Universal Relations in Neutron Stars

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Neutron stars

Neutron stars (NS) are very instersting objects.

They can be considered as the "ground state" of matter:
they are the most compact self-graviting stellar objects
in which the gravitational interaction is balanced by another force
(mainly Fermi pressure of neutrons).
Beyond this configuration the star becomes a BH.

Neutron stars are extreme objects, under many respects:

mass: ~1.2 - 2.0 solar masses

radius: ~10 - 15 km (n.b.: solar radius ~7 10^5 km)

density: up to ~10^{15} g/cm^3 (n.b.: nuclear density ~ 2 10^{14} g/cm^3)

rotation rate: up to ~1000 Hz

magnetic field: up to ~10^{16} G

gravitational field: $\frac{GM}{Rc^2} \sim 0.1 - 0.2$ => promising GW sources!

Neutron stars

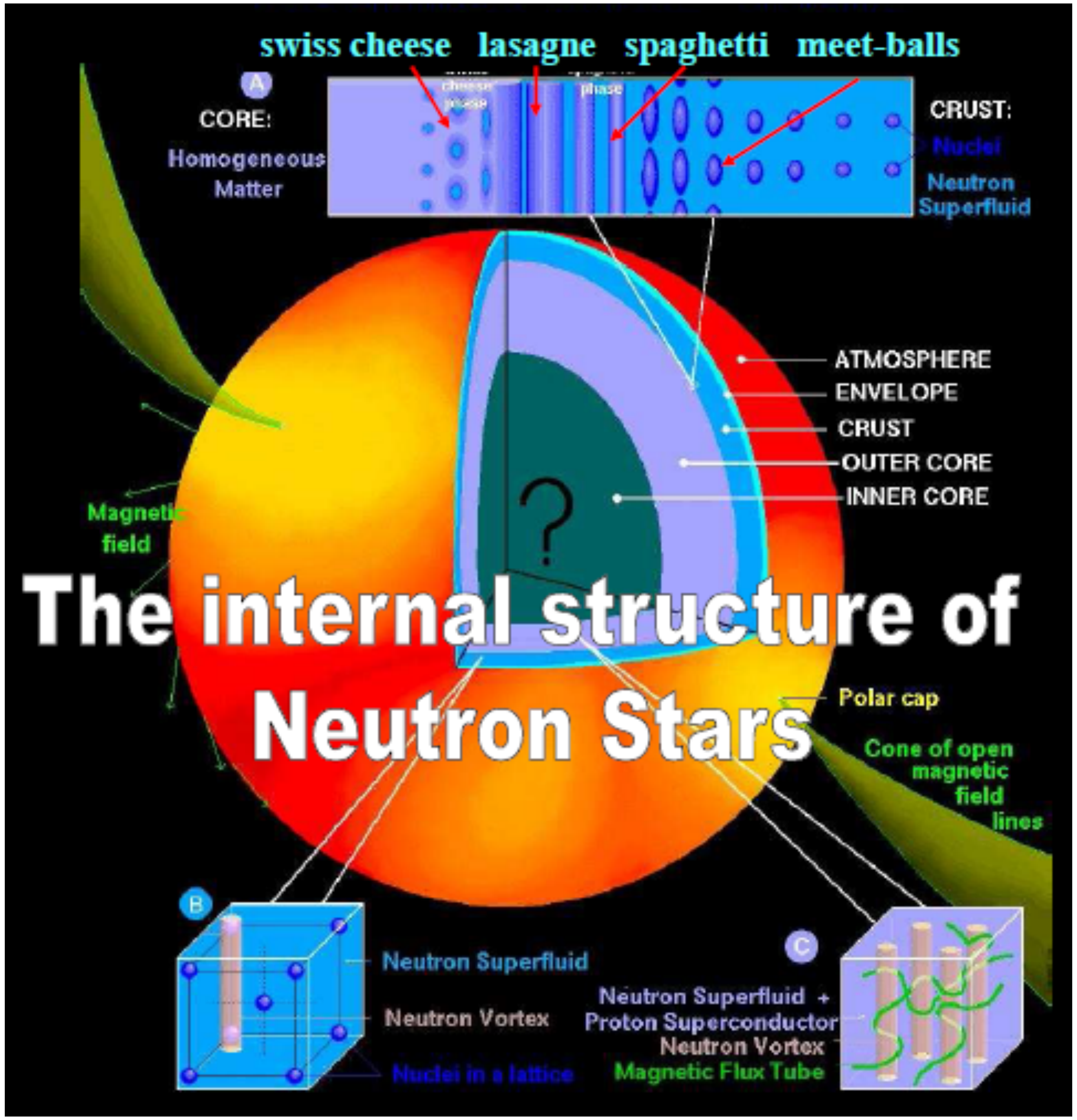
Crust : general agreement on its composition (outer: e, ion lattice; inner: e,n,nuclei)

Outer core : general agreement in its composition (n,p,e,μ)

Inner core: we do not know!

- Supranuclear densities of matter in inner core can not be reproduced in the lab
- Hadron interactions play a crucial role

Credits: D. Page



Our lack of knowledge on the Equation of State (EoS) of inner core reflects our ignorance on the non-perturbative regime of QCD.

We do not even know the particle content:

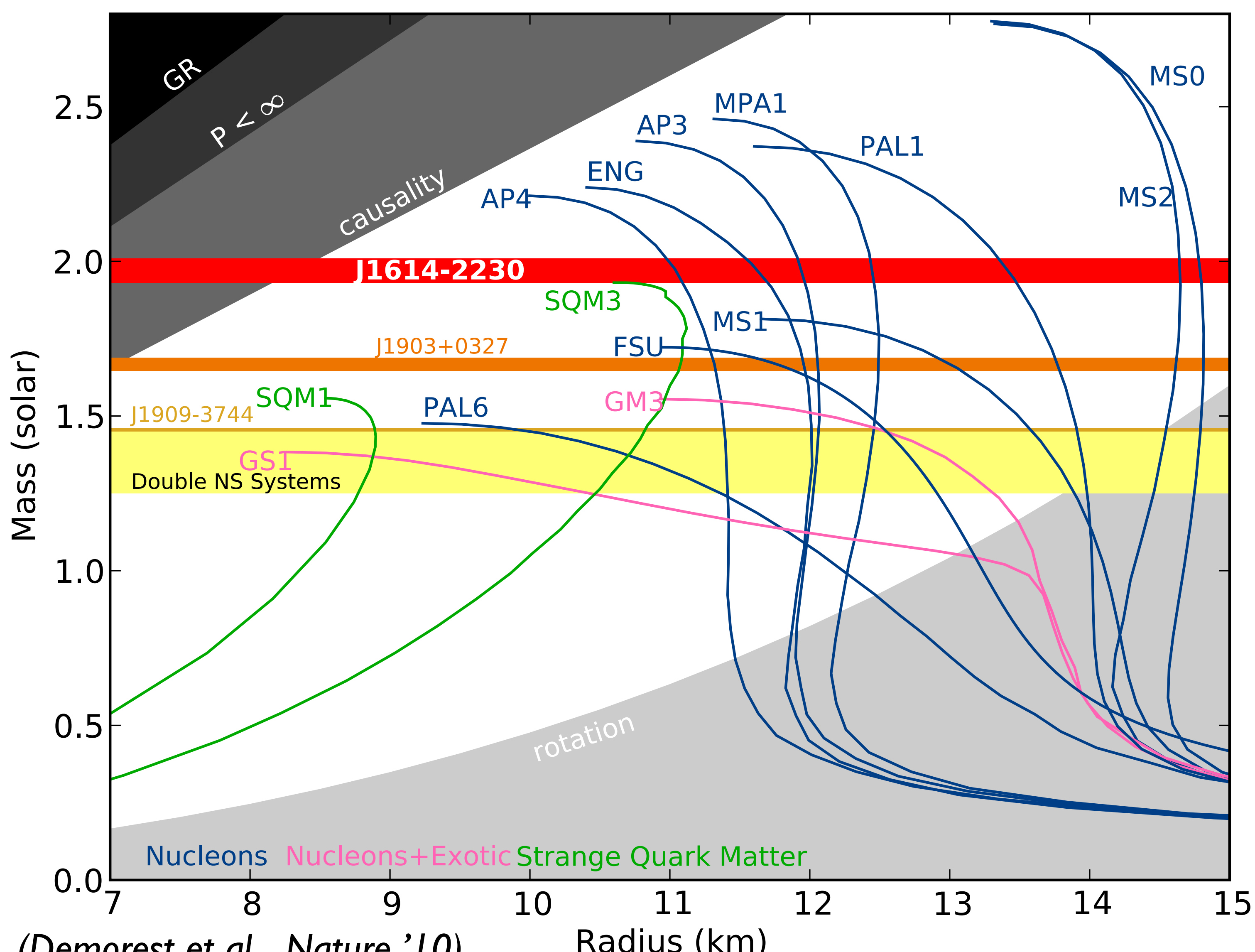
Hadrons? Hyperons? Meson condensates? Deconfined quark matter?

