Black holes in massive gravity

Eugeny Babichev

LPT, Orsay

ARXIV:1503.07529

with Richard Brito

OUTLINE

- ✦Introduction
- ✦Black holes in massive gravity
- ✦Perturbations and (in)stability
- ✦Conclusions

INTRODUCTION

Why modify gravity?

Why modify gravity?
- cosmological constant problems,
- non-renormalizability problem,
- benchmarks for testing General Relativity
- theoretical curiosity.

Many ways to modify gravity:
- f(R), scalar-tensor theories,
- Galileons, Horndeski (and beyond) theory, KGB, Fab-four,
- higher-dimensions,
- DGP,
- Horava, Khronometric
- massive gravity

- Naively, cancellation of the cosmological constant, because of the Yukawa decay;
- Small cosmological constant due to small graviton mass

Old problems of massive gravity

1.

Physical ghost [Boulware&Deser'72]

2.

Extra propagating degrees of freedom.

Difficult to pass basic Solar system gravity tests.

vDVZ discontinuity [van Dam&Veltman'70, Zakharov'70]

Fierz-Pauli massive gravity

Expand the Einstein-Hilbert action:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_{GR} = M_P^2 \int d^4x \sqrt{-g} R = \int d^4x \left(-\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} \right) + \mathcal{O}(h^3)$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\partial_\mu \partial_\nu h - \frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}\partial_\rho \partial_\mu h^\rho_\nu + \frac{1}{2}\partial_\rho \partial_\nu h^\rho_\mu - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h)$$

2 propagating spin: 2 massless gravitons, spin-2

$$x^\alpha \to x^\alpha + \xi^\alpha, \; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}$$

Fierz-Pauli massive gravity

Fierz-Pauli action *(Fierz&Pauli'39)*:

$$S_{PF} = M_P^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

Linearized Einstein-Hilbert term

mass term

$$\cancel{x^\alpha \to x^\alpha + \xi^\alpha, \; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}}$$

5 healthy degrees of freedom (because of a particular choice of the potential, h=0)

for a generic mass term 6 d.o.f., one is necessary Ostrogradski ghost

Non-linear completion ?

Non-linear massive gravity

Introduce an extra metric to construct mass term
(contract indices)

$g_{\mu\nu}$:physical metric, matter couples to it

$f_{\mu\nu}$:an extra metric (may be dynamical or fixed)

Construct a potential, which is
invariant under diffeomorphism (common for two metrics)
+ some technical conditions

Non-linear massive gravity

potential for metric

building block: $\mathbf{g^{-1}f}$

$$S_{int}^{(2)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-f} \; H_{\mu\nu} H_{\sigma\tau} \left(f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right)$$ *(Boulware & Deser'72)*

$$S_{int}^{(3)} \equiv -\frac{1}{8} m^2 M_P^2 \int d^4x \, \sqrt{-g} \; H_{\mu\nu} H_{\sigma\tau} \left(g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)$$ *(Arkani-Hamed et al'03)*

where $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

Vainshtein mechanism

screens extra degree of freedom

Non-linear effects restore General Relativity
close to the source
due to the non-linear effects

[Vainshtein'72]
[EB, Deffayet, Ziour'09'10]

Non-linear massive gravity

Boulware-Deser ghost

Generically there are two propagating scalars: one is a ghost !

(Boulware & Deser'72)

dRGT massive gravity

special case of non-linear massive gravity

Massive gravity without Boulware-Deser ghost
[de Rham, Gabadadze, Tolley'10'11, Hassan & Rosen'12]

$$Y = \sqrt{\mathbf{g}^{-1}\mathbf{f}}$$

g is physical metric;
f is fixed (flat) or extra dynamical metric.

$$S = M_P^2 \int d^4x \sqrt{-g} \left[R[g] - m^2 \sum_{k=0}^{k=4} \beta_k e_k(Y) \right] + \kappa M_P^2 \int d^4x \sqrt{-f}\, \mathcal{R}[f]$$

$$e_0 = 1,\; e_1 = [X],\; e_2 = \frac{1}{2}\left([X]^2 - [X^2]\right),\; e_3 = \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right),$$

$$e_4 = \frac{1}{24}\left([X]^4 - 6[X]^2[X^2] + 3[X^2]^2 + 8[X][X^3] - 6[X^4]\right)$$

