Neutron stars as probes of strong-field gravity

Work over several years (2010-2015) with:

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Lisbon, Jun 12 2015

INVESTIGADOR FCT

QUALIFICAR É CRESCER.

Outline

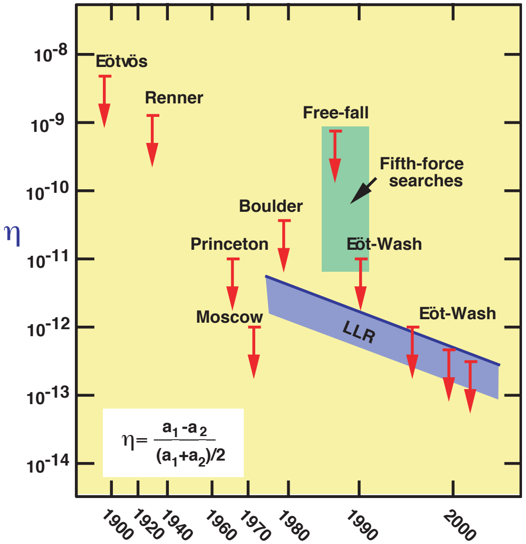
- Weak- vs strong-field tests of GR
- Strong-field tests – of what?
- Stars in GR and in modified gravity
- Examples: scalar-tensor theory
  tensor-multi-scalar theories
  EdGB gravity
- Issues: EOS/gravity degeneracy
  Theory degeneracy
- The post-TOV formalism

Weak-field tests
vs
strong-field tests

The foundations of general relativity

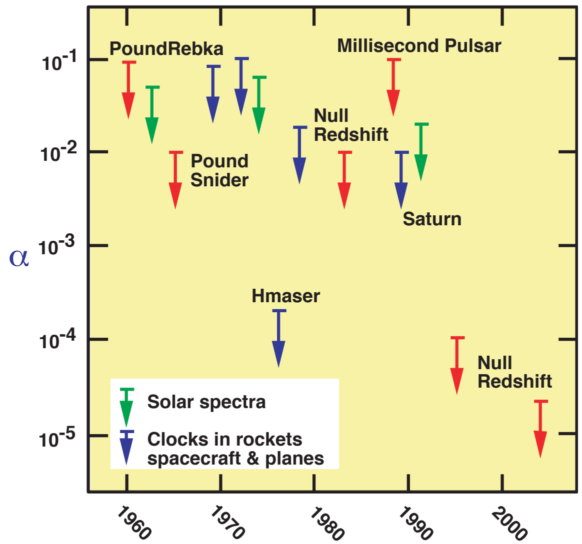
[Will, 1403.7377]

TESTS OF THE
WEAK EQUIVALENCE PRINCIPLE



YEAR OF EXPERIMENT

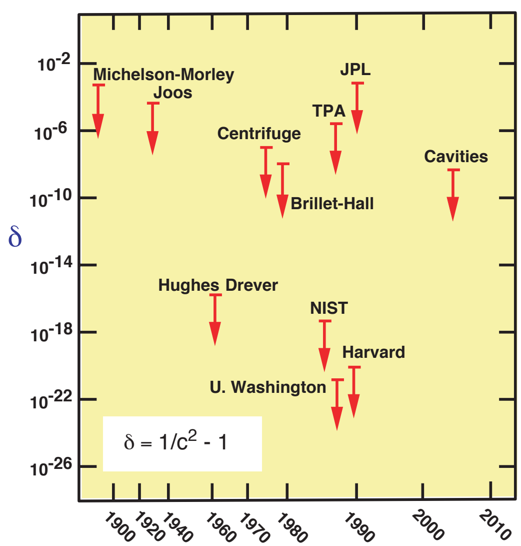
TESTS OF
LOCAL POSITION INVARIANCE



YEAR OF EXPERIMENT

$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$

TESTS OF
LOCAL LORENTZ INVARIANCE



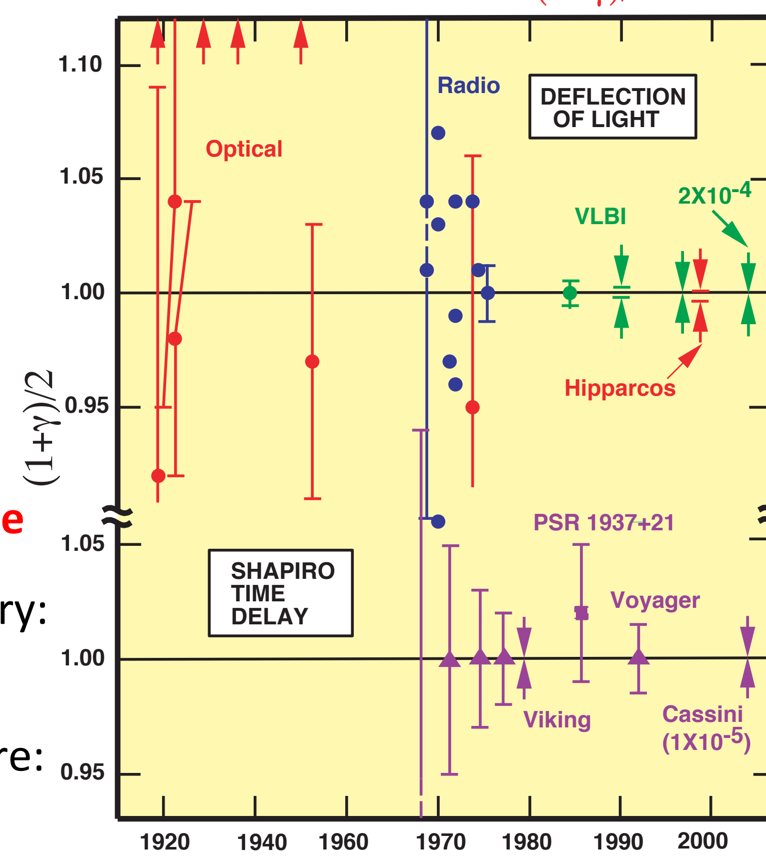
YEAR OF EXPERIMENT

Weak Equivalence Principle+
Local Lorentz Invariance+
Local Position Invariance=
Einstein's Equivalence Principle

Implies gravity is a metric theory:
gravity is **spacetime curvature**

Best test of spacetime curvature:
Cassini bound

THE PARAMETER $(1+\gamma)/2$



YEAR OF EXPERIMENT

Why should we bother looking for modifications of GR?

(Circa 1919)
Journalist: ***"Herr Einstein, what if the theory turned out to be wrong?"***
Einstein: ***"I would feel sorry for the dear Lord. The theory is correct."***

(Circa 1970)
Chandrasekhar to his postdoc Clifford Will:
"Why do you spend so much time testing GR? We know the theory is right."

1) **Theory: GR is not renormalizable**
   It becomes renormalizable if one adds high-order curvature terms to the action

2) **Experiments: dark matter, dark energy**
   Due to modified gravity?

Problem: GR is extremely well tested "in between" these two regimes

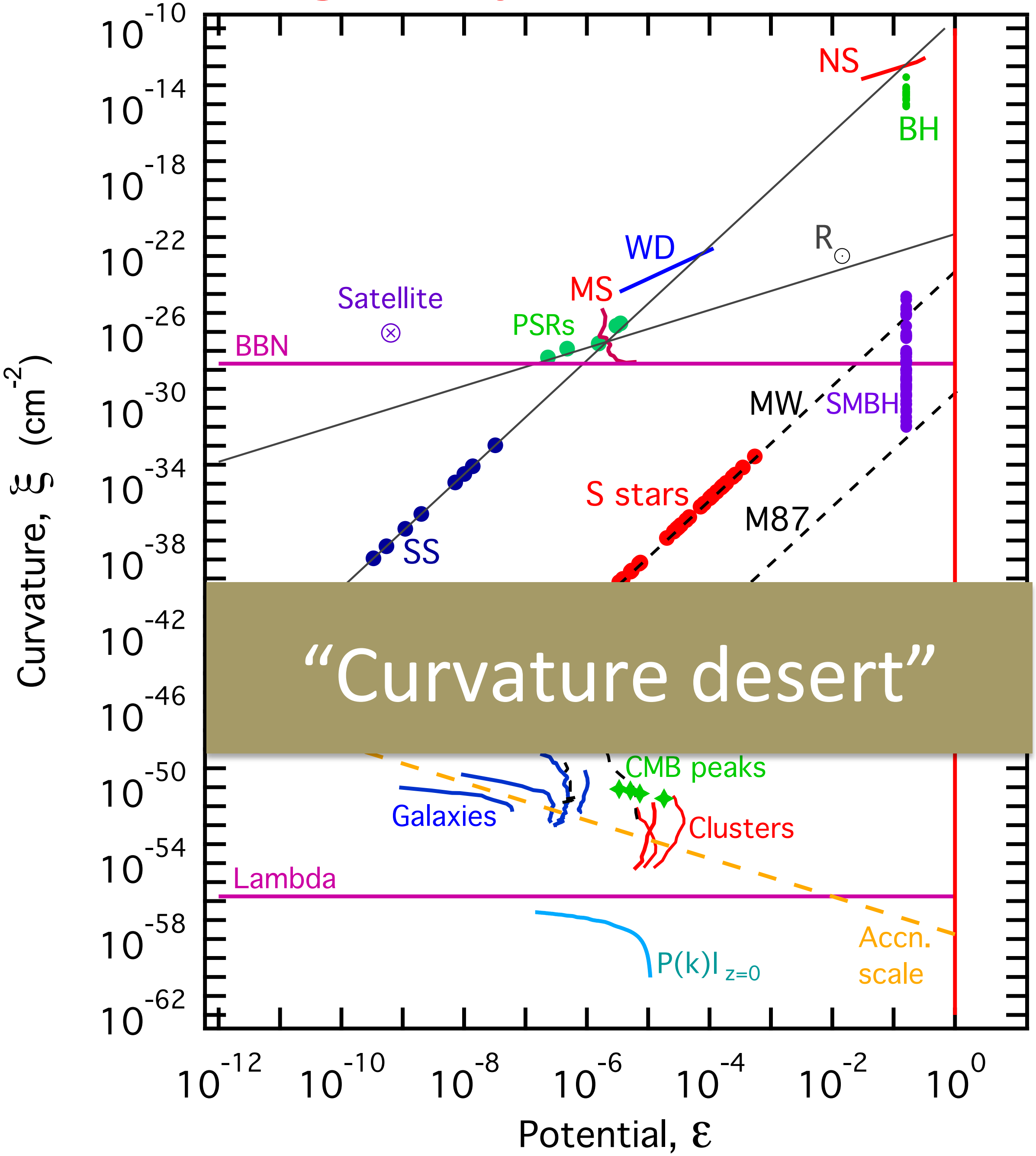
"Short blanket problem" for modifications of GR

Tests of strong gravity

Berti+ 1501.07274

What is “strong” gravity?