(see e.g. *J.M. Lattimer & M. Prakash, Phys. Rept. '07*)

Neutron stars

Nuclear physicists have proposed several EoSs describing matter in the NS core, with different assumptions (particle content, nuclear many body vs. mean field) and different computational techniques.

In the next years, gravitational wave observations (Advanced LIGO/Virgo, ET) and also astrophysical observations (NICER, LOFT) are expected to constrain the EoS



(Demorest et al., Nature '10)

The physical quantities characterizing a NS

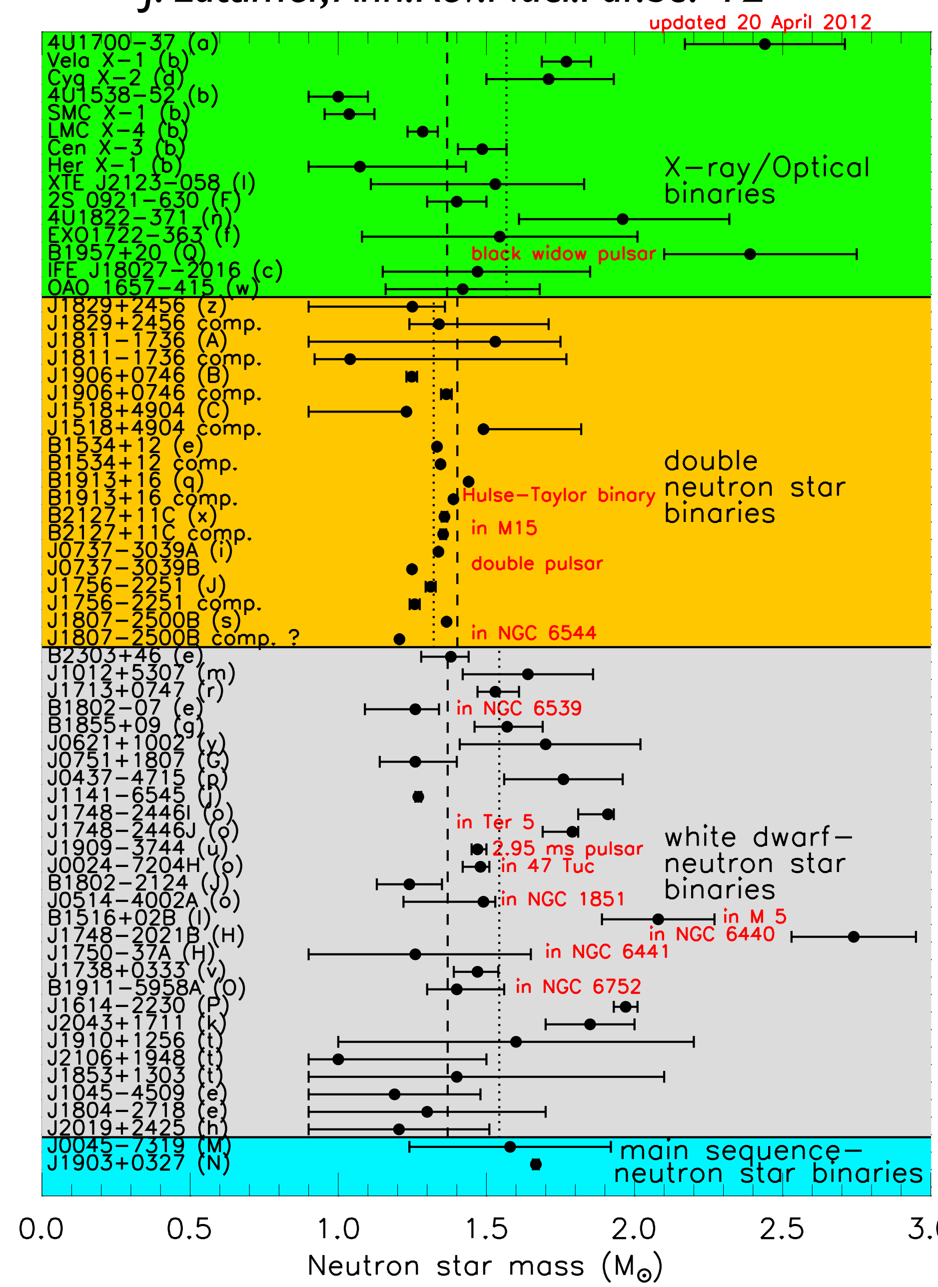
1) Rotation rate $10^{-2}\,Hz \lesssim \nu \leq 716\,Hz$

In the case of pulsars, the easiest and most accurate quantity to measure: we discovered NS by detecting a very regular pulse, associated to their rotation.

2) Mass $1.2 M_\odot \lesssim M \lesssim 2 M_\odot$

Measured with good accuracy in binaries (pulsar timing, Shapiro delay).
It can be uncertain in isolated NS (X-ray emission).

J. Lattimer, Ann.Rev.Nucl.Par.Sc. '12



The physical quantities characterizing a NS

3) Radius $$10\,Km \lesssim R \lesssim 15\,Km$$

Present estimates of NS radius based on observations of the EM emission.

Since the EM signal is affected by complex physics of the NS crust and atmosphere, the measurements of the NS radius are still model-dependent and then not fully reliable.