Equations of motion

$$G^{\mu}{}_{\nu} = m^2 \left(T^{\mu}{}_{\nu} + \Lambda_g \delta^{\mu}_{\nu} \right)$$

$$\mathcal{G}^{\mu}{}_{\nu} = m^2 \left(\frac{\sqrt{-g}}{\sqrt{-f}} \frac{\mathcal{T}^{\mu}{}_{\nu}}{\kappa} + \Lambda_f \delta^{\mu}_{\nu} \right)$$

$G_{\mu\nu}$ is the Einstein tensor for metric $g_{\mu\nu}$

$\mathcal{G}_{\mu\nu}$ is the Einstein tensor for metric $f_{\mu\nu}$

in vacuum

BLACK HOLES

Black holes

Schwarzschild metric

Non-bidiagonal BHs

[Salam & Strathdee'77]
[Isham & Storey'78]
[Koyama, Niz, Tasinato'11]

Ansatz (bi-Eddington-Finkelstein form) [EB & Fabbri'13]:

$$ds_g^2 = -\left(1 - \frac{r_g}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right]$$

Two choices:

$$\begin{cases} r_g = r_f & \text{bi-diagonal} \\ \beta(C-1)^2 - 2\alpha(C-1) + 1 = 0 & \text{non-bidiagonal} \end{cases}$$

For these choices the extra "mass" energy-momentum tensor reduces to effective cosmological constant

Charged Black holes

Electromagnetic field coupled to g

$$ds_g^2 = -\left(1 - \frac{r_g}{r} + \frac{r_Q^2}{r^2} - \frac{r^2}{l_g^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r} - \frac{r^2}{l_f^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right].$$

$$A_\mu = \left\{\frac{Q}{r}, 0, 0, 0\right\}$$

[EB & Fabbri'14]

Black holes

Rotating solutions

Original Kerr metric

$$ds_g^2 = -\left(1 - \frac{r_g r}{\rho^2}\right)\left(dv + a\sin^2\theta d\phi\right)^2$$
$$+ 2\left(dv + a\sin^2\theta d\phi\right)\left(dr + a\sin^2\theta d\phi\right) + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
$$\rho^2 = r^2 + a^2\cos^2\theta$$

f is flat, but unusual form

$$ds_f^2 = C^2\left[-dv^2 + 2dvdr + 2a\sin^2\theta drd\phi + \rho^2 d\theta^2 + \left(r^2 + a^2\right)\sin^2\theta d\phi^2\right]$$

Obtained from $\quad ds_M^2 = -dt^2 + dx^2 + dy^2 + dz^2$

by:

$$t = v - r,\ x + iy = (r - ia)e^{i\phi}\sin\theta,\ z = r\cos\theta$$
$$r \to Cr,\ v \to Cv,\ a \to Ca$$

