Holographic superconductors and the effect of back-reaction

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Collaborations and References

Work done in collaboration with:

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References:

Y. Brihaye and B. Hartmann, Phys.Rev. D81 (2010) 126008
Y. Brihaye and B. Hartmann, JHEP 1009 (2010) 002
Y. Brihaye and B. Hartmann, Phys. Rev. D83 (2011) 126008

Outline

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Gauge/gravity duality

- d-dim String Theory $\Leftrightarrow$ (d-1)-dim SU(N) gauge theory
  (Maldacena; Witten; Gubser, Klebanov & Polyakov (1998))
- Couplings

$$\lambda = (\ell/l_s)^4 = g^2 N \quad , \quad g_s \sim g^2$$

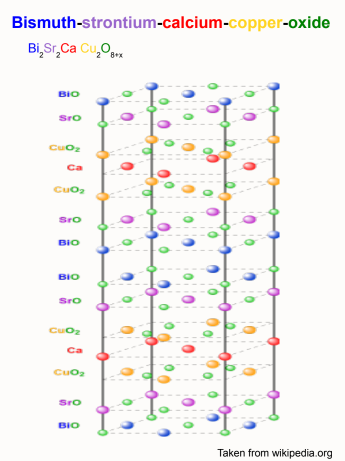
λ: 't Hooft coupling	N: rank of gauge group
ℓ: bulk scale/AdS radius	l_s: string scale
g: gauge coupling	g_s: string coupling

- classical gravity limit:

$$g_s \to 0 \qquad l_s/\ell \to 0$$

dual to **strongly coupled QFT with $N \to \infty$**

Application: high temperature superconductivity



- superconductivity associated to CuO_2-planes
- $T_c \approx 95$ K
- energy gap $\omega_g/T_c = 7.9 \pm 0.5$ (Gomes et al., 2007)

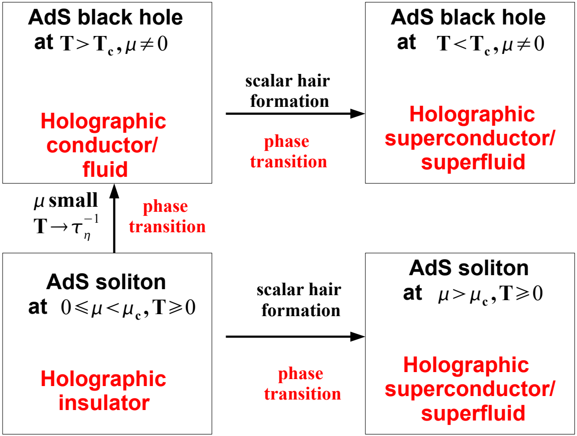
  $\Rightarrow$ compare to BCS value:

  $$\omega_g/T_c = 3.5$$

  $\Rightarrow$ **Strongly coupled** electrons in high-T_c superconductors?

Holographic phase transitions

T: temperature, μ: chemical potential



Probe limit vs. Backreaction

- $G = 0$ ($e = \infty$) "**probe limit**": fixed space-time background
  $\Rightarrow$ coupled scalar & gauge field equations
- $G \neq 0$ ($e < \infty$): **backreaction** of matter fields on space-time
  $\Rightarrow$ coupled gravity, scalar & gauge field equations
- results in systems of coupled, nonlinear ordinary or partial differential equations that have to be solved **numerically**

The model

Gauss-Bonnet gravity in d-dimensional Anti-de Sitter (AdS_d)

$$S = \frac{1}{2\gamma} \int d^d x \sqrt{-g} \left(R - 2\Lambda + \frac{\alpha}{2} \mathcal{L}_{\rm GB} + 2\gamma \mathcal{L}_{\rm m} \right) + S_{\rm ct}$$

Gauss-Bonnet Lagrangian

$$\mathcal{L}_{\rm GB} = \left(R^{MNKL} R_{MNKL} - 4 R^{MN} R_{MN} + R^2 \right)$$