Actually, a reliable measurement of the NS radius would be very important, since it would stongly constrain the EoS (see e.g. *J. Lattimer & M. Prakash, Phys. Rept. '07*)

Hopes for the next years:

- GW detection could allow us to measure R: for instance, by measuring the characteristic frequencies in the GW emission from a NS-NS coalescence (*R. Oechslin & H.-T. Janka, PRL '07; N. Stergioulas et al., MNRAS '11; A. Bauswein et al., PRL '12, PRD '14; K. Hotokezaka et al., PRD '13; K. Takami et al., PRL '14*)
- future space-based X-ray observer (NICER, LOFT) could be sensitive enough to allow a more accurate modeling of the EM signal => reliable measure of R

The physical quantities characterizing a NS

4) **Momentum of inertia** $10^{38}\,kg\,m^2 \lesssim I \lesssim 3\cdot 10^{38}\,kg\,m^2$

Presently, no measure of the NS momentum of inertia, only estimates based on our theoretical understanding

Hopes for the next years:

Measure I from the frame dragging effects in double pulsars

J.M. Lattimer & B.F. Schutz, ApJ, '05: *"We estimate that the moment of inertia of star A in the recently discovered double pulsar system PSR J0737-3039 may be determined after a few years of observation to something like 10% accuracy."*

This was too optimistic.
But the next generation of radio telescopes (FAST, SKA) could allow to measure the frame dragging in double pulsars - and then the momentum of inertia of NS - in the next few years

(see e.g. *M. Kramer & N. Wex, CQG, '09*)

The physical quantities characterizing a NS

5) **Tidal deformability (Love number)** $Q_{ij} = \lambda C_{ij} = \frac{2}{3} k_2 R^5 C_{ij}$

$$10^{36}\, g\, cm^2\, s^2 \lesssim \lambda \lesssim 10^{37}\, g\, cm^2\, s^2$$

Presently, no measure of the NS tidal deformability, only estimates based on our theoretical understanding

Hopes for the next years:

the GW signal emitted by NS-NS coalescing binaries, detected by Advanced LIGO/Virgo, can allow to measure tidal deformability

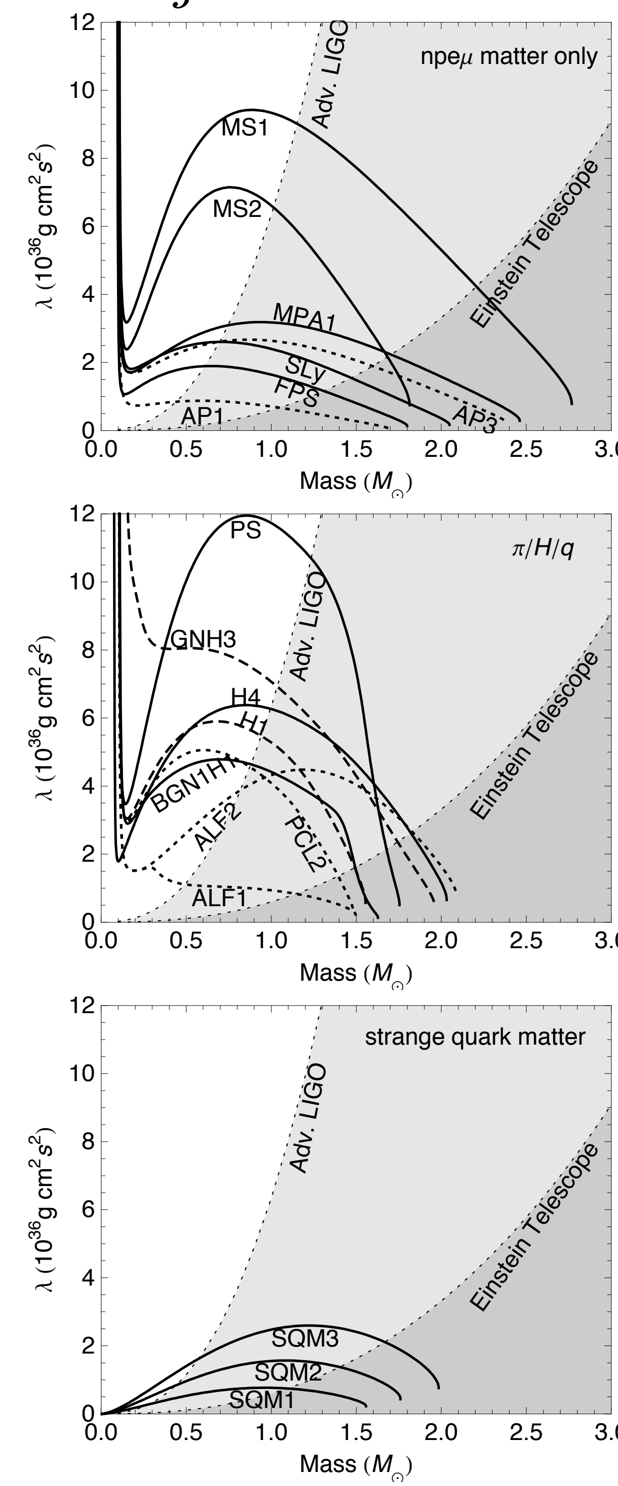
$$h_{PN}(x) = \mathcal{A}(x) e^{i[\Psi_{PP}(x)+\Psi_T(x)]}$$

$$x = (\pi m f)^{2/3} \qquad m + m_1 + m_2 \qquad \nu = m_1 m_2 / m^2 \qquad \mathcal{M} = m\nu^{3/5}$$

$$\mathcal{A}(x) = \sqrt{\frac{5}{24}} \frac{\mathcal{M}^{5/6}}{\pi^{2/3} d} f^{-7/6} \left[1 + \beta_1 x + \beta_2 x^2 + \ldots\right]$$

$$\Psi_{PP}(x) = 2\pi f t_c - \phi_c + \frac{3}{128\nu x^{5/2}} \left[1 + \alpha_2 x + \alpha_3 x^{3/2} + \alpha_4 x^2 + \ldots\right]$$

$$\Psi_T(x) = -\frac{117\tilde{\lambda}}{8\nu m^5} x^{5/2} \left[1 + \tilde{\alpha}_2 x + \tilde{\alpha}_3 x^{3/2} + \ldots\right] \qquad \tilde{\lambda} = \frac{m_1 + 12 m_2}{26 m_2}$$