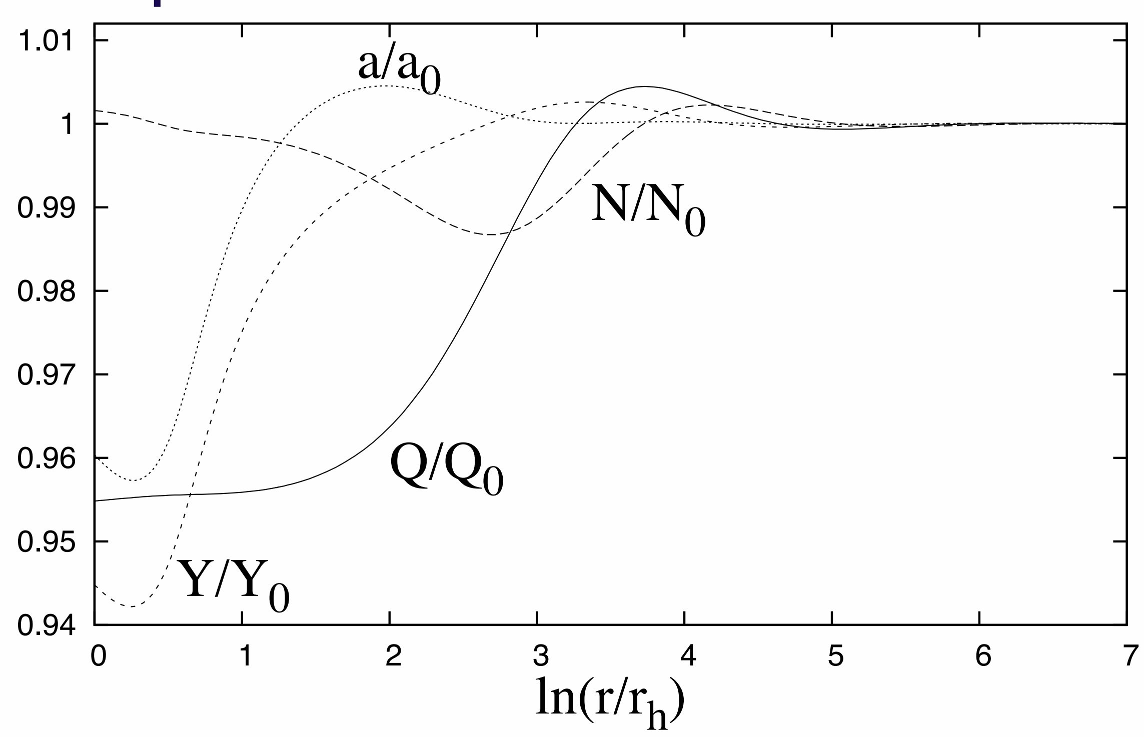
Hairy black holes

Asymptotically AdS hairy solutions exist `[Volkov,'12]`

$$g_{\mu\nu}dx^\mu dx^\nu = -Q^2\,dt^2 + N^{-2}\,dr^2 + R^2 d\Omega^2\,,$$
$$f_{\mu\nu}dx^\mu dx^\nu = -a^2\,dt^2 + b^2\,dr^2 + U^2 d\Omega^2\,,$$

Numerical integration of a system of coupled ODEs

$$\begin{cases} N' = \mathcal{F}_1(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4) \\ Y' = \mathcal{F}_2(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4) \\ U' = \mathcal{F}_3(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4). \end{cases}$$