$S_{\rm ct}$: boundary counterterm
$\gamma = 8\pi G$: gravitational coupling
$\Lambda = -(d-1)(d-2)/(2\ell^2)$: cosmological constant
ℓ: AdS radius
α: Gauss–Bonnet coupling

The model

Lagrangian of **charged complex scalar field**:

$$\mathcal{L}_{\rm m} = -\frac{1}{4} F_{MN} F^{MN} - (D_M\psi)^* D^M\psi - m^2\psi^*\psi \ , \quad M, N = 0, 1, 2, 3, d-1$$

U(1) field strength tensor

$$F_{MN} = \partial_M A_N - \partial_N A_M$$

covariant derivative

$$D_M\psi = \partial_M\psi - ieA_M\psi$$

e: electric charge
m: mass of scalar field

Outline

The Ansatz

- For $d \leq (3+1)$ Gauss-Bonnet terms do not contribute
- Metric in $d = (4+1)$

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(dx^2 + dy^2 + dz^2\right)$$

- **Electric field** only

$$A_M dx^M = \phi(r)dt$$

- Gauge freedom: scalar field chosen to be **real**

$$\psi = \psi(r)$$

Equations

$$
\begin{aligned}
f' &= 2r\frac{2r^2/\ell^2 - f}{r^2 - 2\alpha f} \\
&- \gamma\frac{r^3}{2fa^2}\left(\frac{2e^2\phi^2\psi^2 + f(2m^2a^2\psi^2 + \phi'^2) + 2f^2a^2\psi'^2}{(r^2-2\alpha f)}\right) \\
a' &= \gamma\frac{r^3(e^2\phi^2\psi^2 + a^2f^2\psi'^2)}{af^2(r^2-2\alpha f)} \\
\phi'' &= -\left(\frac{3}{r} - \frac{a'}{a}\right)\phi' + 2\frac{e^2\psi^2}{f}\phi \\
\psi'' &= -\left(\frac{3}{r} + \frac{f'}{f} + \frac{a'}{a}\right)\psi' + \underbrace{\left(m^2 - \frac{e^2\phi^2}{fa^2}\right)}_{m^2_{\text{eff}}}\frac{\psi}{f}
\end{aligned}
$$

Conditions at the horizon

- Horizon $r = r_h$

$$f(r_h) = 0 \quad , \quad a(r_h) \text{ finite}$$

- Regularity of matter fields on horizon

$$\phi(r_h) = 0 \ , \ \psi'(r_h) = \left. \frac{m^2 \psi r^2}{4r - \gamma r^3 \left(m^2 \psi^2 + \phi'^2/(2a^2)\right)} \right|_{r=r_h}$$

Conditions on the AdS boundary

- Electric potential

$$\phi(r \gg 1) = \mu - q/r^2$$

μ: chemical potential
q: charge density

- Scalar field

$$\psi(r \gg 1) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}$$

with

$$\lambda_\pm = 2 \pm \sqrt{4 - 3(\ell_{\rm eff}/\ell)^2} \ , \ \ \ell_{\rm eff}^2 = \frac{2\alpha}{1 - \sqrt{1 - 4\alpha/\ell^2}}$$

Parameters

- Equations invariant under rescalings

$$(1): \quad r \to \lambda r \ , \ t \to \lambda t \ , \ \ell \to \lambda \ell \ , \ e \to e/\lambda \ , \ \alpha \to \lambda^2 \alpha$$
$$(2): \quad \phi \to \lambda \phi \ , \ \psi \to \lambda \psi \ , \ \gamma \to \gamma/\lambda^2 \ , \ e \to e/\lambda$$

$\Rightarrow e = 1, \ell = 1$ **without loss of generality**

- choose $m^2 = -\frac{3}{\ell^2} > m^2_{\rm BF} = -\frac{4}{\ell^2}$

  (close to, but **above** Breitenlohner-Freedman (BF) bound)