Strong-field probes: black holes and neutron stars



Gravitational field

$$\epsilon \equiv \frac{GM}{rc^2}$$

Curvature (Kretschmann scalar)

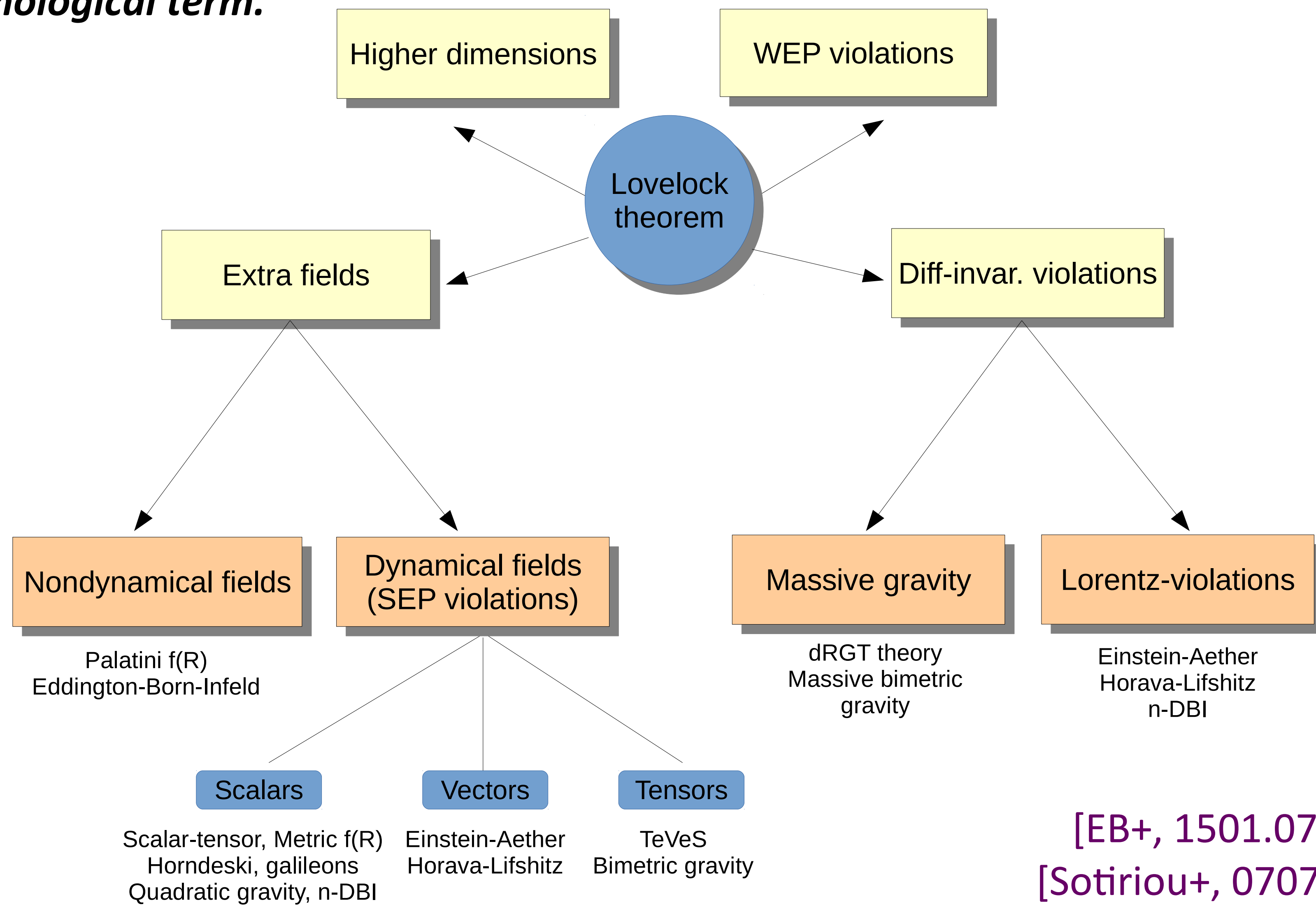
$$\xi = \left(R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}\right)^{1/2} = \sqrt{48}\,\frac{GM}{r^3 c^2}$$

Strongest test of GR:
PSR J0348+0432, P=2.46hr,
$v/c = 2\times10^{-3}$
[Antoniadis+, 1304.6875]

Figure: [Baker+, 1412.3455]

A guiding principle to modify GR: Lovelock's theorem

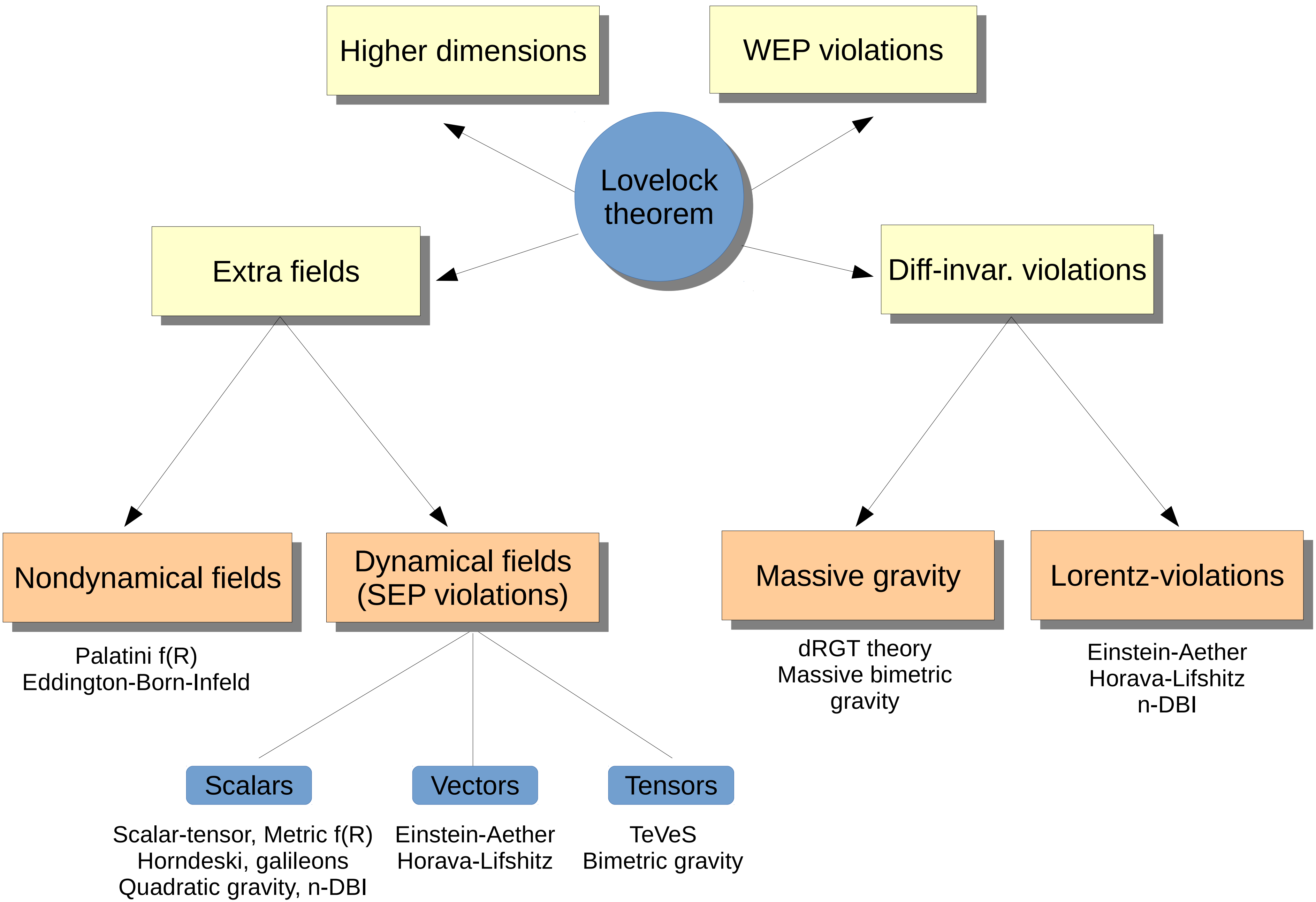
"In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term."



[EB+, 1501.07274]
[Sotiriou+, 0707.2748]

…and about what?

Lovelock theorem as a map for the modified gravity zoo



Tests of general relativity – against what?