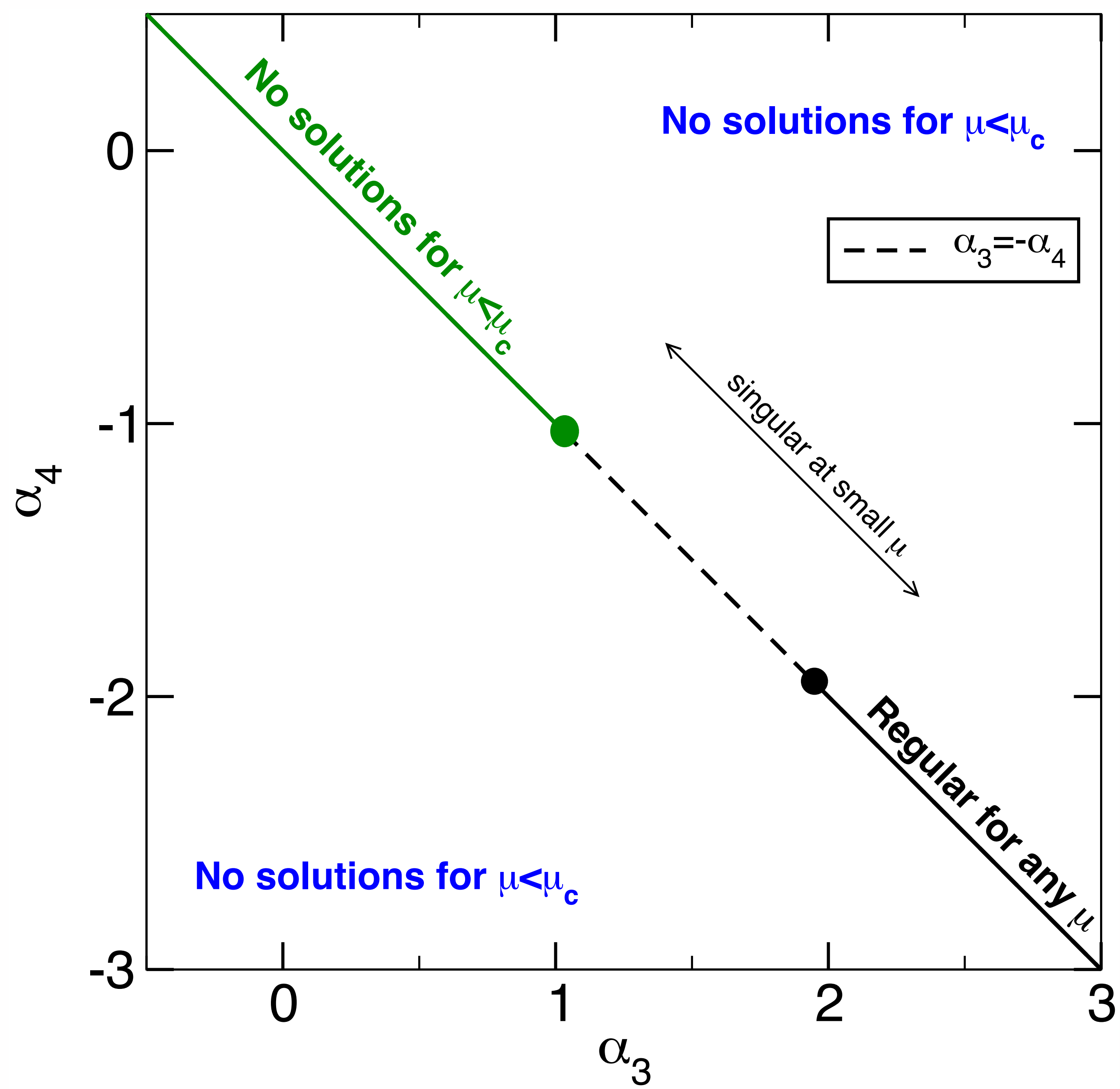


Hairy black holes

Asymptotically flat hairy solutions [Brito, Cardoso, Pani'13]

$$N = 1 - \frac{C_1}{2r} + \frac{C_2(1+r\mu)}{2r} e^{-r\mu}\,,$$
$$Y = 1 - \frac{C_1}{2r} - \frac{C_2(1+r\mu)}{2r} e^{-r\mu}\,,$$
$$U = r + \frac{C_2(1+r\mu+r^2\mu^2)}{\mu^2 r^2} e^{-r\mu}\,,$$

Yukawa decay



For generic potential only for large BH mass.

$$r_s \sim 1/H$$

PERTURBATIONS

Perturbations

spherically symmetric ansatz for perturbations

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h^{(g)}_{\mu\nu}, \; f_{\mu\nu} = f^{(0)}_{\mu\nu} + h^{(f)}_{\mu\nu}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) \; .$$

$$h^{\mu\nu}_{(g)} = e^{\Omega v} \begin{pmatrix} h^{vv}_{(g)}(r) & h^{vr}_{(g)}(r) & 0 & 0 \\ h^{vr}_{(g)}(r) & h^{rr}_{(g)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2 \sin^2\theta} \end{pmatrix} \qquad (\Omega > 0)$$

$h^{\mu\nu}_{(-)} \equiv h^{\mu\nu}_{(g)} - C^2 h^{\mu\nu}_{(f)}$ i.e. $h^{vv}_{(-)}(r) = h^{vv}_{(g)}(r) - h^{vv}_{(f)}(r)$

Perturbations

spherically symmetric ansatz for perturbations

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h^{(g)}_{\mu\nu}, \; f_{\mu\nu} = f^{(0)}_{\mu\nu} + h^{(f)}_{\mu\nu}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) \; .$$

$$h^{\mu\nu}_{(f)} = \frac{e^{\Omega v}}{C^2} \begin{pmatrix} h^{vv}_{(f)}(r) & h^{vr}_{(f)}(r) & 0 & 0 \\ h^{vr}_{(f)}(r) & h^{rr}_{(f)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2 \sin^2\theta} \end{pmatrix} \qquad (\Omega > 0)$$

$h^{\mu\nu}_{(-)} \equiv h^{\mu\nu}_{(g)} - C^2 h^{\mu\nu}_{(f)}$ i.e. $h^{vv}_{(-)}(r) = h^{vv}_{(g)}(r) - h^{vv}_{(f)}(r)$

Spherical Perturbations

Regularity at horizons and infinity !