AdS black hole without scalar hair

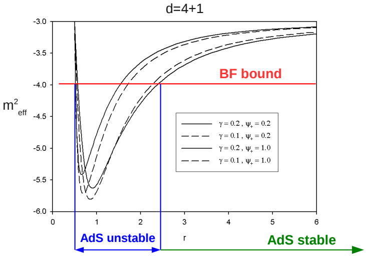
$$\psi(r) \equiv 0 \ , \ \ \phi(r) = \frac{q}{r_h^2} - \frac{q}{r^2} \ , \ \ a(r) \equiv 1$$

$$f(r) = \frac{r^2}{2\alpha}\left(1 - \sqrt{1 - \frac{4\alpha}{\ell^2}\left(1 - \frac{r_h^4}{r^4}\right) - \frac{4\alpha\gamma q^2}{r^6}\left(1 - \frac{r^2}{r_h^2}\right)}\right)$$

(Boulware, Deser, 1985; Cai, 2002)

Formation of scalar hair on AdS black hole

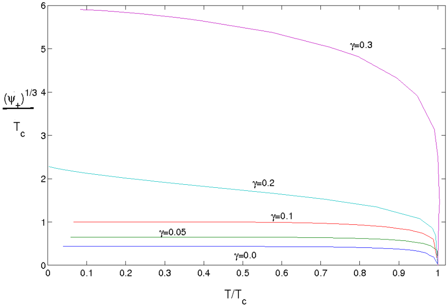
- Pioneering example: electrically charged AdS black hole close to $T = 0$ unstable to form **scalar hair** (Gubser, 2008)
- Example (Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)



Holographic superconductors: backreaction

(Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)

- Value of condensate increases with increasing γ



Holographic superconductors: critical temperature T_c

- for $\alpha = 0$ (Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)

$$T_\mathrm{c} \approx 0.198 \cdot \exp(-10.6\gamma) q^{1/3}$$

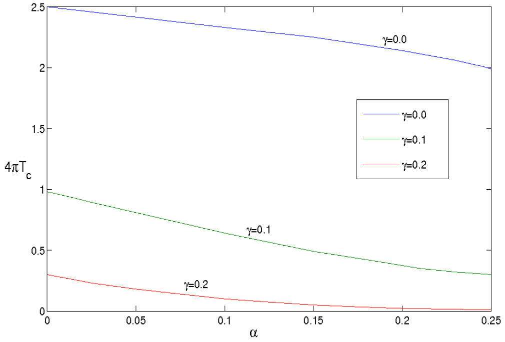
- ω_g changes little with γ (Gregory, Kanno, Soda (2009); Barclay, Gregory, Kanno, Sutcliffe (2010))

$$\Rightarrow \omega_g / T_c \text{ rises exponentially with } \gamma$$

Holographic superconductors: critical temperature T_c

(Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)

- for $\alpha \neq 0$:



Outline

Black strings and solitons

- $d = (3+1) \Rightarrow$ classical gravity, i.e. GR
- compactify one direction
  $\Rightarrow$ boundary theory "lives" on $\mathbb{R}^2 \times S^1$
  $\Rightarrow$ "richer" set of solutions
  - solutions with horizons: **black strings**
  - globally regular solutions: (cigar-shaped) **solitons**
- rotate solution around symmetry axis
  $\Rightarrow$ additional magnetic field

Ansatz: metric

- Metric of a (3+1)-dimensional **black string (BS)**

$$ds^2 = -b(\rho)dt^2 + \frac{1}{f(\rho)}d\rho^2 + \rho^2 \left(g(\rho)dt - d\varphi\right)^2 + p(\rho)dz^2$$

with $f(\rho_h) = b(\rho_h) = 0$ at horizon $\rho = \rho_h$

- Metric of a (3+1)-dimensional **soliton (S)**

$$ds^2 = -p(\rho)dt^2 + \frac{1}{f(\rho)}d\rho^2 + b(\rho)\left(g(\rho)dt - d\eta\right)^2 + \rho^2 dz^2$$