- Action principle
- Well-posed
- Testable predictions
- Cosmologically viable, BHs, neutron stars

$$\mathcal{L} = f_0(\phi)R$$
$$-\omega(\phi)\partial_a\phi\partial^a\phi - M(\phi) + \mathcal{L}_{\rm mat}\left[\Psi, A^2(\phi)g_{ab}\right]$$
$$+f_1(\phi)(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})$$
$$+f_2(\phi)R_{abcd}{}^{*}R^{abcd}$$ + Lorentz-violating terms…

Alternative theories usually:

Introduce more fields (scalars, vectors) or higher-curvature terms

Need strong-field tests! Challenge pillars of general relativity:

- Equivalence principle
- Lorentz invariance (Einstein-aether, TeVeS…)
- Parity conservation…

[Gair+,1212.5575; Clifton+, 1106.2476]

Properties of (some) modified gravity theories

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [34]	[35–37]
Multiscalar	S	✗	✓	✓	✓	✓	✓ [38]	[39]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [40, 41]	[42]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[43]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [44]	[45]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[46–49]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[48–51]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([52])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[17]
Galileon	S	✗	✓	✓	✓	✓	✓?	[17, 53]
Nondynamical fields								
Palatini $f(R)$	–	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	–	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVeS	SVT	✗	✓	✓	✓	✓	?	[37]
$f(R)\mathcal{L}_m$	?	✗	✓	✓	✓	✗	?	
$f(T)$	?	✗	✓	✗	✓	✓	?	[54]

[EB+, arXiv:1501.07274]

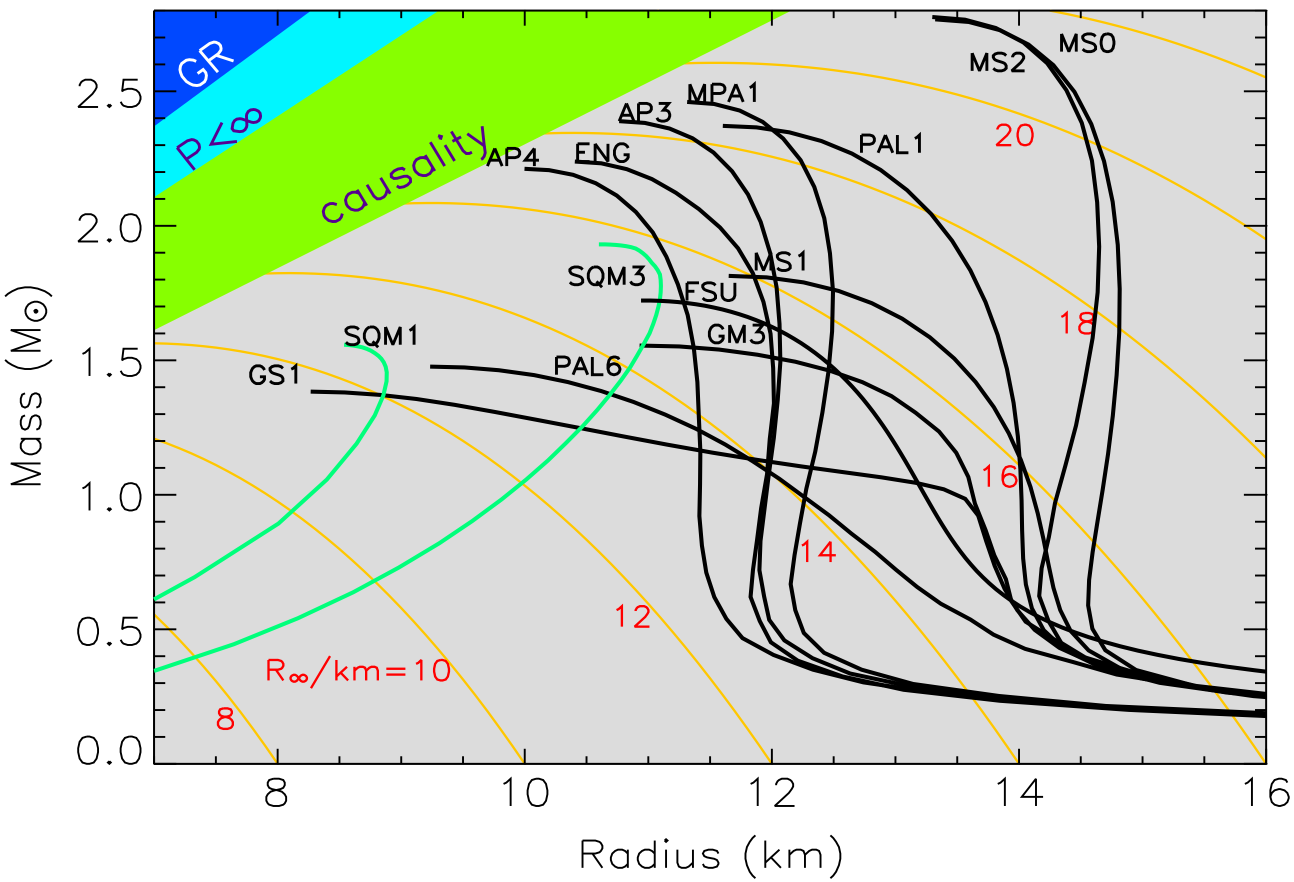
Compact stars in general relativity and modified gravity

“Internal” tests: neutron stars

Strong-field signatures:
high curvatures in interior, spontaneous scalarization…

Observables? Consider the Hartle-Thorne expansion in $\Omega/(M/R^3)^{1/2}$

Zero order in rotation: **M(R)** - mass-radius relation
Radii hard to measure, both in binaries and in isolated systems



Neutron stars as EOS probes

Strong-field signatures:
high curvatures in interior, spontaneous scalarization…

Observables? Consider the Hartle-Thorne expansion in $\Omega/(M/R^3)^{1/2}$

Zero order in rotation: **M(R)** - mass-radius relation
Radii hard to measure, both in binaries and in isolated systems

Corrections:

Moment of inertia I may be measurable in binary pulsars
[Lattimer-Schutz, Kramer, Wex…]

Tidal "Love number" may be measurable in binary inspirals
[Mora-Will, Berti-Iyer-Will, Read, Hinderer, Lang, Binnington, Poisson, Vines, Damour, Nagar, Bernuzzi, Villain, Favata, Yagi, Yunes…]

Quadrupole Q or higher-order moments: light curves or QPOs
[Laarakkers-Poisson, Berti-Stergioulas, BWMB, Baubock+, Pappas…]

Stellar oscillations

Neutron stars in (some) modified gravity theories

Theory	Structure			Collapse	Sensitivities	Stability	Geodesics
	NR	SR	FR				
Extra scalar field							
Scalar-Tensor	[109–114]	[112, 115, 116]	[117–119]	[120–127]	[128]	[129–139]	[118, 140]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[141–153]	[154]	[155]	[156, 157]	?	[158, 159]	?
Quadratic gravity							
Gauss-Bonnet	[160]	[160]	[77]	?	?	?	?
Chern-Simons	$\equiv$ GR	[25, 40, 161–163]	?	?	[162]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[164, 165]	?	?	[166]	[43, 44]	[158]	?
Khronometric/ Hořava-Lifshitz	[167]	?	?	?	[43, 44]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[168, 169]	?	?	?	?	?	?
Galileon	[170]	[170]	?	[171, 172]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[173–177]	?	?	?	–	?	?
Eddington-Born-Infeld	[178–184]	[178, 179]	?	[179]	–	[185, 186]	?

[EB+, arXiv:1501.07274]

A "theory of theories"

$$\mathcal{L} = f_0(|\phi|)R - \gamma(|\phi|)\partial_a\phi^*\partial^a\phi - V(|\phi|) + f_1(|\phi|)R^2 + f_2(|\phi|)R_{ab}R^{ab} + f_3(|\phi|)R_{abcd}R^{abcd} + f_4(|\phi|)R_{abcd}{}^*R^{abcd} + \mathcal{L}_{\rm mat}[\Psi, A^2(|\phi|)g_{ab}], \qquad (2)$$

	f_0	f_1	f_2	f_3	f_4	ω	V	γ	A	$\mathcal{L}_{\rm mat}$
General relativity	κ	0	0	0	0	0	0	1	1	perfect fluid
Scalar-tensor (Jordan frame) [24]	$F(\phi)$	0	0	0	0	0	$V(\phi)$	$\gamma(\phi)$	1	perfect fluid
Scalar-tensor (Einstein frame) [23]	κ	0	0	0	0	0	$V(\phi)$	2κ	$A(\phi)$	perfect fluid
$f(R)$ [36]	κ	0	0	0	0	0	$\kappa\frac{Rf_{,R}-f}{16\pi\bar{G}f_{,R}^2}$	2κ	$f_0^{-1/2}=f_{,R}^{-1/2}$	perfect fluid
Quadratic gravity [47]	κ	$\alpha_1\phi$	$\alpha_2\phi$	$\alpha_3\phi$	$\alpha_4\phi$	0	0	1	1	perfect fluid
EDGB [48]	κ	$e^{\beta\phi}$	$-4f_1$	f_1	0	0	0	1	1	perfect fluid
Dynamical Chern-Simons [59]	κ	0	0	0	$\beta\phi$	0	0	1	1	perfect fluid
Boson stars [71]	κ	0	0	0	0	ω	$\frac{m^2}{2}\lvert\phi\rvert^2$	1	1	0

[Yunes & Stein, 1101.2921]
[Pani+, 1109.0928]

Example #1: scalar-tensor theory

Pani+EB 1405.4547; Silva+ 1410.2511, 1411.6286

Scalar-tensor theory and spontaneous scalarization

- Action (in the "Einstein frame"):

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g^\star}\left[R^\star - 2g^{\star\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V(\varphi)\right] + S_M[\Psi, A^2(\varphi)g^\star_{\mu\nu}]$$

- Gravity-matter coupling:

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$$\alpha(\varphi) = \alpha_0 + \beta_0(\varphi - \varphi_0) + \ldots$$

- Field equations:

$$G^\star_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^\star_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^\star_{\mu\nu}V(\varphi) + 8\pi T^\star_{\mu\nu}\,,$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}\,,$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993); PRD 54, 1474 (1996); EB+,

Scalarization threshold: a back-of-the-envelope derivation

$$\Box_{g^*}\varphi = -4\pi\alpha(\varphi)T^*$$

$$\alpha(\varphi) = \beta_0\varphi$$

$$-T^* = A^4(\epsilon^* - 3p^*) \sim \frac{3}{4\pi R^2}\frac{m}{R} \quad \text{for} \quad r < R$$

$$\nabla^2\varphi = \mathrm{sign}(\beta_0)\left[\frac{3|\beta_0|(m/R)}{R^2}\right]\varphi = \mathrm{sign}(\beta_0)\kappa^2\varphi$$