T. Hinderer et al., PRD '10

6) **Quadrupole moment, higher order tidal moments, etc.**

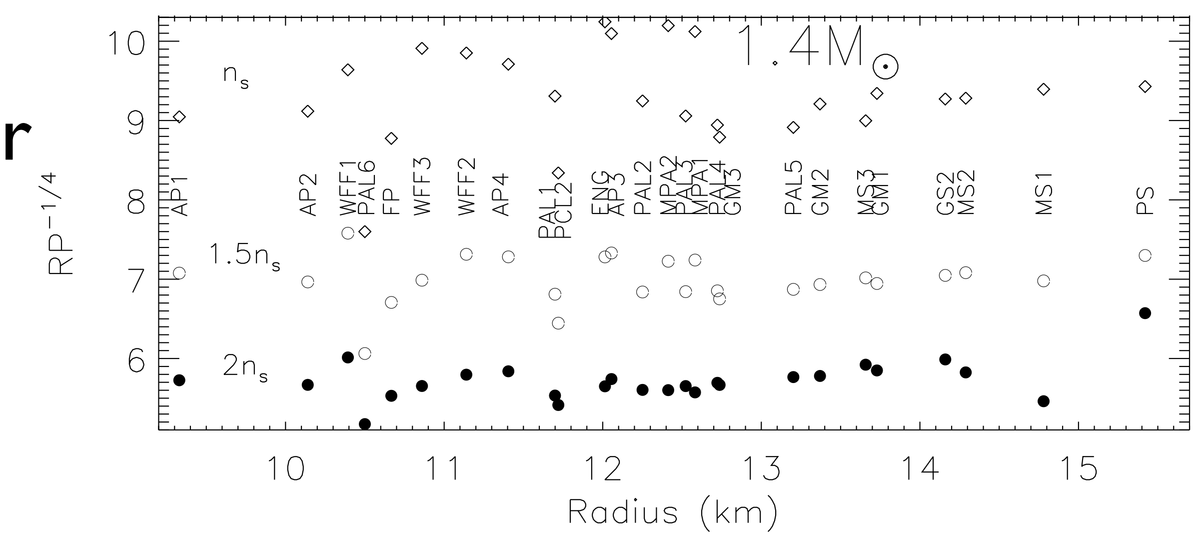
Direct measure does not appear feasible, but enter in models of NS processes

Universal relations in NS

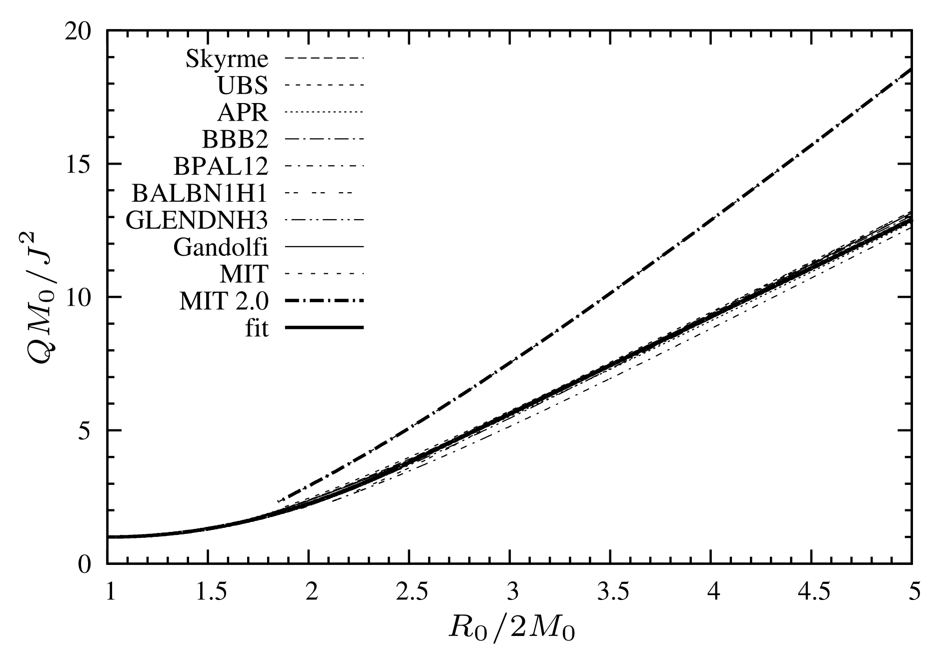
Very recently, a powerful tool to study NS and to extract information on the EoS from present and future observations, has been found: some quantities characterizing NSs satisfy universal relations.
In particular, the so-called I-Love-Q relations.

Preliminary works:

J.M. Lattimer, M. Prakash, ApJ '01:
NS radius depends on the pressure at nuclear saturation density, regardless from the EoS



M. Urbanec et al., Proc. Int. Astron. Un. '13:
Spin induced quadrupole moment, normalized with rotation, depends on the NS compactness C=M/R, regardless from the EoS



In this talk I will only discuss the NS structure, but similar relations involve its oscillations (*gravitational wave asteroseismology*)

Universal relations in NS

I-Love-Q relations

K.Yagi & N.Yunes, Science '13; ibid., PRD '13

I=J/Ω: momentum of inertia
λ: tidal deformability
Q: spin-induced quadrupole moment

$$\bar{I} = \frac{I}{M^3}$$

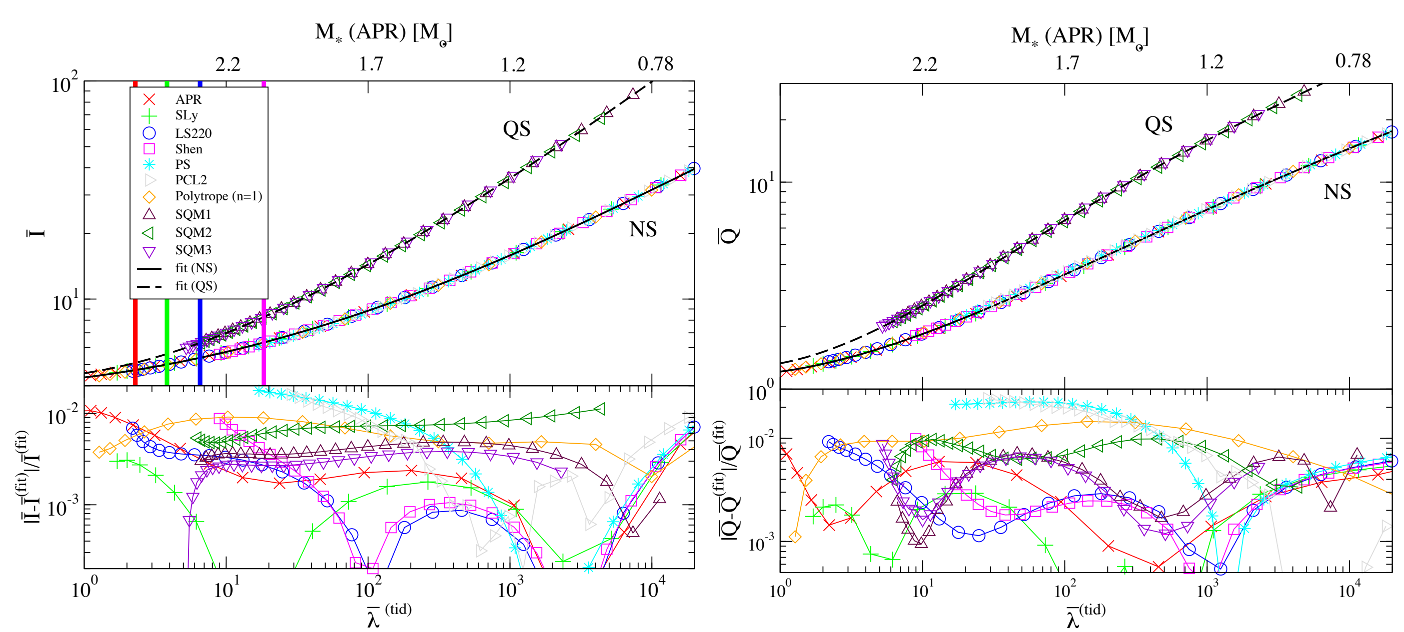
$$\bar{\lambda} = \frac{\lambda}{M^5}$$

$$\bar{Q} = \frac{Q}{MJ^2}$$

$$y_i, x_i = \bar{I}, \bar{\lambda}, \bar{Q}$$

$$\ln y_i = a_i + b_i \ln x_i + c_i (\ln x_i)^2 + d_i (\ln x_i)^3 + e_i (\ln x_i)^4$$

Do not depend on the EoS
do not depend on M



Universal relations in NS

I-Love-Q relations

Can be useful in many respects

- Observational astrophysics:
  The observation of one of these three quantities would give us the other two.
  The knowledge of all of them allow to reconstruct the spacetime near the NS, and, for instance, model its X-ray emission.