with $f(\rho_0) = b(\rho_0) = 0$ at $\rho = \rho_0$ and η **periodic** with period

$$\tau_\eta = \frac{4\pi}{\sqrt{b'(\rho_0)f'(\rho_0)}}$$

Ansatz: matter fields

- U(1) gauge field for black strings

$$A_M dx^M = \phi(\rho) dt + A(\rho) d\varphi$$

- U(1) gauge field for AdS solitons

$$A_M dx^M = \phi(\rho) dt + A(\rho) d\eta$$

- Gauge freedom: scalar field chosen to be **real**

$$\psi = \psi(\rho)$$

Conditions on the AdS boundary $\rho \gg 1$

- boundary theory "lives" on $\mathbb{R}^2 \times S^1$
- U(1) potential

$$\phi(\rho \gg 1) = \mu - q/\rho \quad , \quad A(\rho \gg 1) = \sigma - \tilde{q}/\rho$$

μ: chemical potential — σ: superfluid velocity
q: electric charge density — $\tilde{q}$: magnetic charge density

- Scalar field for $m^2 = -2/\ell^2 > m^2_{\rm BF} = -9/(4\ell^2)$

$$\psi(\rho \gg 1) = \frac{\psi_-}{\rho} + \frac{\psi_+}{\rho^2}$$

Holographic superfluids: order of phase transition

- Consider small perturbation around solution describing fluid phase (A_0, ϕ_0) in **probe limit**

$$\psi = \varepsilon\psi_0 + O(\varepsilon^2) \quad , \quad A = A_0 + \varepsilon^2 \delta A + O(\varepsilon^4) \quad , \quad \phi = \phi_0 + \varepsilon^2 \delta\phi + O(\varepsilon^4)$$

- Compare **free energy** in Grand Canonical ensemble $\Omega = -TS_{\rm os}$ of the two phases

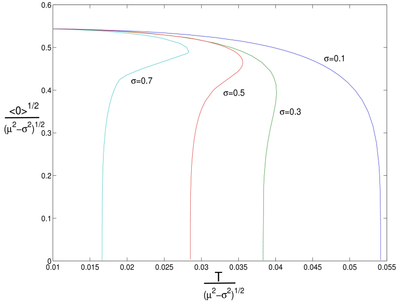
$$\frac{\delta\Omega}{T^3 V} = - \quad \varepsilon^4 \int_{\rho_h}^{\infty} d\rho\, \rho^2 \left(\mathcal{F}_1 (\delta\phi)'^2 - \mathcal{F}_2 (\delta A)'^2 + \mathcal{F}_3 (\delta\phi)'(\delta A)'\right) + \quad \varepsilon^4 \left[\tilde{q}\delta\sigma - q\delta\mu\right] + O(\varepsilon^6) \ .$$

where $\mathcal{F}_i$, $i = 1, 2, 3$ depend on metric functions and are positive on $[\rho_h, \infty[$

Holographic superfluids: order of phase transition

(Y. Brihaye & B. Hartmann, JHEP 1009 (2010) 002)

- Probe limit $\gamma = 0$, fluid/BS superfluid phase transition

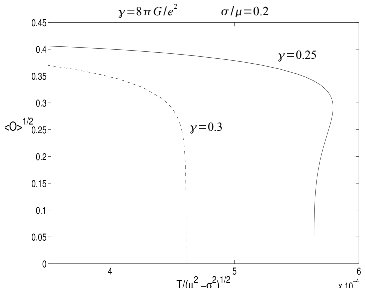


- 2nd order phase transition for σ small
- 1st order phase transition for σ large

Holographic superfluids: order of phase transition

(Y. Brihaye & B. Hartmann, Phys. Rev. D 83 (2011) 126008)