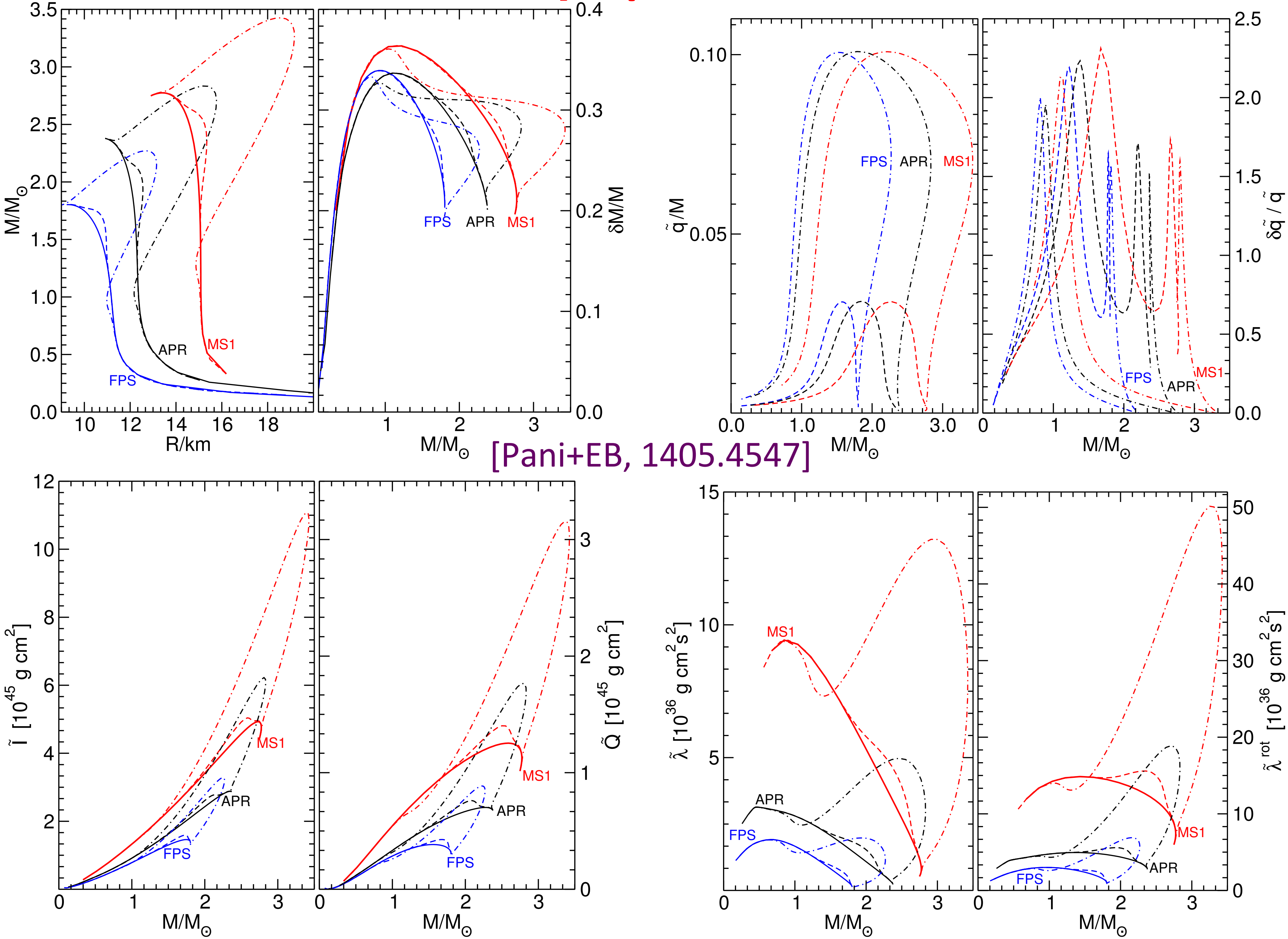
$$\beta_0 < 0 \Longrightarrow \varphi_{\text{inside}} = \varphi_{\text{c}}\frac{\sin(\kappa r)}{\kappa r}$$

$$\varphi_{\text{c}} = \frac{\varphi_0}{\cos(\kappa R)} \gg \varphi_0 \qquad \boxed{\kappa R \sim \pi/2}$$

$$\boxed{m/R \sim 0.2 \Longrightarrow \beta \sim -4}$$

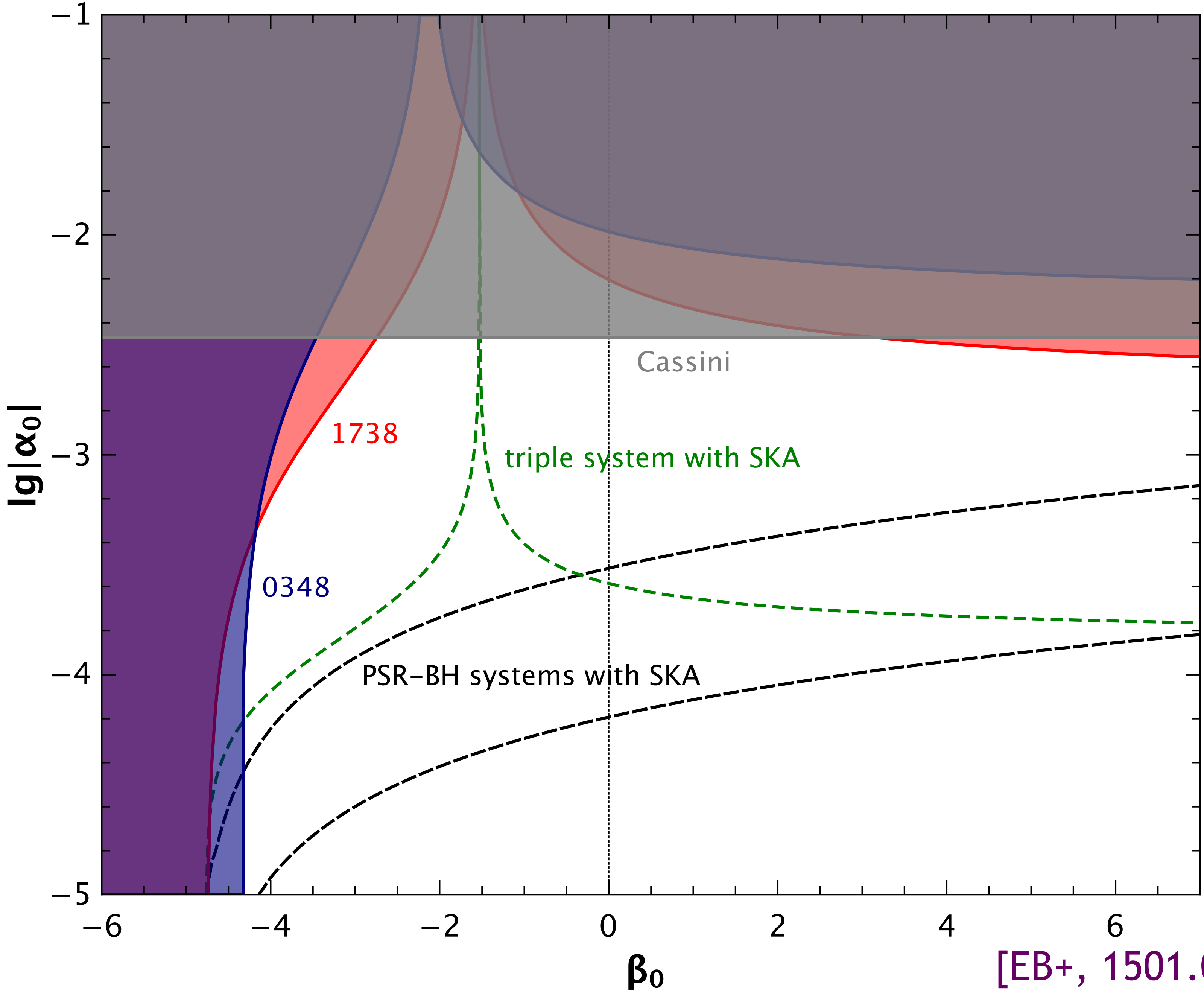
[Damour+Esposito-Farese, PRL 70, 2220

Scalar-tensor theory: spontaneous scalarization



[Pani+EB, 1405.4547]

Binary pulsar bounds on spontaneous scalarization



[EB+, 1501.07274]

Scalarization is alive – but for how long?

1) Could scalarization leave imprints in crustal oscillations?
   No - pulsar bounds are too strong

2) Can the EOS dependence save us?
   No - too mild

3) Can anisotropy save us?
   Possibly so - if you believe in anisotropy…

4) Multiscalarization?
   Work in progress…

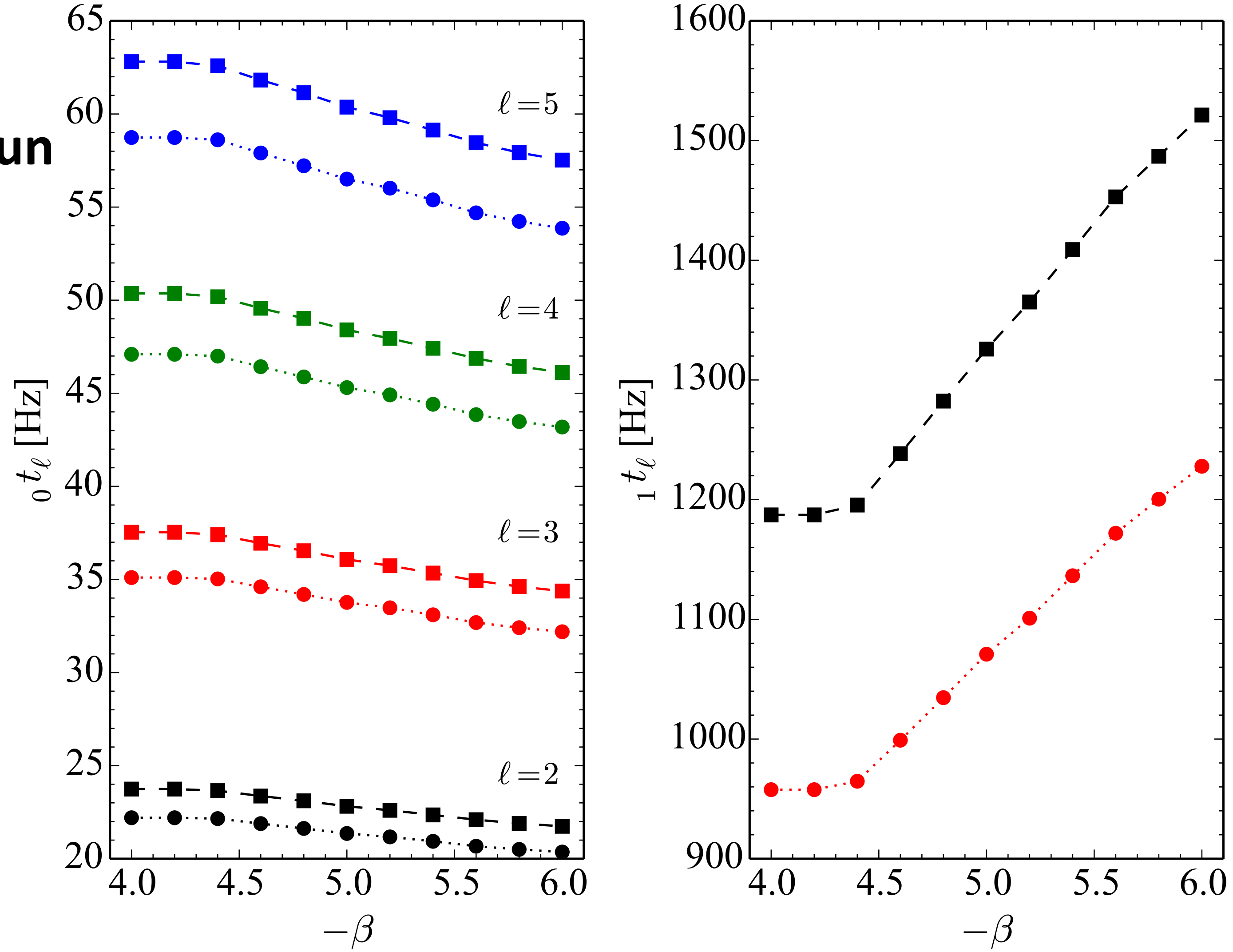
Signatures in crustal (torsional) oscillations?