- Gravitational wave astronomy:
  Break the degeneracy between quadrupole moment and rotation rate in the gravitational waveform of NS-NS inspiral

- Fundamental physics:
  Allow for tests of General Relativity with NS, which are EoS-independent.
  A violation of the relations (checked through independent measurements of I, λ, Q) could be a clue of new physics (deviations from GR)

Universal relations in NS

This idea attracted a lot of interest:

- K. Yagi & N. Yunes, Science 341, 365 (2013)
- K. Yagi & N. Yunes, Phys. Rev. D88, 023009 (2013)
- M. Urbanec, J.C. Miller, Z. Stuchlik, Proc. Int. Astron. Un. 291, 536 (2013)
- A. Maselli, V. Cardoso, V. Ferrari, L. G., P. Pani, Phys. Rev. D88, 023007 (2013)
- A. Maselli, L. G., V. Ferrari Phys. Rev. D88, 104040 (2013)
- M. Baubok, E. Berti, D. Psaltis. F. Ozel, Astroph. J. 777, 68 (2013)
- B. Haskell, R. Ciolfi, F. Pannarale, L. Rezzolla, Mon. Not. Roy. Astron. Soc. Lett. 438, L71 (2014)
- G. Pappas, T.A. Apostolatos, Phys. Rev. Lett. 121101, 112 (2014)
- D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Astroph. J. Lett. 781, L6 (2014)
- S. Chakrabarti, T. Delsate, N. Gurlebeck, J. Seinhoff, Phys. Rev. Lett. 112, 201102 (2014)
- L.C. Stein, K. Yagi, N. Yunes, Astroph. J. 788, 15 (2014)
- K. Yagi, Phys. Rev. D89, 043011 (2014)
- K. Yagi, K. Kyutoku, G. Pappas, N. Yunes, T.A. Apostolatos, Phys. Rev. D89, 124013 (2014)
- D.D. Doneva, S.S. Yazadjiev, K.V. Staykov, K.D. Kokkotas, arXiv:1408.1641
- P. Pani, E. Berti, Phys. Rev. D90, 024025 (2014)
- G. Martinon, A. Maselli, L. G., V. Ferrari, Phys. Rev. D90, 064026 (2014)
- K. Yagi, L.C. Stein, G. Pappas, N. Yunes, T.A. Apostolatos, arXiv:1406.7587
- ...

Universal relations in NS

Extensions of the I-Love-Q relations

- Including the compactness (and then the radius) in the game

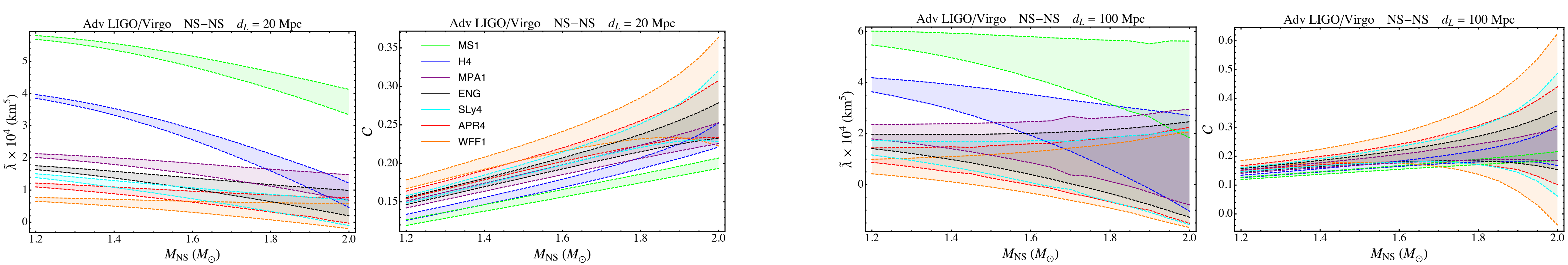
Universal relations exist between the NS compactness C=M/R and the tidal deformability λ with an error ~2%:

$$C = 3.71\times 10^{-1} - 3.91\times 10^{-2}\ln\bar{\lambda} + 1.056\times 10^{-3}(\ln\bar{\lambda})^2\,.$$

(A. Maselli, V. Cardoso, V. Ferrari, L.G., P. Pani, PRD 88, 023007 '13)

They can allow to translate a measure of λ in a measure of R.

Is this helpful in extracting information on the EoS? Not really...



(A. Maselli, L.G., V. Ferrari, PRD 88, 104040 '13)

Once we extract λ from the gravitational waveform, we do not need to find R: λ is more effective than C to discriminate between different EoS.

Universal relations in NS

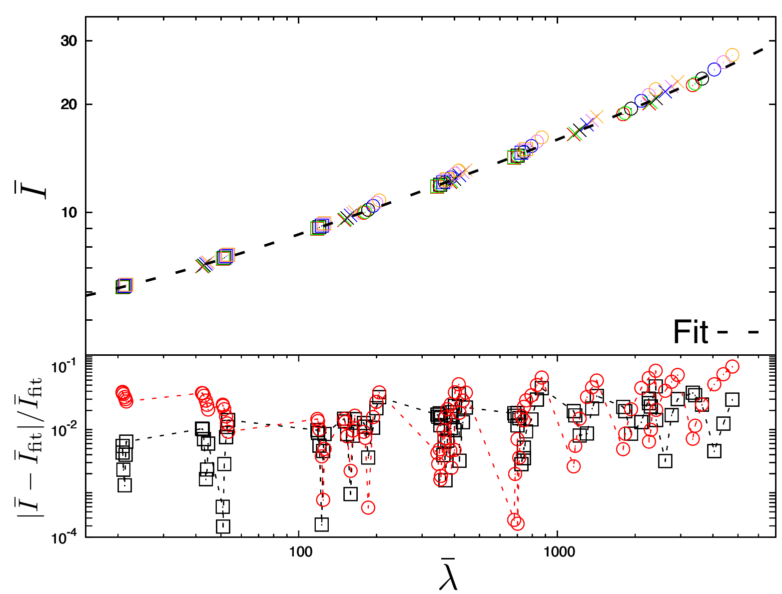
Extensions of the I-Love-Q relations

- Extending to the late-inspiral phase

In the late-inspiral stage, the equation $Q_{ij}=-\lambda C_{ij}$ with constant λ is violated:

the Love number becomes a *Love function* $\lambda(r)$ *(A. Maselli, L.G., F. Pannarale, V. Ferrari, PRD '12)*

Hovever, the violation of the I-λ relation due to this is mild and can be corrected:



(A. Maselli, V. Cardoso, V. Ferrari, L.G., P. Pani, PRD 88, 023007 '13)

In addition, it is not clear how to measure I in this stage.

