{h^{vv}_{(-)}}{2} \end{pmatrix}$$

$$\mathcal{A} \equiv \frac{C^2\left(\beta(C-1)^2 - 1\right)}{C-1}$$

$$\delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) = -\delta T^{\mu}{}_{\nu}$$

Non-bidiagonal case

perturbed stress tensor and constraints

$$\nabla^{\nu}_{(f)}\delta\left(\frac{\sqrt{-g}}{\sqrt{-f}}\mathcal{T}^{\mu}{}_{\nu}\right) \propto \nabla^{\nu}_{(g)}\delta T^{\mu}{}_{\nu} = 0$$

$$\frac{\mathcal{A}\left(r_g - r_f\right)}{4r^2}\, e^{\Omega v}\left\{-\left(r h^{\theta\theta}_{(-)}\right)', h^{vv}_{(-)}, 0, 0\right\} = 0.$$

Solution of the above equation

$$h^{vv}_{(-)} = 0,\ h^{\theta\theta}_{(-)} = \frac{c_0}{r}$$

Non-bidiagonal case

finding the solution for perturbations

$$h^{vv}_{(-)} = 0,\ h^{\theta\theta}_{(-)} = \frac{c_0}{r}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) .$$

Homogeneous solution + particular (inhomogeneous) solution

$$h^{\mu\nu}_{(g,f)} = h^{\mu\nu(g,f)}_{GR} + h^{\mu\nu(g,f)}_{(m)}$$ general solution

$$h^{\mu\nu}_{GR} = -\nabla^{\mu}\xi^{\nu} - \nabla^{\nu}\xi^{\mu}$$ homogeneous solution

Non-Bi-diagonal case

solution for perturbations

$$h_{GR}^{\mu\nu(f)} = 0$$

$$h_{GR}^{\mu\nu(g)} = e^{\Omega v} \begin{pmatrix} 0 & \Omega c_1 & 0 & 0 \\ \Omega c_1 & c_0\left(\Omega - \frac{r_g}{2r^2}\right) & 0 & 0 \\ 0 & 0 & c_0 r^{-3} & 0 \\ 0 & 0 & 0 & c_0 \csc^2(\theta) r^{-3} \end{pmatrix}$$

$$h_{(m)}^{rr(g)} = \frac{\mathcal{A}(r_g - r_f)e^{\Omega v}}{4\Omega} m^2 h_{(-)}^{\theta\theta},$$

$$h_{(m)}^{rr(f)} = -\kappa^{-1} h_{(m)}^{rr(g)}.$$

Since at $r \to \infty$ $v = t + r$ the perturbations are not regular at infinity.

NO unstable modes

Non-bidiagonal solution is stable against radial perturbations

Bidiagonal case

linearized equations for perturbations

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(g)}_{\alpha\beta} + \frac{m^2}{2}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0,$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(f)}_{\alpha\beta} - \frac{m^2}{2\kappa}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0$$

$$h^{(-)\mu}{}_{\nu} = h^{\mu}_{\nu} - \tilde{h}^{\mu}_{\nu}$$

$$h^{(+)\mu}{}_{\nu} = h^{\mu}_{\nu} + \kappa \tilde{h}^{\mu}_{\nu}$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\left(\nabla_\mu \nabla_\nu h - \nabla_\nu \nabla_\sigma h^\sigma_\mu - \nabla_\mu \nabla_\sigma h^\sigma_\nu + \Box h_{\mu\nu} - g_{\mu\nu}\Box h + g_{\mu\nu}\nabla_\alpha \nabla_\beta h^{\alpha\beta} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h_{\lambda_\sigma}\right)$$

Bidiagonal case

linearized equations for perturbations

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(-)}_{\alpha\beta} + \frac{m'^2}{2}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0,$$
$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(+)}_{\alpha\beta} = 0$$

$$h^{(-)\mu}{}_{\nu} = h^{\mu}_{\nu} - \tilde{h}^{\mu}_{\nu}$$
$$h^{(+)\mu}{}_{\nu} = h^{\mu}_{\nu} + \kappa \tilde{h}^{\mu}_{\nu}$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\left(\nabla_\mu \nabla_\nu h - \nabla_\nu \nabla_\sigma h^\sigma_\mu - \nabla_\mu \nabla_\sigma h^\sigma_\nu \right.$$
$$\left. + \Box h_{\mu\nu} - g_{\mu\nu}\Box h + g_{\mu\nu}\nabla_\alpha \nabla_\beta h^{\alpha\beta} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h_{\lambda_\sigma}\right)$$