Why?

Possible smoking gun for scalarization if crustal quakes are associated with QPOs in magnetars

Left: fundamental

Right: overtone



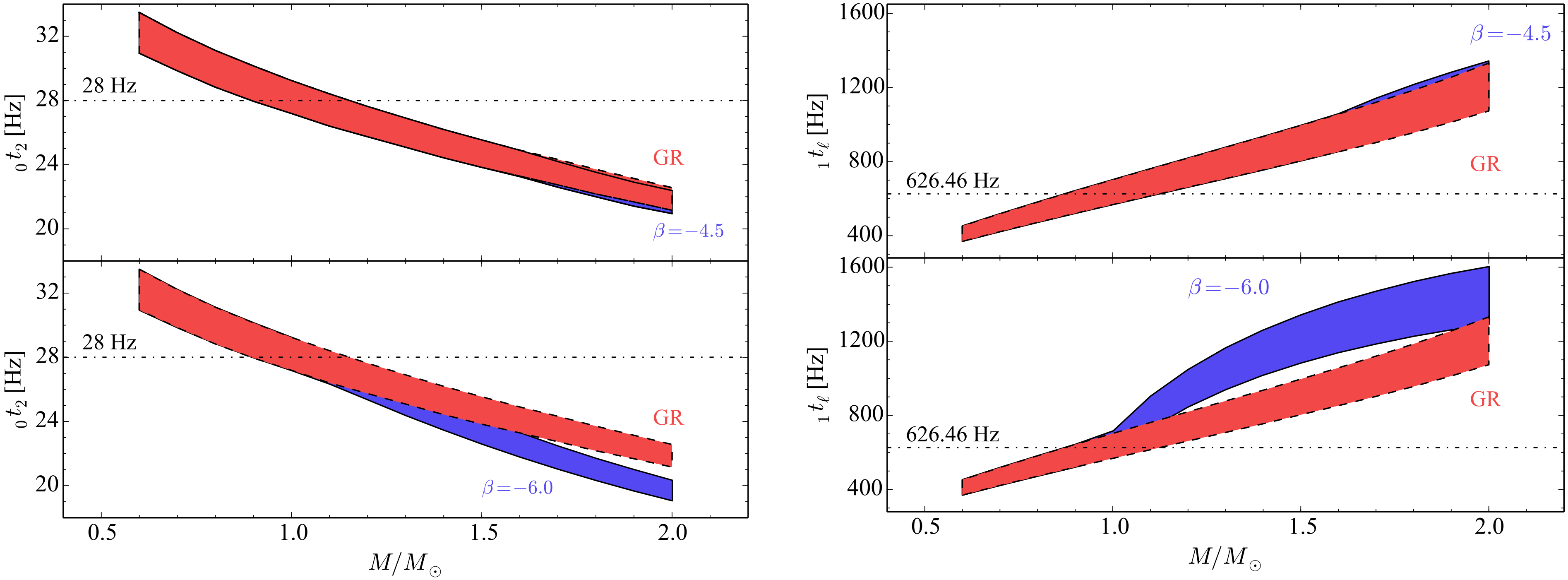
Circles: APR+DH

Squares: APR+KP

DH (Douchin-Haensel), KP (Kobyakov-Pethick): different crustal EOS

[Schumaker-Thorne, MNRAS 253, 457 (1983); Silva+, arXiv:1410.2511]

Crustal microphysics dominates over gravity modifications



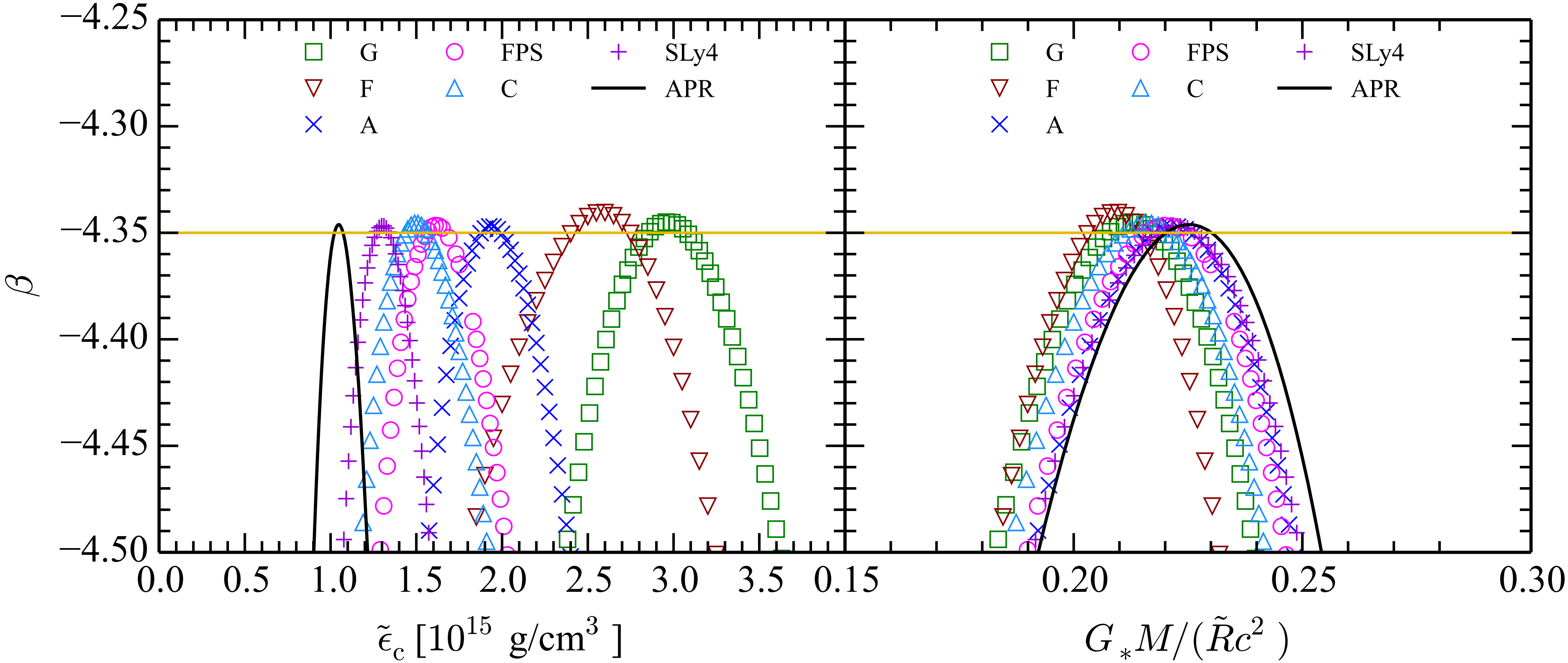
Bands bracket uncertainties in crustal EOS
Left: fundamental mode, right: overtone
Dotted horizontal lines: measured QPOs

Red: GR modes as a function of mass

Blue: ST modes for models that are
- **marginally allowed (top)**
- **excluded (bottom)**

[Silva+, 1410.2511]

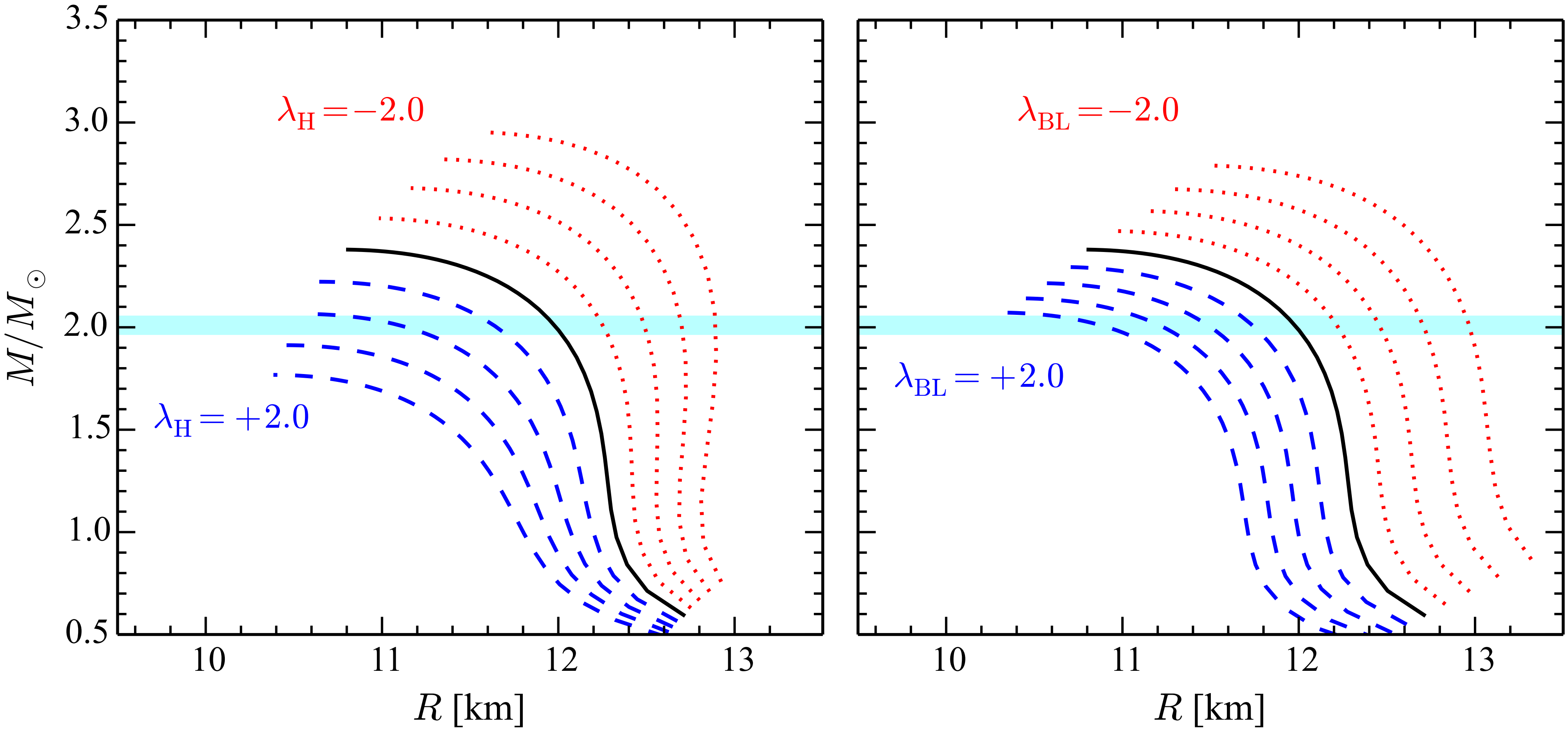
Does the EOS affect the scalarization threshold?