Non-bidiagonal case

General solution for perturbations

$$h^{\mu\nu}_{(g,f)} = h^{\mu\nu(g,f)}_{GR} + h^{\mu\nu(g,f)}_{(m)}$$

$$h^{\mu\nu}_{GR} = -\nabla^{\mu}\xi^{\nu} - \nabla^{\nu}\xi^{\mu}$$

Non-Bi-diagonal case

explicit solution for perturbations

$$h_{GR}^{\mu\nu(f)} = 0$$

$$h_{GR}^{\mu\nu(g)} = e^{\Omega v} \begin{pmatrix} 0 & \Omega c_1 & 0 & 0 \\ \Omega c_1 & c_0 \left(\Omega - \frac{r_g}{2r^2}\right) & 0 & 0 \\ 0 & 0 & c_0 r^{-3} & 0 \\ 0 & 0 & 0 & c_0 \csc^2(\theta) r^{-3} \end{pmatrix}$$

$$h_{(m)}^{rr(g)} = \frac{\mathcal{A}(r_g - r_f) e^{\Omega v}}{4\Omega} m^2 h_{(-)}^{\theta\theta},$$

$$h_{(m)}^{rr(f)} = -\kappa^{-1} h_{(m)}^{rr(g)}.$$

Since at $r \to \infty$ $v = t + r$ the perturbations are not regular at infinity.

NO unstable modes
Non-bidiagonal solution is stable against radial perturbations

[EB & Fabbri'14]

Perturbations for bidiagonal case

$$h^{(-)\mu}{}_{\nu} = h^{\mu}_{\nu} - \tilde{h}^{\mu}_{\nu}$$

$$h^{(+)\mu}{}_{\nu} = h^{\mu}_{\nu} + \kappa\tilde{h}^{\mu}_{\nu}$$

$$m' \equiv m\sqrt{1+1/\kappa}$$

$$\boxed{\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(+)}_{\alpha\beta} = 0}$$

$h^{(-)}_{\mu\nu}$ is massive

$h^{(+)}_{\mu\nu}$ is massless

$$\nabla^{\nu} h^{(-)}_{\mu\nu} = h^{(-)} = 0$$

$$\Box h^{(-)}_{\mu\nu} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h^{(-)}_{\lambda_\sigma} = m'^2 h^{(-)}_{\mu\nu}$$

Bi-diagonal case

Gregory-Laflamme instability

EFI-93-02

January 1993

BLACK STRINGS AND p-BRANES ARE UNSTABLE

Ruth Gregory

Enrico Fermi Institute, University of Chicago

5640 S.Ellis Ave, Chicago, IL 60637, U.S.A.

Raymond Laflamme

$\delta g_{ab} = h_{ab}$, the Lichnerowicz equation, is essentially a wave equation

$$\Delta_{\mathrm{L}} h_{ab} = \left(\delta_a^c \delta_b^d \Box + 2R_a{}^c{}_b{}^d\right) h_{cd} = 0. \quad (1.1)$$

Because of the symmetries of the background $\mathrm{Sch}_4 \times \mathbb{R}$ metric, this reduces to a four-dimensional Lichnerowicz operator plus a ∂_z^2 piece. Performing a Fourier decomposition of h_{ab} in the fifth dimension yields

$$\Delta_{\mathrm{L}} h_{ab} = \left(\Delta_4 - m^2\right) h_{ab} = 0. \quad (1.2)$$

focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra $10 - D$ dimensions.