Universal relations in NS

Extensions of the I-Love-Q relations

- Including the magnetic field

A strong magnetic field B affects the quadrupole moment Q.
The standard I-Q relation considers the *rotation induced* Q;
if the contribution of B to Q is not negligible, i.e.,
strong B / small Ω
the universal relation is violated.

At lowest order,

$$\bar{Q} \approx 4.9\,\bar{I}^{1/2} + 10^{-3}\bar{I}\left(\frac{B_p}{10^{12}\,\mathrm{G}}\right)^2\left(\frac{P}{1\,\mathrm{s}}\right)^2 \quad \text{(poloidal field)}$$

$$\bar{Q} \approx 4.9\,\bar{I}^{1/2} - 3\times 10^{-5}\bar{I}\left(\frac{\langle B\rangle}{10^{12}\,\mathrm{G}}\right)^2\left(\frac{P}{1\,\mathrm{s}}\right)^2 \quad \text{(toroidal field)}$$

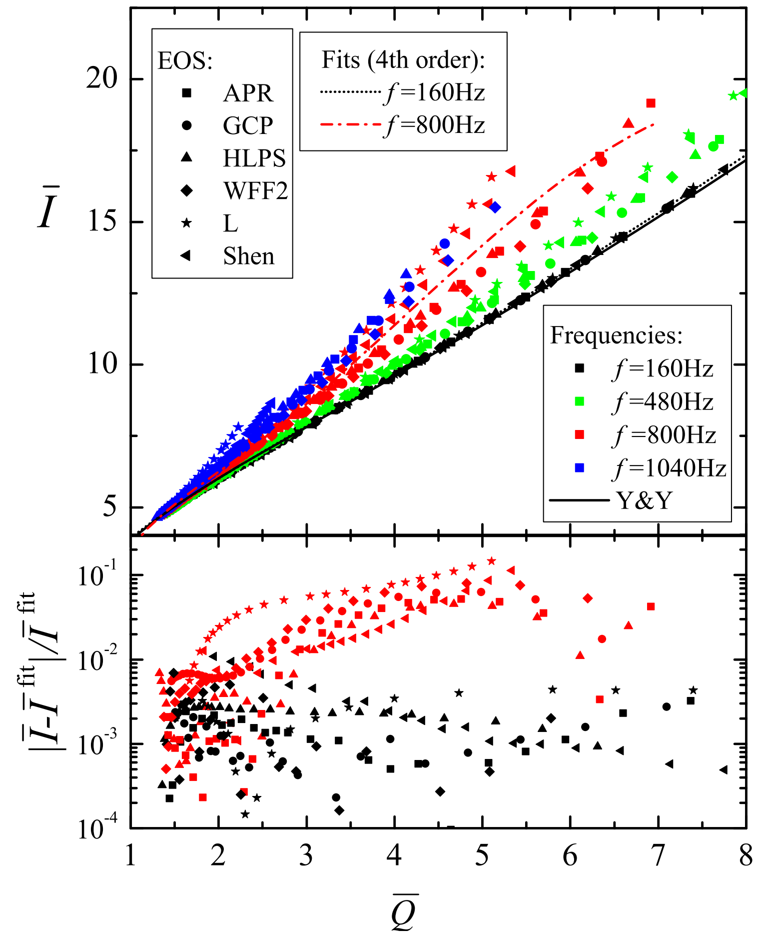
B. Haskell, R. Ciolfi, F. Pannarale, L. Rezzolla, Mon. Not. Roy. Astron. Soc. Lett. 438, L71 (2014)

For instance, for P>10s and B>10^{12} G, for particular field configurations, universality can be lost.

Universal relations in NS

Extensions of the I-Love-Q relations

- Including rotation



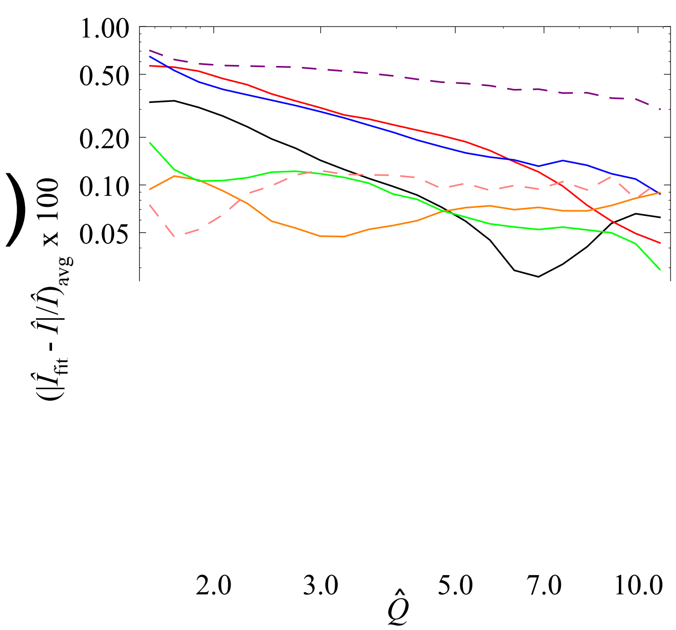
Preliminary results for rapidly rotating NS were showing a *breakdown* of I-Q universality

(D.D. Doneva, S.S.Yazadjiev, N. Stergioulas,K.D. Kokkotas,ApJ 781, L6 '14)

However, it later became clear that this was due to the choice of paramterers

(S. Chakrabarti,T. Delsate, N. Gurlebeck, J.Steinhoff):

Instead of expressing the momentum of inertia in terms of the rotation rate (making a polynomimal fit of $\ln \bar{I} = \bar{I}(\ln \bar{Q}, f)$) they chose to express it in terms of the angular momentum, a=J/M² (making a polynomial fit of $\ln \bar{I} = \bar{I}(\ln \bar{Q}, a)$)



and this was universal!