Dependence of β on EOS is too mild
for ordinary models of high-density nuclear matter

[Silva+, 1411.6286]

A (not so exotic?) way out: anisotropy



λ = degree of anisotropy

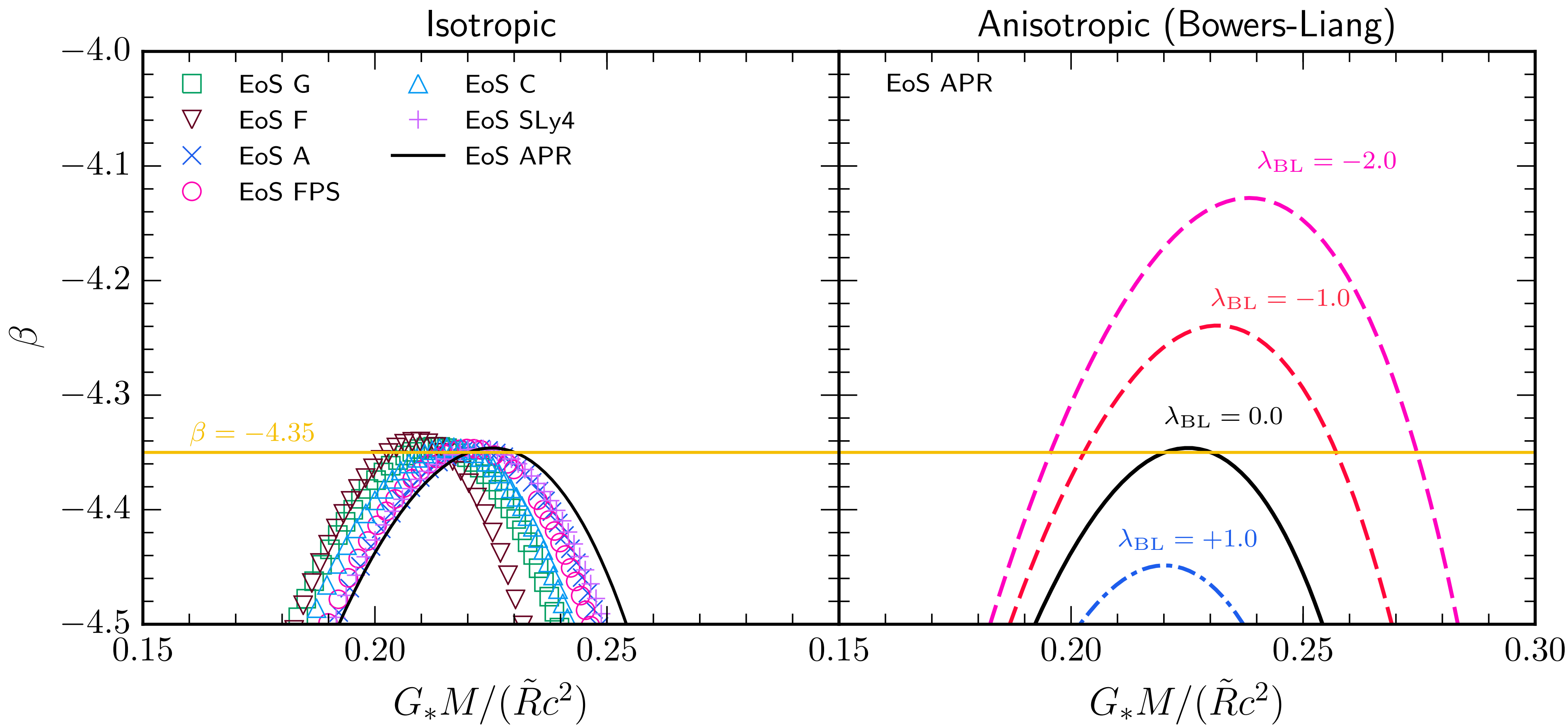
Two models: Horvat+ (left), Bowers-Liang (right)

λ<0: (tangential pressure)>(radial pressure)

[Silva+, 1411.6286]

[Adam+, 1503.03095; Kamiak-Broderick-Afshordi, 1503.03898]

Anisotropy and scalarization threshold

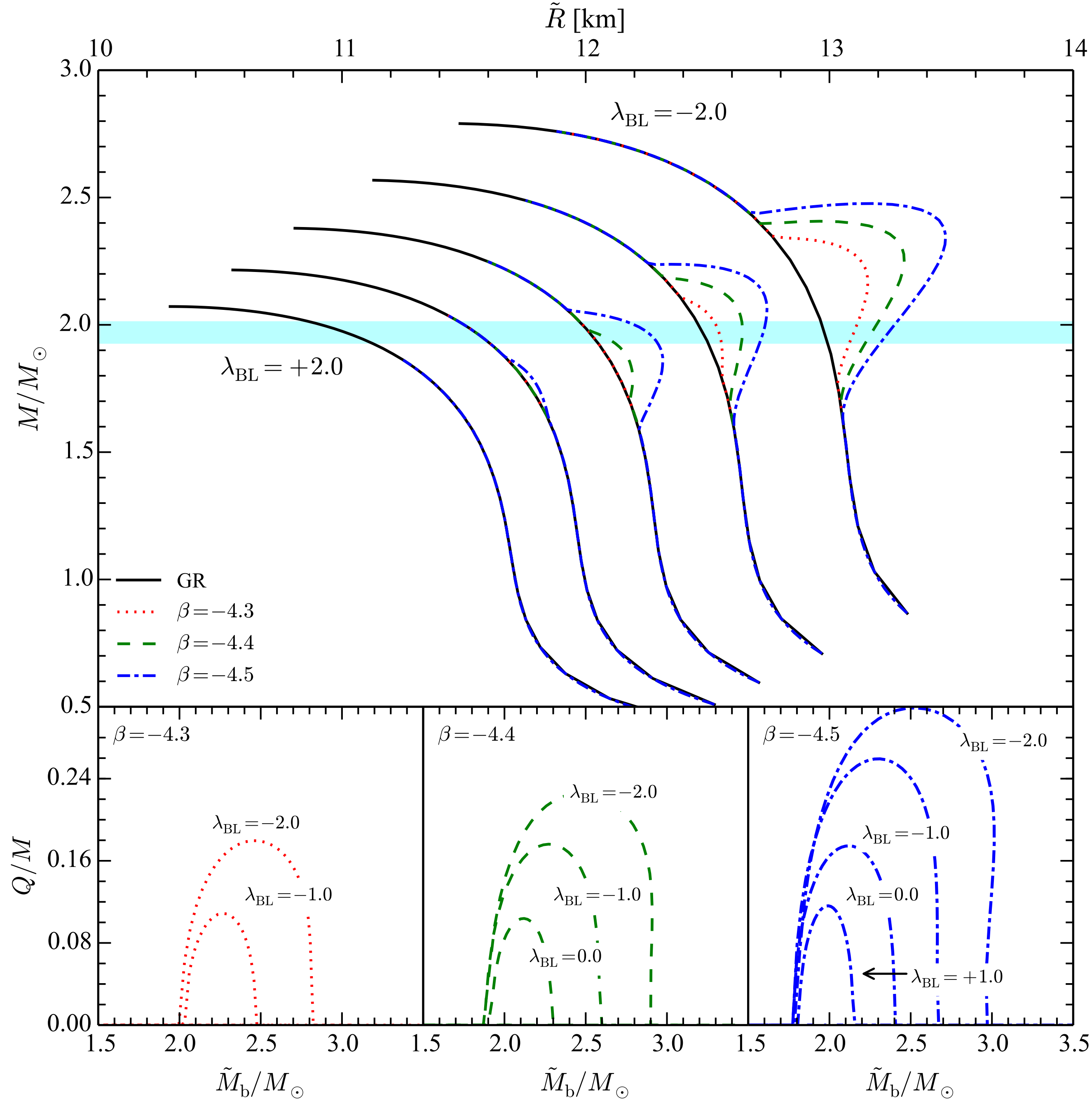


λ = degree of anisotropy

Aside: in the limit λ=-2π the Bowers-Liang model for constant-density stars has R=2M – and the low-order multipole moments also tend to those of Kerr!

[Yagi-Yunes+, 1502.04131]

Anisotropy boosts effects of scalarization



Example #2: tensor-multi-scalar theories

Horbatsch+ 1505.07462

Multiscalarization?