Bi-diagonal case

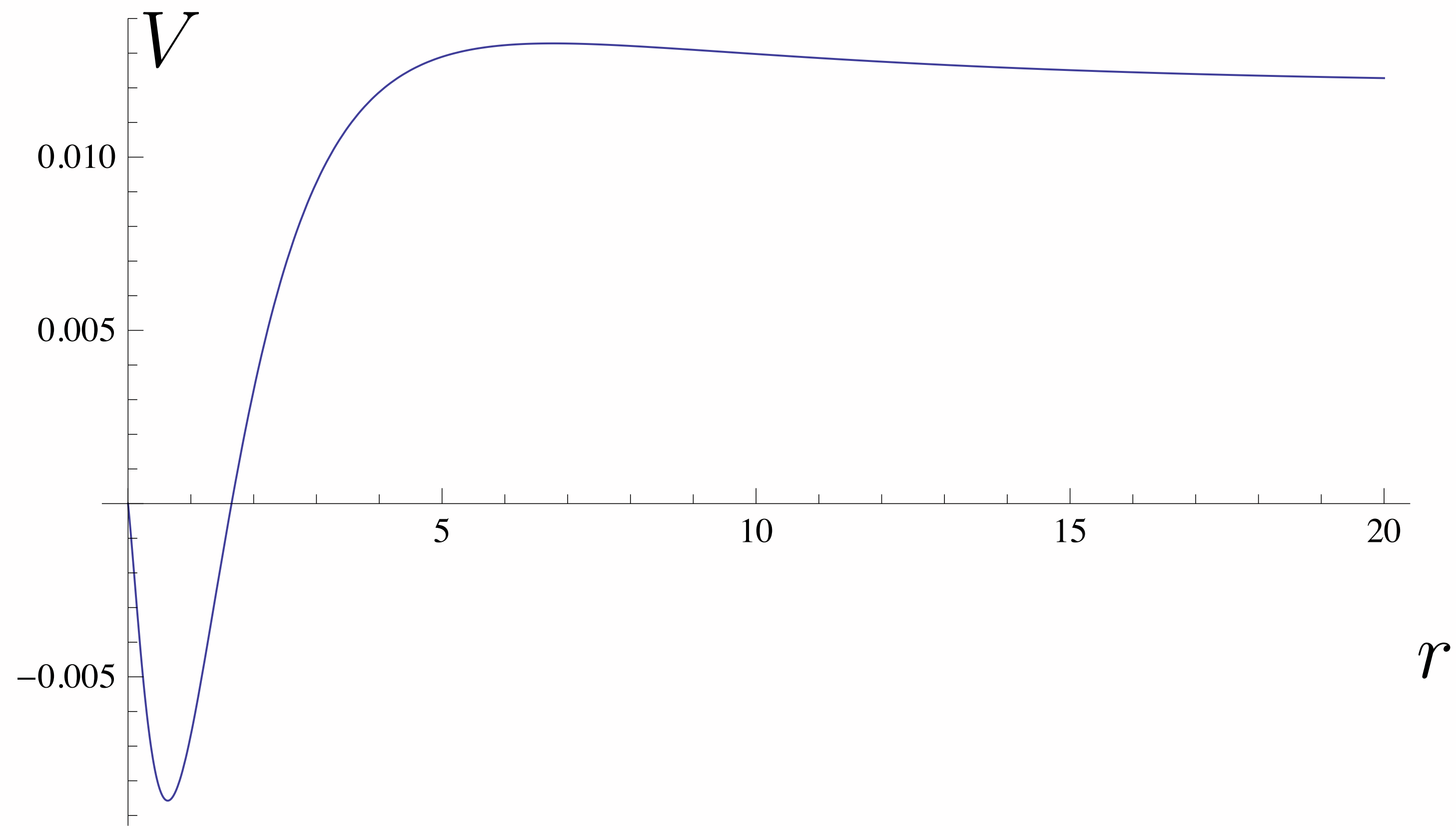
GL instability

A system of equations of second order plus 2 constraints on $H_{tt},\ H_{tr},\ H_{rr},\ K$

Playing with equations we can obtain a single equation on φ_0 (a combination of H_{tt}, H_{rr} and H_{tr})

$$\frac{d^2}{dr_*^2}\varphi_0 + \left[\omega^2 - V(r)\right]\varphi_0 = 0$$

Unstable $(\Omega > 0)$ mode, satisfying boundary conditions?



$$V_0 = \left(1 - \frac{r_g}{r}\right)\left[\frac{2M}{r^3} + m'^2 + \frac{24M(M-r)m'^2 + 6r^3(r-4M)m'^4}{\left(2M + r^3 m'^2\right)^2}\right]$$