Universal relations in NS

Extensions of the I-Love-Q relations

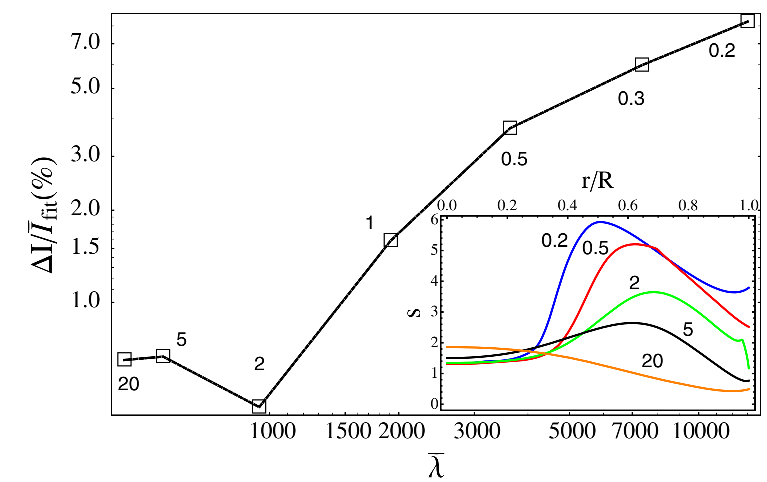
- Hot, newly born NS

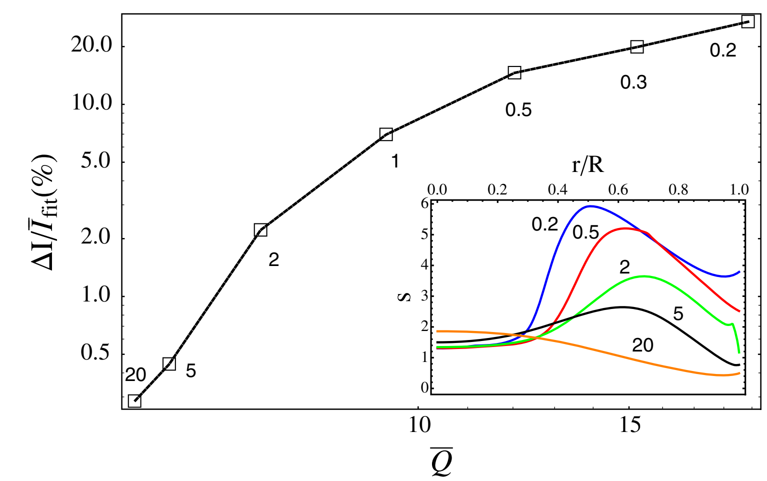
(*G. Martinon, A. Maselli, L. G., V. Ferrari, Phys. Rev. D90, 064026 '14*)

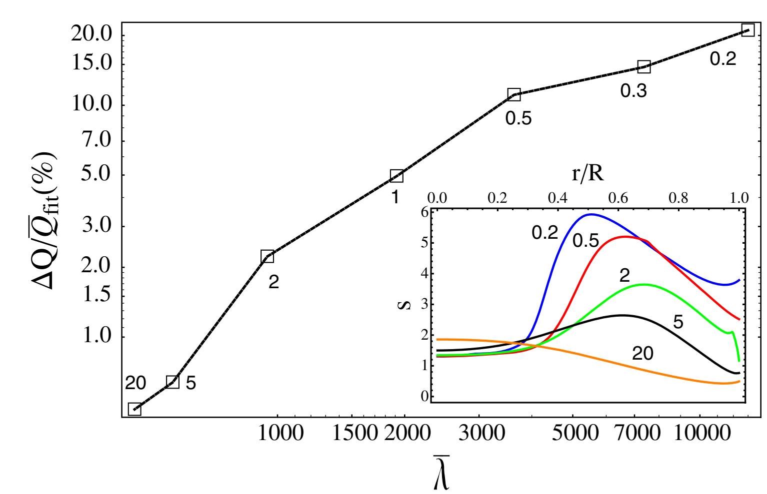
To study hot, newly born proto-neutron stars
we included finite temperature effects.
Not only a "hot" EoS, but a
non-barothropic EoS, with profiles of entropy
and lepton number obtained by numerical evolutions

The entropy gradient spoils the I-λ-Q universality

But this only occurs in the ~first second after bounce







Universal relations in NS

Extensions of the I-Love-Q relations

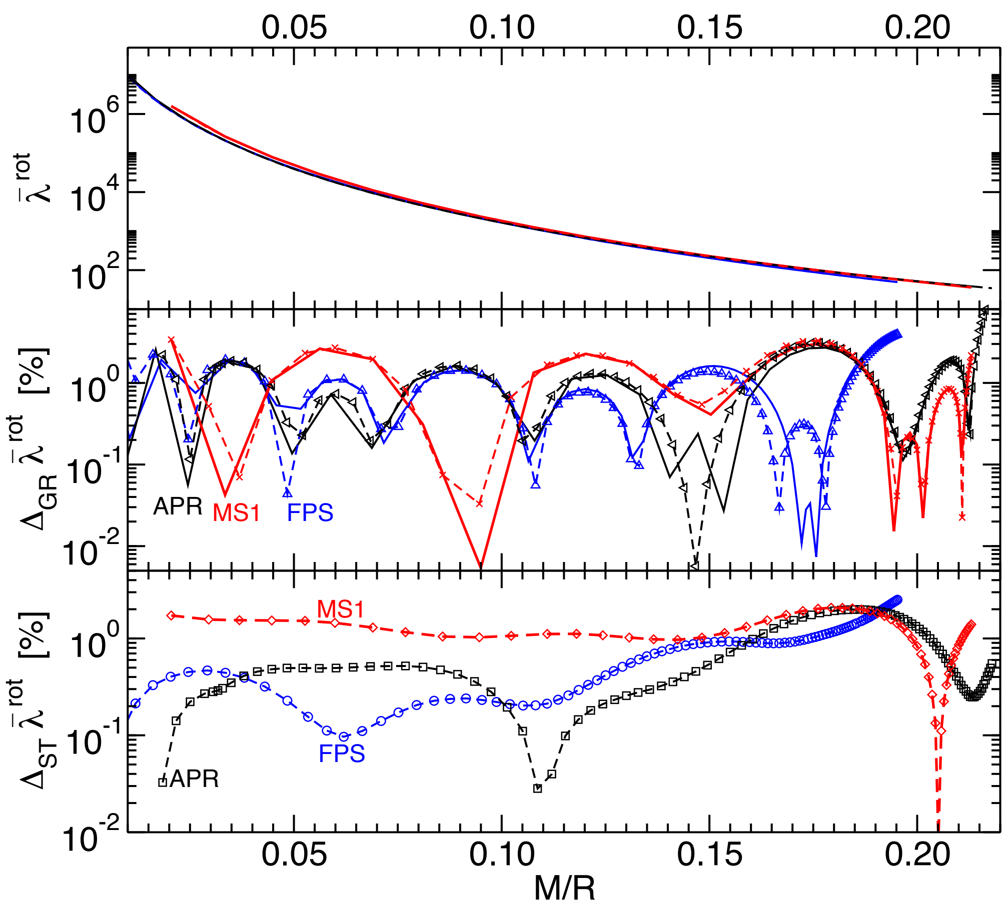
- Alternative theories of gravity

(P. Pani, E. Berti, Phys. Rev. D90, 024025 '14; D.D. Doneva, S.S.Yazadjiev, K.V. Staykov, K.D. Kokkotas, arXiv:1408.1641)

One of the motivation of the I-Love-Q relations:
to perform test of GR which do not depend on the (poorly known) EoS.
But how are the relations violated in specific alternative theories of gravity?

Scalar-tensor gravity:

Even in the most interesting scenario, a NS with spontaneous scalarization (in the narrow window of parameters compatible with pulsar observations), the I-Love-Q relations are satisfied (even though with reduced accuracy).



I-Love-Q useless to discriminate GW from scalar-tensor gravity!