Damour/Esposito-Farese, CQG 9, 2093 (1992)

$$S = \frac{1}{4\pi G_\star}\int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}g^{\mu\nu}\gamma_{AB}(\phi)\partial_\mu\phi^A\,\partial_\nu\phi^B - B(\phi)\right)$$
$$+ S_{\rm m}[A^2(\phi)g_{\mu\nu};\Psi]\,,$$

Two-scalar model:

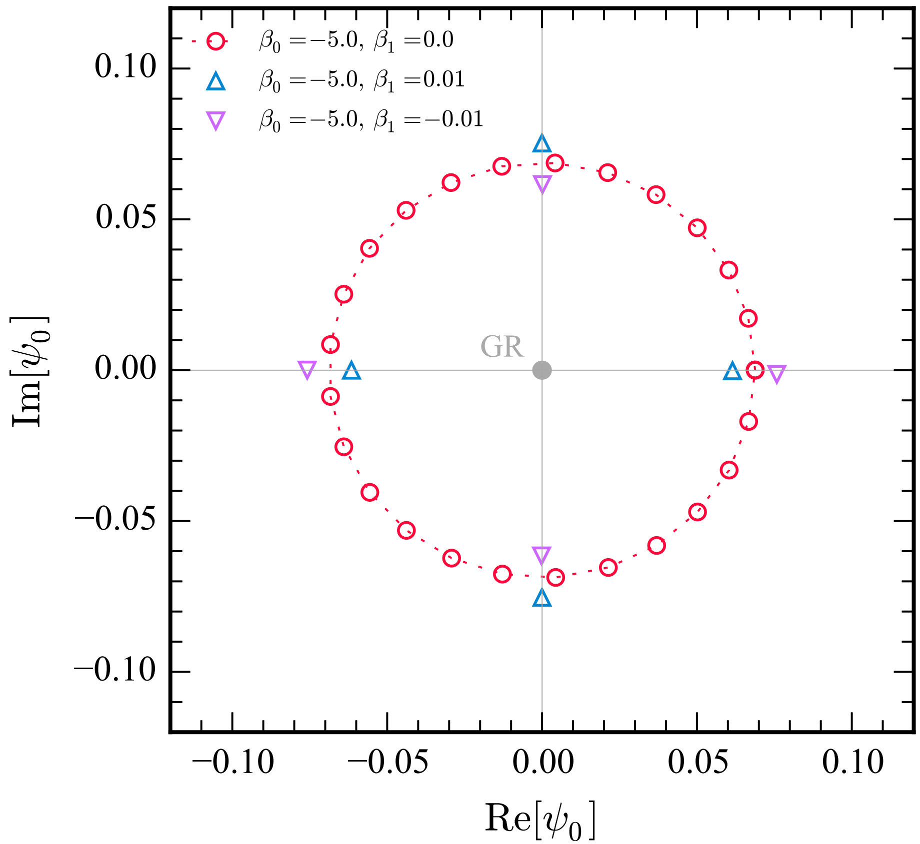
$$S = \frac{1}{4\pi G_\star}\int d^4x\sqrt{-g}\left(\frac{R}{4} - g^{\mu\nu}\gamma(\varphi,\bar\varphi)\nabla_\mu\bar\varphi\,\nabla_\nu\varphi - B(\varphi,\bar\varphi)\right)$$
$$+ S_{\rm m}[A^2(\varphi,\bar\varphi)g_{\mu\nu};\Psi]\,,$$

$$\gamma(\varphi,\bar\varphi) = \frac{1}{2}\left(1 + \frac{\bar\varphi\varphi}{4\mathbf{r}^2}\right)^{-2} \qquad \psi = \varphi e^{\mathrm{i}\theta_1/2}$$

$$\log A(\psi,\bar\psi) = \alpha\psi + \bar\alpha\bar\psi + \frac{1}{2}\beta_0\psi\bar\psi + \frac{1}{4}\beta_1\psi^2 + \frac{1}{4}\beta_1\bar\psi^2 + \ldots\,.$$

α=0: symmetry breaking

$$\log A(\psi, \bar{\psi}) = \alpha\psi + \bar{\alpha}\bar{\psi} + \frac{1}{2}\beta_0\psi\bar{\psi} + \frac{1}{4}\beta_1\psi^2 + \frac{1}{4}\beta_1\bar{\psi}^2 + \ldots .$$

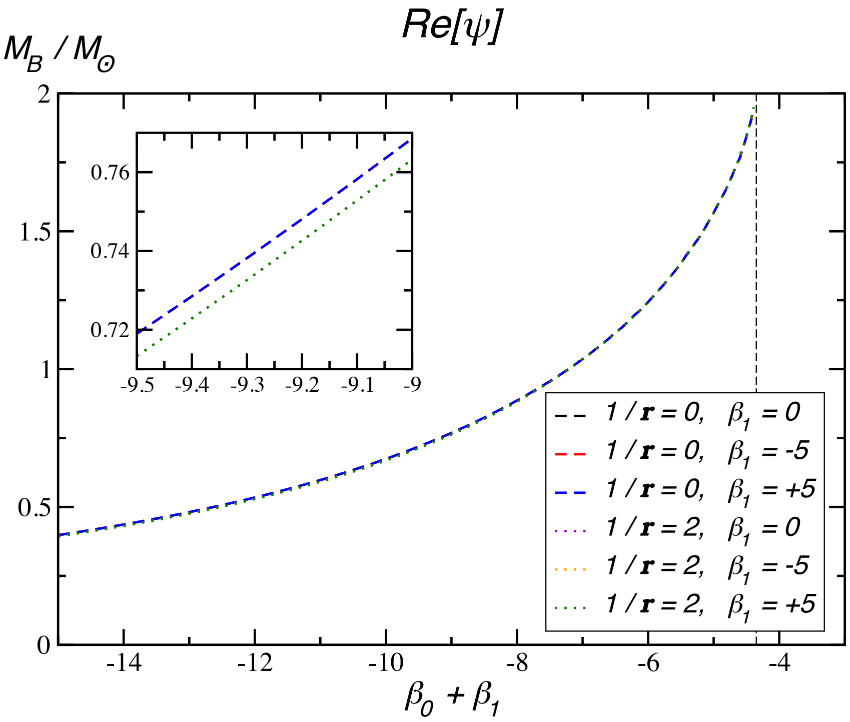


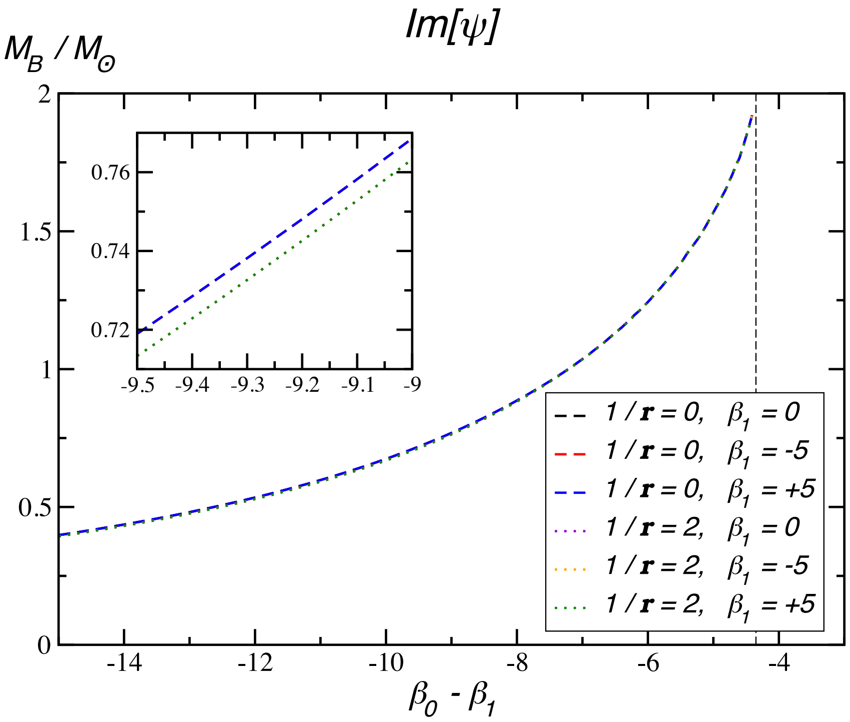
$$A(\psi, \bar{\psi}) = \exp\left(\frac{1}{2}\beta_0\psi\bar{\psi}\right)$$

$$\log A(\psi, \bar{\psi}) = \frac{1}{2}\left[(\beta_0 + \beta_1)\mathrm{Re}[\psi]^2 + (\beta_0 - \beta_1)\mathrm{Im}[\psi]^2\right]$$