$$m' \equiv m\sqrt{1 + 1/\kappa}$$

$h^{(-)}_{\mu\nu}$ is massive

$h^{(+)}_{\mu\nu}$ is massless

Perturbations

massive modes of bi-diagonal solution

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(-)}_{\alpha\beta} + \frac{m'^2}{2}\left(h^{(-)}_{\mu\nu} - g_{\mu\nu} h^{(-)}\right) = 0$$

$$\nabla^{\nu} h^{(-)}_{\mu\nu} = h^{(-)} = 0$$

$$\Box h^{(-)}_{\mu\nu} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h^{(-)}_{\lambda_\sigma} = m'^2 h^{(-)}_{\mu\nu}$$

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BLACK STRINGS AND p-BRANES ARE UNSTABLE

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ABSTRACT

We investigate the evolution of small perturbations around black strings and branes which are low energy solutions of string theory. For simplicity we focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra $10 - D$ dimensions.

Bi-diagonal case

Gregory-Laflamme instability

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BLACK STRINGS AND p-BRANES ARE UNSTABLE

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$\delta g_{ab} = h_{ab}$, the Lichnerowicz equation, is essentially a wave equation

$$\Delta_{\rm L} h_{ab} = \left(\delta_a^c \delta_b^d \Box + 2R_{a\ b}^{\ c\ d}\right) h_{cd} = 0. \qquad (1.1)$$

Because of the symmetries of the background $\mathrm{Sch}_4 \times \mathbb{R}$ metric, this reduces to a four-dimensional Lichnerowicz operator plus a ∂_z^2 piece. Performing a Fourier decomposition of h_{ab} in the fifth dimension yields

$$\Delta_{\rm L} h_{ab} = \left(\Delta_4 - m^2\right) h_{ab} = 0. \qquad (1.2)$$

focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra $10 - D$ dimensions.

Gregory-Laflamme instability

The solutions to this equations has been studied already.
In the context of Gregory-Laflamme instability.

Five-dimensional black string:

$$ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 + dz^2$$

Fourier decomposition around the infinite 5th flat dimension
$h^{(4)}_{\mu\nu}$ satisfy the same massive spin two equations with $m'^2 = k^2$