Universal relations in NS

Extensions of the I-Love-Q relations

- Shape quadrupole: the eccentricity of rotating NS

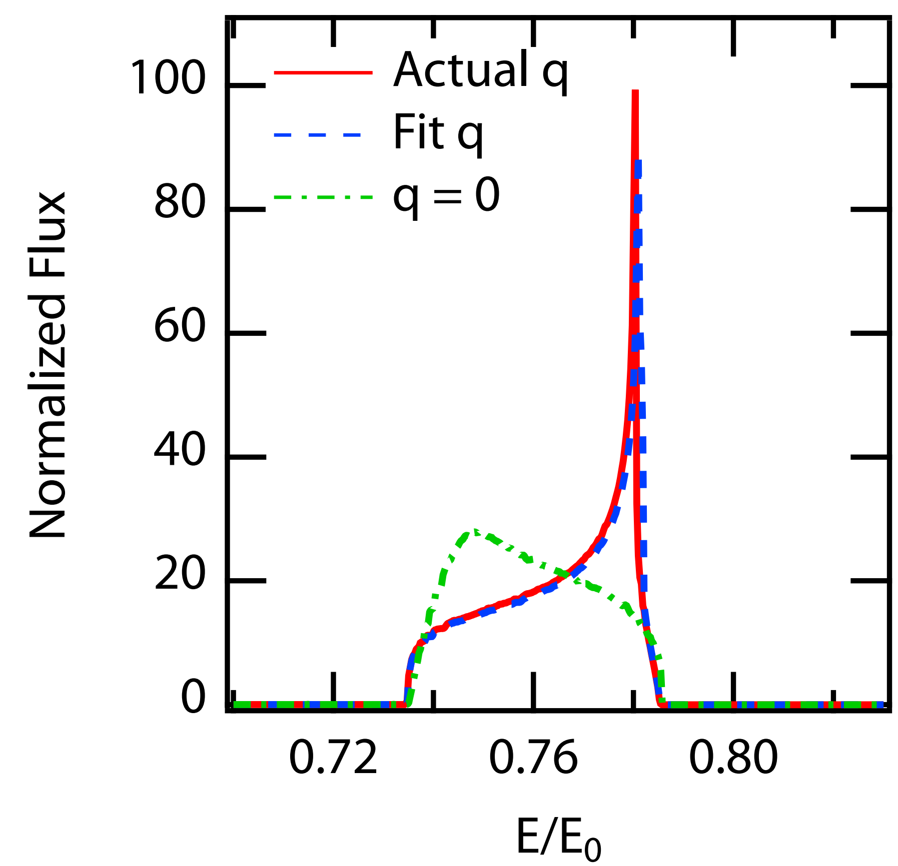
(M. Baubok, E. Berti, D. Psaltis. F. Ozel, Astroph. J. 777, 68 (2013))

The NS eccentricity can be expressed (for slow rotation) as a function
e=e(C,J,Q)

Then, using the universal relation between Q and C, one can express it as
e=e(C,J)

This can be very helpful,
for instance to model the profiles of emission lines.

When future X-ray missions as NICER and LOFT will be taking data, a reliable model of the emission could allow to finally measure the NS radius!
(D. Psaltis, F. Ozel, D. Chakrabarty, Astrophys. J. 787, 136 '14)



Universal relations in NS

Extensions of the I-Love-Q relations

- Higher order moments

G. Pappas, T.A. Apostolatos, Phys. Rev. Lett. 121101, 112 '14; L.C. Stein, K. Yagi, N. Yunes, Astroph. J. 788, 15 '14; K. Yagi, Phys. Rev. D89, 043011; K. Yagi, K. Kyutoku, G. Pappas, N. Yunes, T.A. Apostolatos, Phys. Rev. D89, 124013, '14

Universal relations have been extended to higher order moments, such as the octupole and hexadecapole multipole moments $Q \rightarrow S_3, M_4$ and the higher-order Love numbers $\lambda \rightarrow \lambda_3$

- l=3,4 multipole moments can be important to extract the NS radius from the X-ray signal detected by NICER, LOFT for rapidly rotating NS

- l=3 tidal deformability can be important for parameter estimation in 3rd generation GW detector (ET, LIGO III)

Universal relations in NS

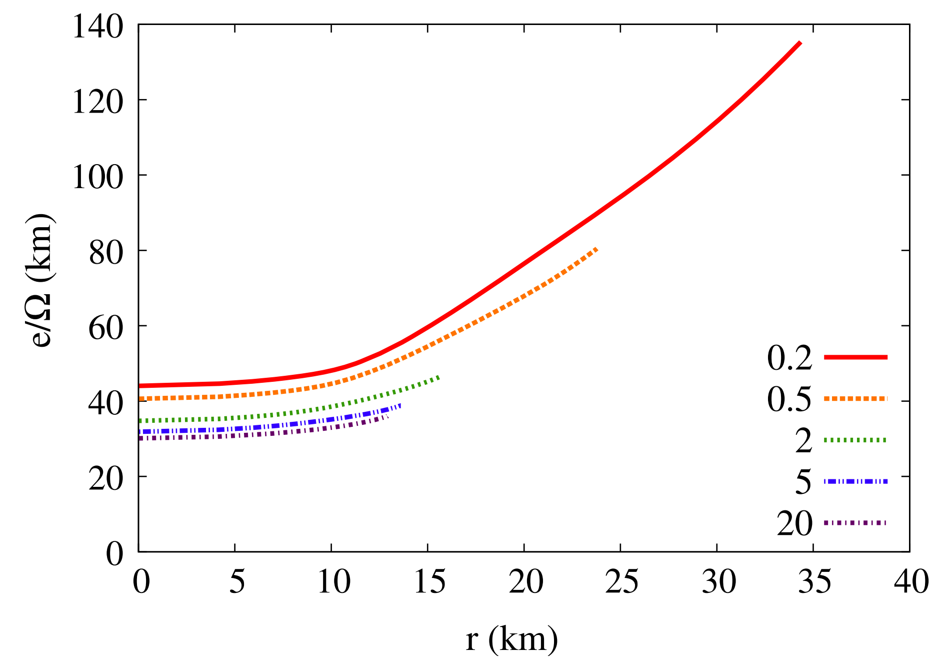
Why do universal relations hold?

(K. Yagi, L.C. Stein, G. Pappas, N. Yunes, T.A. Apostolatos, arXiv:1406.7587)

- I, Q are mainly determined by the structure of the *outer* region of the NS, that is the region we understand better, in which the proposed EoS are similar

- universal relations emerge as the eccentricity of constant density profiles is nearly constant: e(r) ~ const. (self-similarity)
  In cold NS the eccentricity is constant, expecially in the outer region, and this reduces the parameter space. This is an *emergent symmetry*.

For instance, in hot, young PNS
self-similarity is violated in ~ the 1st s after bounce,
exactly when the I-Love-Q relations do not hold



(*G. Martinon, A. Maselli, L. G., V. Ferrari, Phys. Rev. D90, 064026 '14*)

Conclusions

- Universal relations seem to be an emergent simmetry appearing in cold, old NS

- They relate various quantities characterizing the NS:
momentum of inertia, Love number, rotation-induced quadrupole moment (I-λ-Q)
but also compactness, higher order Love number, higher multipole moments,
eccentricity, etc., irrespectively from the EoS (and from the mass)

- Can be useful to break degeneracies in observations both in GW and X-ray,
eventually extracting information on the NS EoS from such observations

- Can also be useful to discriminate GR from alternative theories
(but scalar-tensor theories behave as GR in this respect)

- The original formulation can be extended to include rotating star, with
and appropriate choice of the parameter describing rotations,
and to describe the last stages of the collapse

- Universality can be spoiled by strong magnetic fields and by thermal effects