- fluid/BS superfluid phase transition away from probe limit



- 1st order for σ large and γ small
- 2nd order for $\sigma \geq 0$ and γ large

Holographic superfluids: order of phase transition

- Order of PT correct? $\Rightarrow$ **free energy** $\Omega = -TS_{\rm os}$
- Relation between fall-off of metric functions and Ω
  - Black strings (BS)

$$\left(\frac{\Omega}{V_2}\right)_{\rm BS} = c_t - 2c_z$$

  - Solitonic solutions (C)

$$\left(\frac{\Omega}{V_2}\right)_{\rm C} = c_t + c_z$$

where

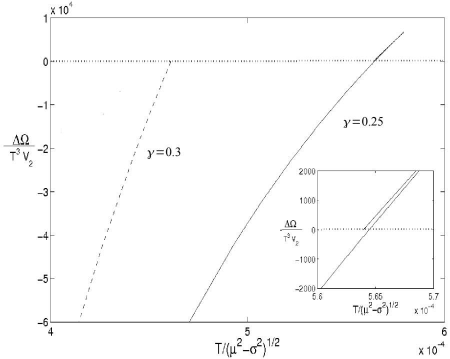
$$f(\rho \gg 1) = \rho^2 + \frac{(c_t + c_z)}{\rho} + O(\rho^{-2}) \ , \ \ g(\rho \gg 1) \sim O(\rho^{-3}) \ ,$$

$$p(\rho \gg 1) = \rho^2 + \frac{c_z}{\rho} + O(\rho^{-2}) \ , \ \ b(\rho \gg 1) = \rho^2 + \frac{c_t}{\rho} + O(\rho^{-2})$$

Holographic superfluids: free energy

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

- $\gamma \neq 0$, fluid/BS superfluid phase transition



Holographic superfluids: phase diagrams

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

- **Free energy** of different phases for static case
  - BS: black string $\rightarrow$ fluid

$$\left(\frac{\Omega}{V_2}\right)_{\rm BS} = -\rho_h^3 \left(1 + \frac{\gamma\mu^2}{2\rho_h^2}\right) \quad , \quad T = \frac{3\rho_h}{4\pi}\left(1 - \frac{\gamma\mu^2}{6\rho_h^2}\right)$$

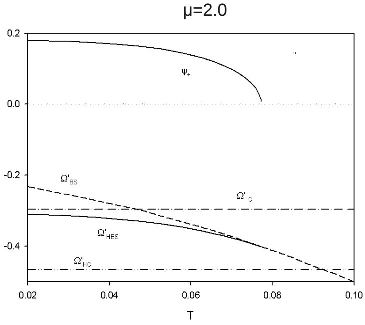
  - C: soliton $\rightarrow$ insulator

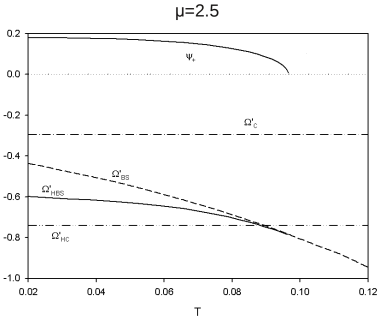
$$\left(\frac{\Omega}{V_2}\right)_{\rm C} = -\left(\frac{4\pi}{3\tau_\eta}\right)^3 \stackrel{\tau_\eta=2\pi}{\Longrightarrow} \left(\frac{\Omega}{V_2}\right)_{\rm C} = (-2/3)^3 \approx -0.29$$

  - HBS: hairy black string $\rightarrow$ superfluid *numerical*
  - HC: hairy soliton $\rightarrow$ superfluid *numerical*

Holographic superfluids: phase diagrams

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)





Holographic superfluids: phase diagrams

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

- Insulator-fluid phase transition for $T = 0$ at

$$\mu = \frac{2}{3} 4^{-1/3} \sqrt{6} \gamma^{-1/2} \approx 1.0287 \gamma^{-1/2}$$