Modes regular at the future horizon, not growing at infinity

Bi-diagonal case

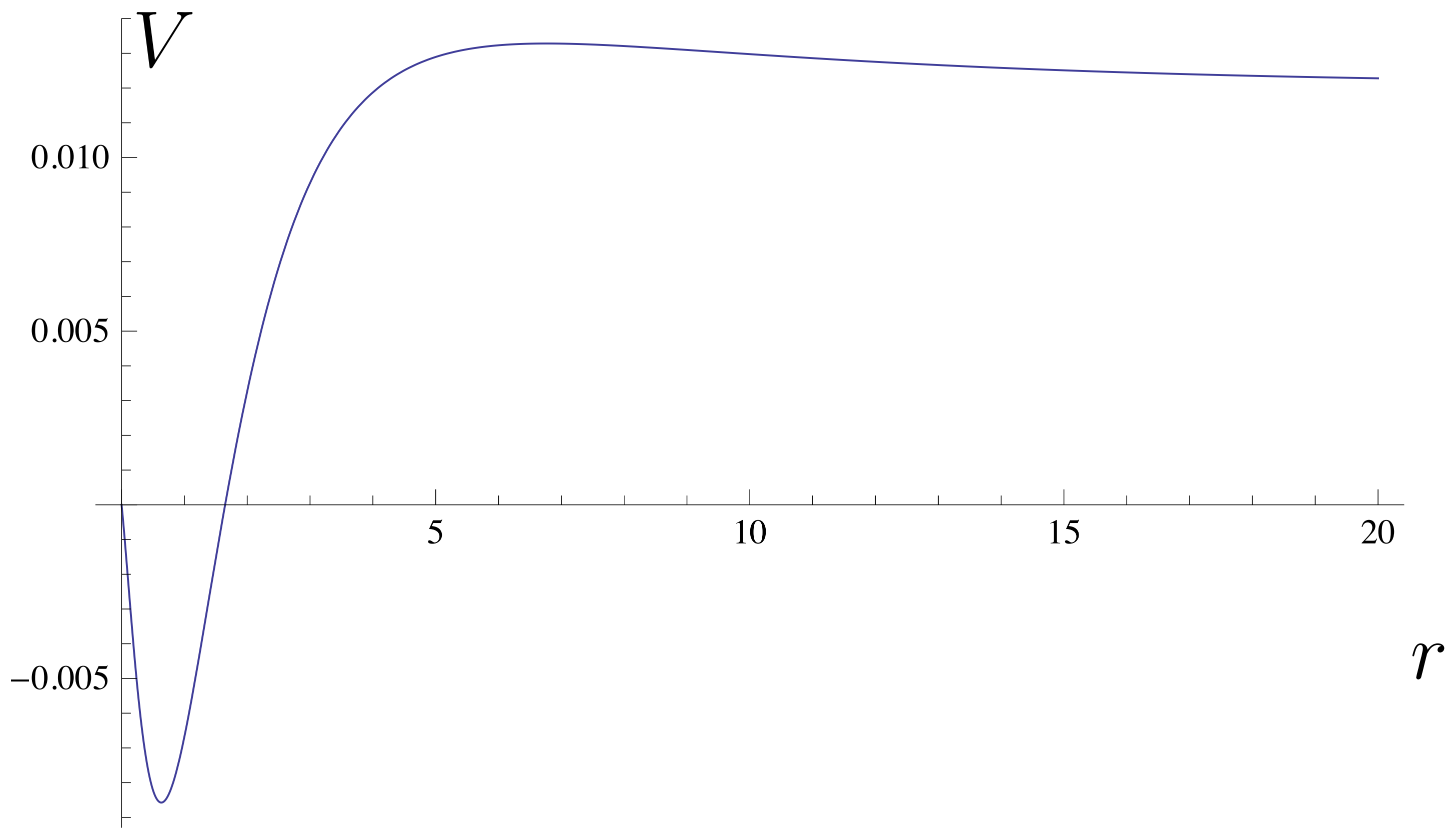
GL instability

A system of equations of second order plus 2 constraints on $H_{tt},\ H_{tr},\ H_{rr},\ K$

Playing with equations we can obtain a single equation on φ_0 (a combination of H_{tt}, H_{rr} and H_{tr})

$$\frac{d^2}{dr_*^2}\varphi_0 + \left[\omega^2 - V(r)\right]\varphi_0 = 0$$

Unstable ($\Omega > 0$) mode, satisfying boundary conditions?



$$V_0 = \left(1 - \frac{r_g}{r}\right)\left[\frac{2M}{r^3} + m'^2 + \frac{24M(M-r)m'^2 + 6r^3(r-4M)m'^4}{(2M + r^3 m'^2)^2}\right]$$