Bi-diagonal case: Instability

$$\boxed{0 < m' < \frac{\mathcal{O}(1)}{r_S}}$$

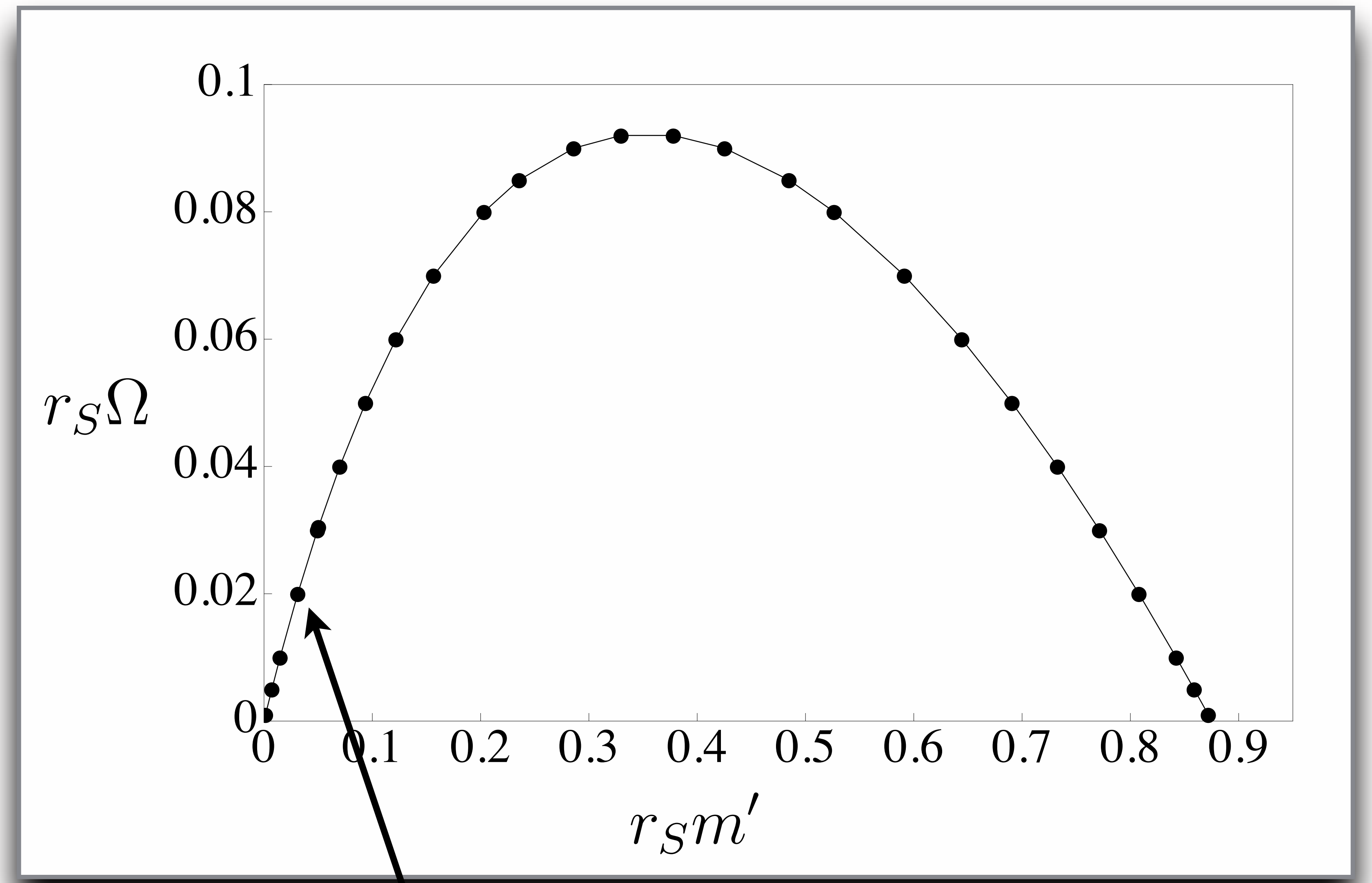
Instability

Confirmed independently by [Brito, Cardoso, Pani'13]

Instability of black holes

rate of instability

Rate of instability



for $m' \sim H \rightarrow \boxed{\tau \sim H^{-1}}$

Very slow instability !