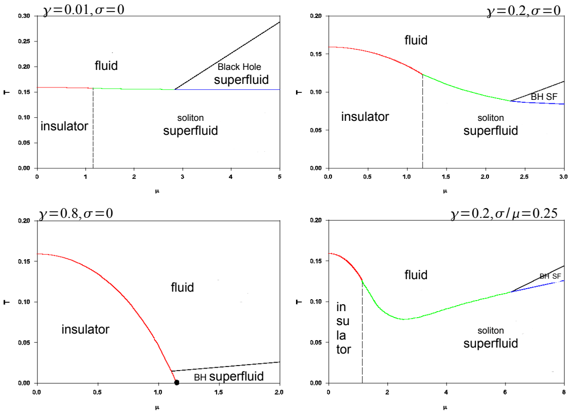
- Insulator-fluid phase transition for $\mu = 0$ at

$$T = 1/(2\pi) = 1/\tau_\eta$$

(not surprising, compare Surya, Schleich & Witt (2001))

Holographic superfluids: phase diagrams

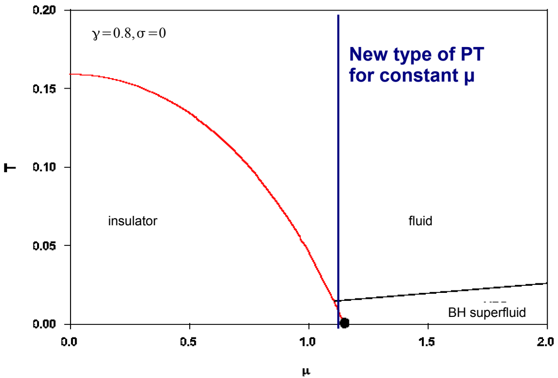
(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)



Holographic superfluids: New phase transitions

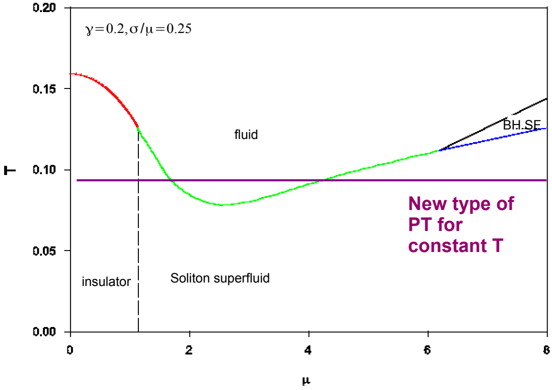
(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)

similar result in (4+1)-dim (Horowitz & Way (2010))



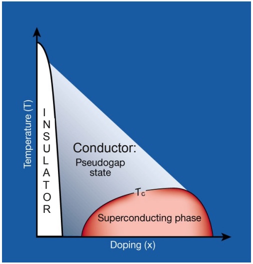
Holographic superfluids: New phase transitions

(Y. Brihaye & B. Hartmann, Phys. Rev. D83 (2011) 126008)



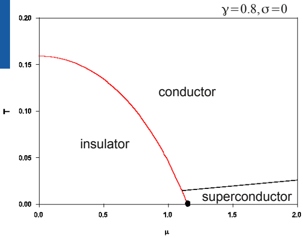
Does this work???

$\sigma = 0 \Rightarrow$ Insulator/conductor/superconductor interpretation



(a) Principle phase diagram of a cuprate superconductor

(from: http://www.pha.jhu.edu/~vstanev1/)



(b) taken from Brihaye and B. Hartmann, Phys. Rev. D, 2011

Outline

Summary

- Backreaction important since ...
  - ... order of phase transition changes
  - ... leads to new type of phase transitions
  - ... lowers critical temperature T_c and increase ω_g/T_c
- Backreaction does not suppress condensation ...
  ... not even when Gauss-Bonnet terms involved
- compactifying one coordinate $\rightarrow$ up to four phases
- To do: (a) yet higher order curvature corrections (Lovelock etc.), (b) phase diagrams for larger γ and σ, (c) ...

Still sceptical about Holography?

Thank you for your attention!