Bi-diagonal case: Instability

$$0 < m' < \frac{\mathcal{O}(1)}{r_S}$$

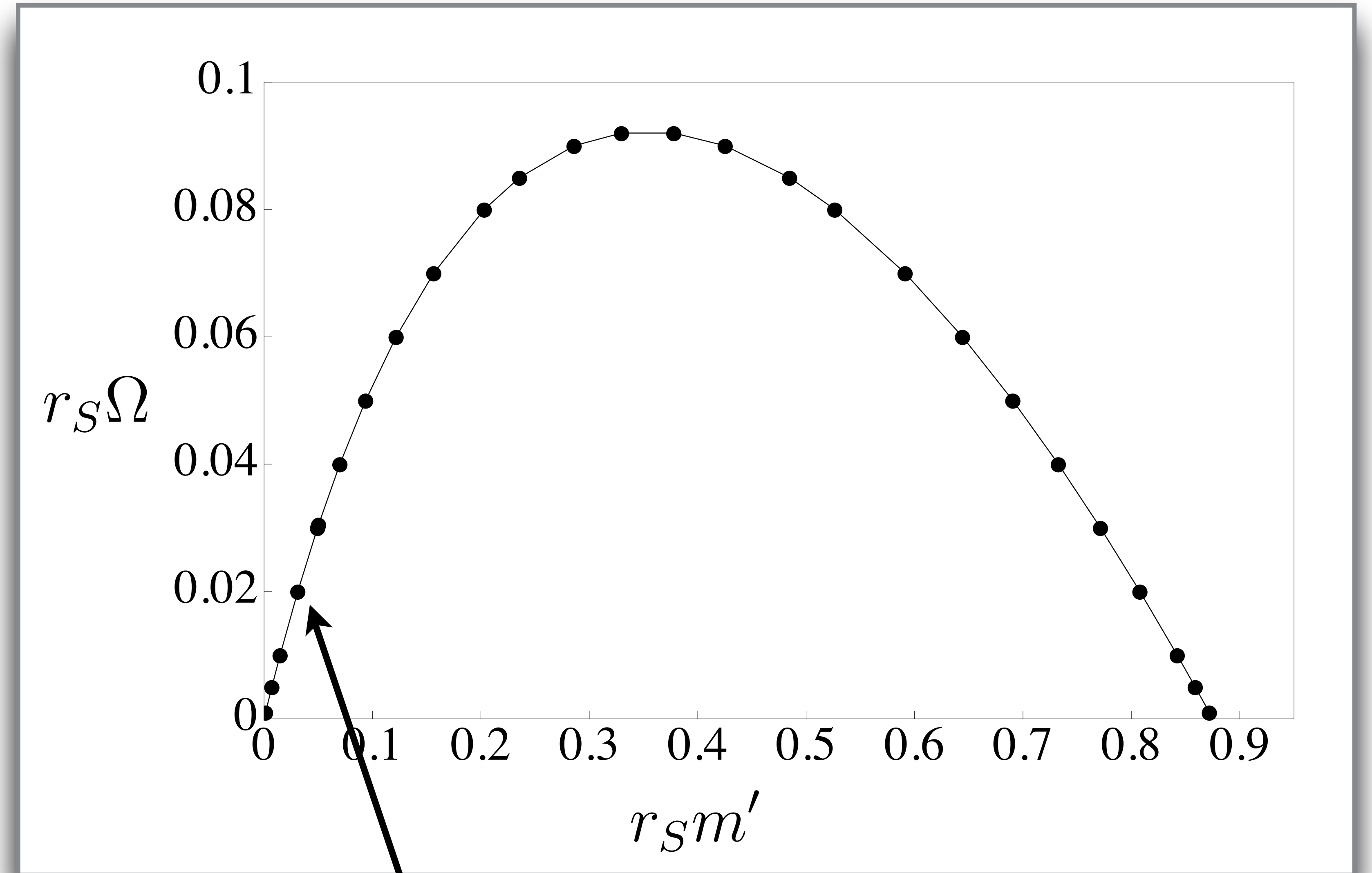
Instability

Confirmed independently by Brito, Cardoso, Pani arXiv:1304.6725

Instability of black holes

rate of instability

Rate of instability



for $m' \sim H \rightarrow \boxed{\tau \sim H^{-1}}$

Very slow instability !

Approximately linear dependance

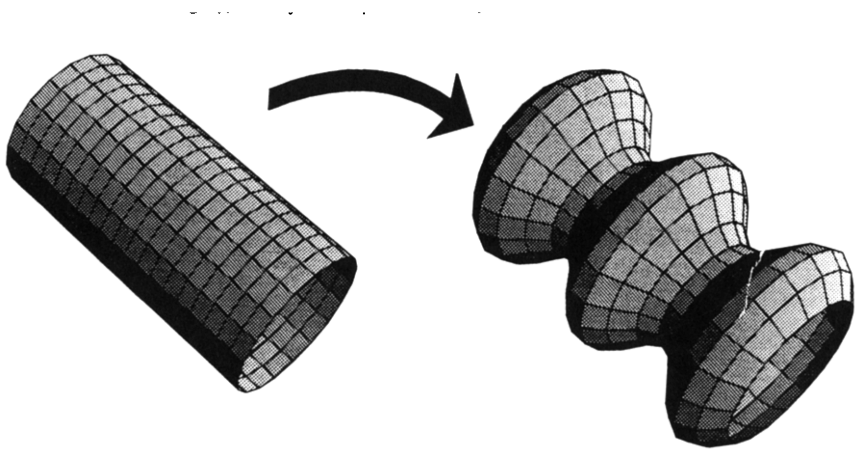
$$r_S \ll 1/m'$$

$$\Omega = m'$$

Instability of black holes

rate of instability

What is the fate of such black holes?



5D Gregory-Laflamme instability

Black holes with massive hair *(Brito,Cardoso,Pani'13) ?*
Only very massive hairy black holes

CONCLUSIONS

- It is possible to construct non-bidiagonal solutions in massive gravity, which are analogues of corresponding GR solutions (Schwarzschild, charged, rotating)
- The non-bidiagonal black holes in massive gravity are stable against radial perturbations
- The bi-diagonal black holes are unstable
- The rate of instability is extremely small
- The fate of black holes? The endpoint of gravitational collapse?
- Do perturbations around black holes contain ghosts?