Approximately linear dependance

$$r_S \ll 1/m'$$

$$\Omega = m'$$

Instability of black holes

extended result

Proportional metrics $f_{\mu\nu} = \omega^2 g_{\mu\nu}$

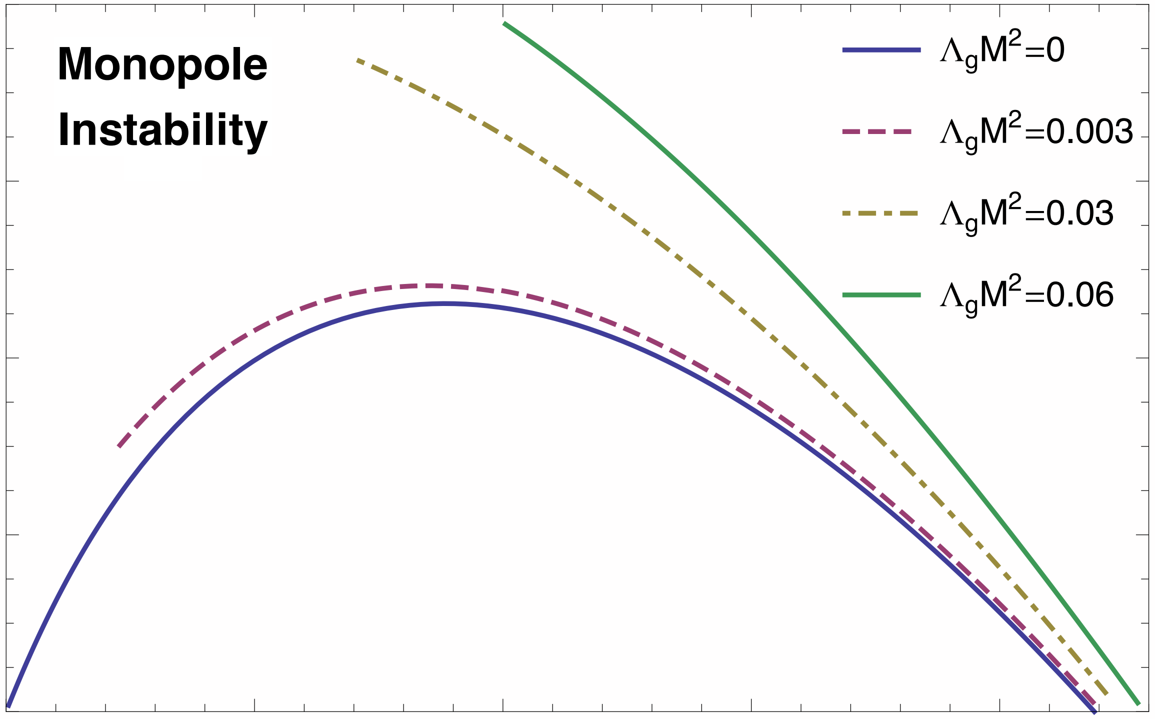
By appropriate choice of the parameters of the mass term the extended bi-Schwarzschild solution exists.

The mass of perturbations is modified by a factor depending on $\alpha_3,\ \alpha_4,\ \omega$

The result is the same: there is instability in the range $0 < \hat{m} < \dfrac{\mathcal{O}(1)}{r_S}$

[EB & Fabbri'13]

de Sitter: instability is still there
[Brito, Cardoso, Pani'13]



Superradiant instability

The Kerr solution in massive gravity is prone to another kind of instability, related to the superradiant scattering of bosonic fields with spinning BHs.

"Natural" instability: linear massive spin-2 perturbations. Similar to instability, which occur for massive scalar and vector perturbations of rotating BHs in GR.

Constraints on graviton mass

$$m_g \lesssim 5 \times 10^{-23} \mathrm{eV}$$

[Brito, Cardoso, Pani'13]

CONCLUSIONS

- It is possible to construct non-bidiagonal solutions in massive gravity, which are analogues of corresponding GR solutions (Schwarzschild, charged, rotating).
- There are hairy massive gravity black holes
- The non-bidiagonal black holes in massive gravity are stable against radial perturbations.
- The bi-diagonal spherically symmetric BHs are unstable due to the helicity-0 mode instability. The rate of instability is extremely small.
- Superradiant instability for rotating BHs in massive gravity.
- The fate of unstable BHs? The endpoint of gravitational collapse?
- Rotating hairy BHs?
- dS hairy black holes?
- Do perturbations around black holes contain ghosts?
- Non-bidiagonal solutions: superradiant instability?