“Independent” biscalarization

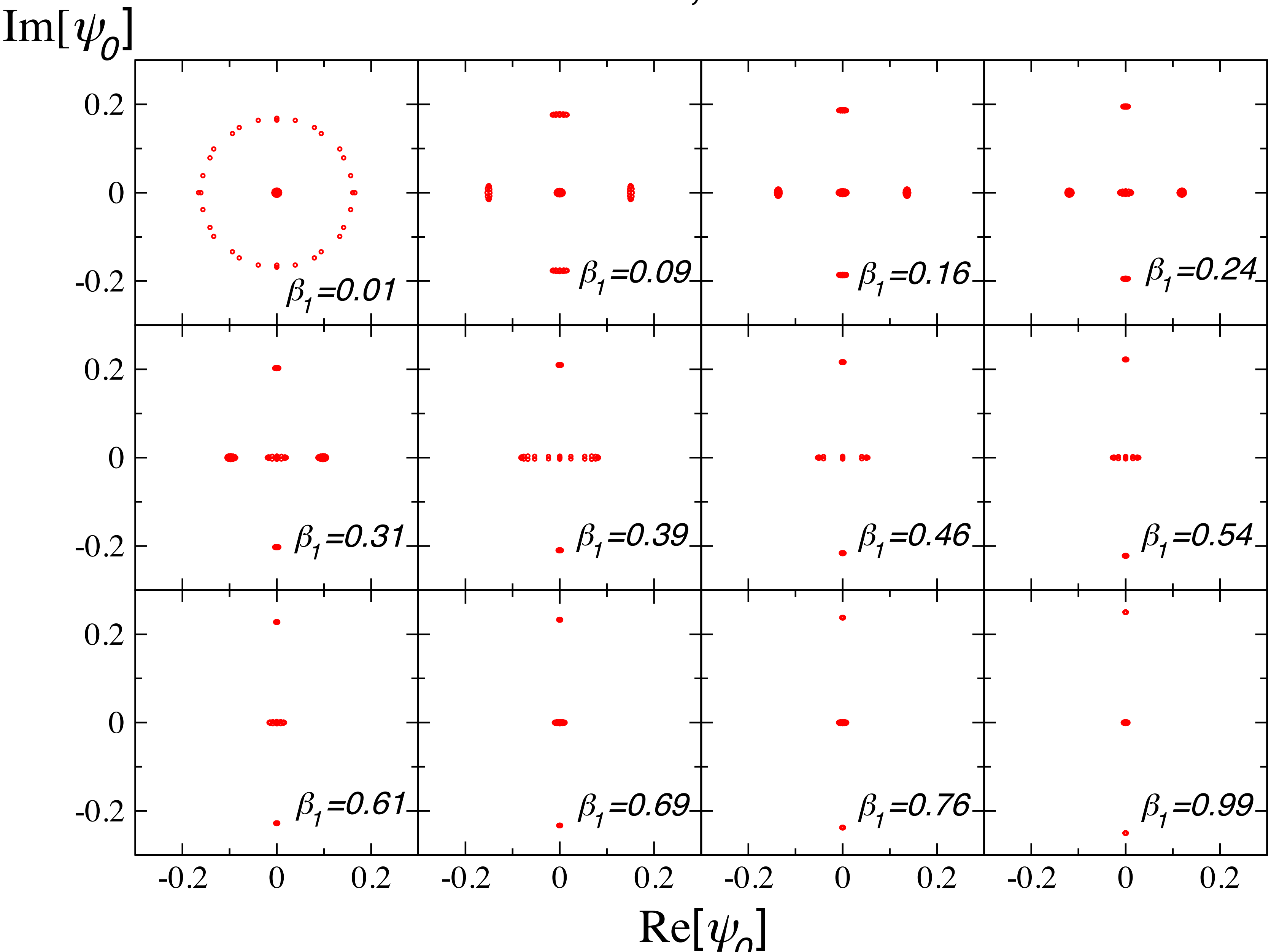
$$\log A(\psi, \bar{\psi}) = \frac{1}{2}\left[(\beta_0 + \beta_1)\mathrm{Re}[\psi]^2 + (\beta_0 - \beta_1)\mathrm{Im}[\psi]^2\right]$$





"True" biscalarization (to be continued…)

$|\alpha| = 0.001, \quad 1/\mathbf{r} = 0$



Example #3: EdGB

Pani+ arXiv:1109.0928

A "theory of theories"

$$\mathcal{L} = f_0(|\phi|)R - \gamma(|\phi|)\partial_a\phi^*\partial^a\phi - V(|\phi|) + f_1(|\phi|)R^2 + f_2(|\phi|)R_{ab}R^{ab} + f_3(|\phi|)R_{abcd}R^{abcd} + f_4(|\phi|)R_{abcd}{}^*R^{abcd} + \mathcal{L}_{\rm mat}[\Psi, A^2(|\phi|)g_{ab}], \quad (2)$$

[Yunes & Stein, 1101.2921]
[Pani+, 1109.0928]

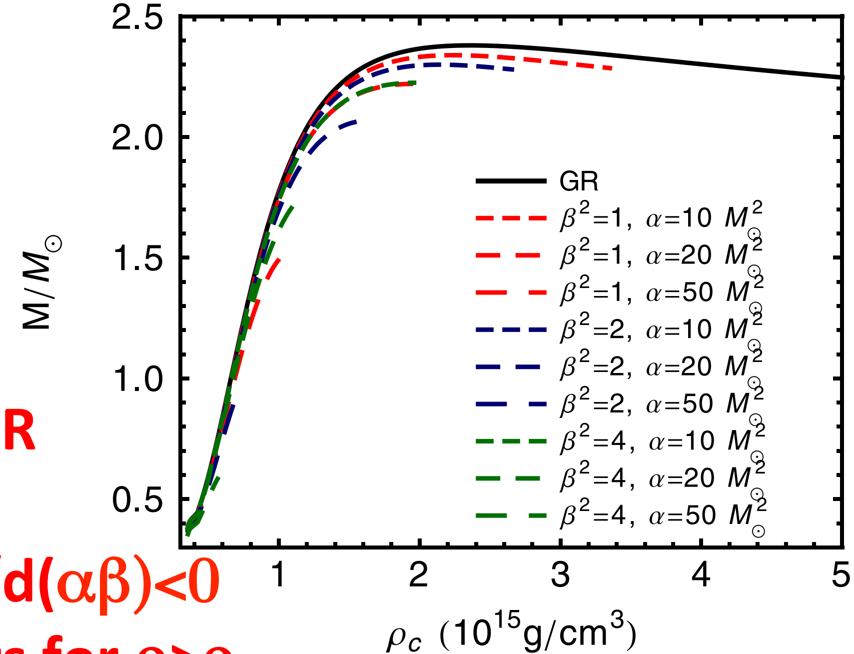
	f_0	f_1	f_2	f_3	f_4	ω	V	γ	A	$\mathcal{L}_{\rm mat}$
General relativity	κ	0	0	0	0	0	0	1	1	perfect fluid
Scalar-tensor (Jordan frame) [24]	$F(\phi)$	0	0	0	0	0	$V(\phi)$	$\gamma(\phi)$	1	perfect fluid
Scalar-tensor (Einstein frame) [23]	κ	0	0	0	0	0	$V(\phi)$	2κ	$A(\phi)$	perfect fluid
$f(R)$ [36]	κ	0	0	0	0	0	$\kappa\frac{Rf_{,R}-f}{16\pi\bar{G}f_{,R}^2}$	2κ	$f_0^{-1/2} = f_{,R}^{-1/2}$	perfect fluid
Quadratic gravity [47]	κ	$\alpha_1\phi$	$\alpha_2\phi$	$\alpha_3\phi$	$\alpha_4\phi$	0	0	1	1	perfect fluid
EDGB [48]	κ	$e^{\beta\phi}$	$-4f_1$	f_1	0	0	0	1	1	perfect fluid
Dynamical Chern-Simons [59]	κ	0	0	0	$\beta\phi$	0	0	1	1	perfect fluid
Boson stars [71]	κ	0	0	0	0	ω	$\frac{m^2}{2}\|\phi\|^2$	1	1	0

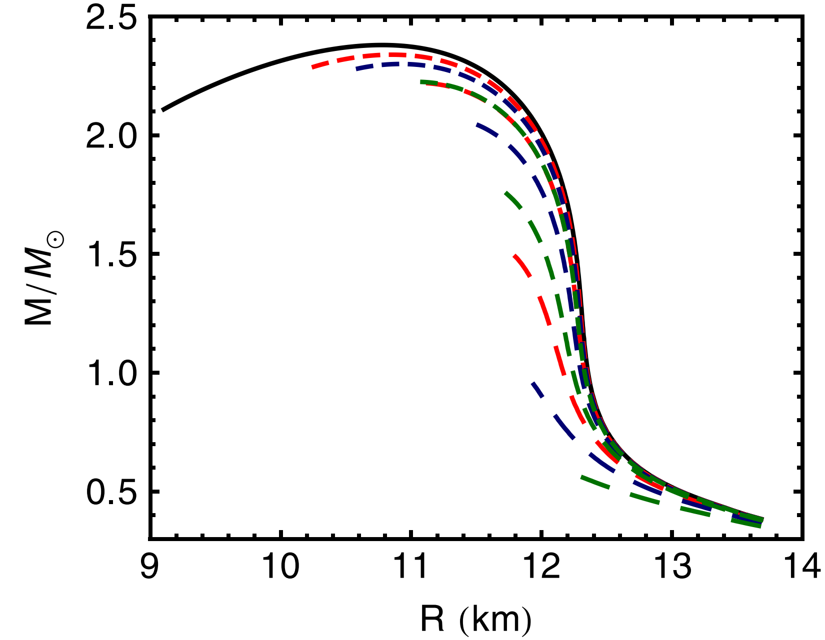
EDGB: $f_1 \equiv \dfrac{\alpha}{16\pi}e^{\beta\Phi}, \qquad 16\pi f_1(\Phi) \sim \alpha + \boxed{\alpha\beta}\Phi$

Set $\alpha > 0$; natural string theory choice is $\beta = \sqrt{2}$

[e.g. Kanti+, hep-th/9511071]

Stellar structure in Einstein-dilaton-Gauss-Bonnet theory

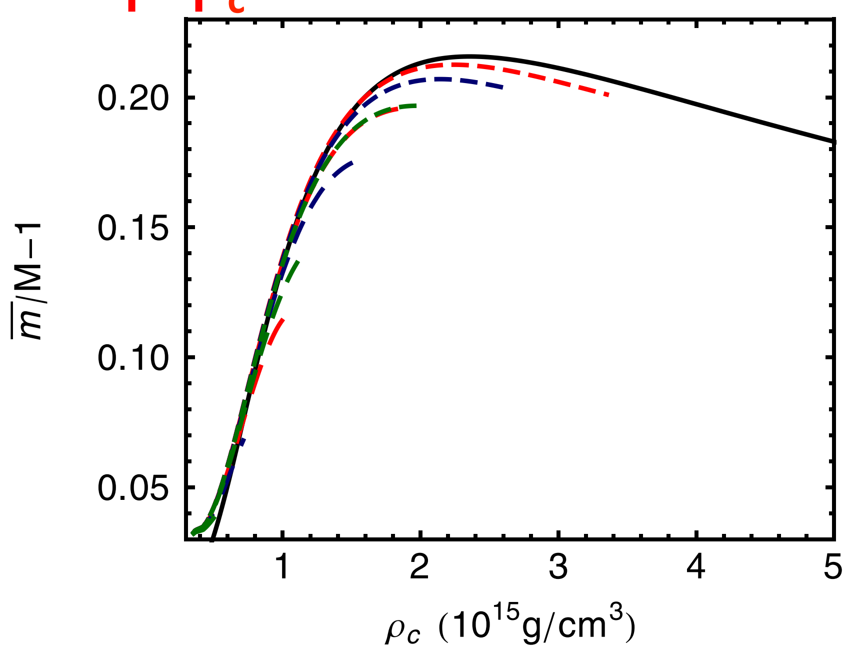


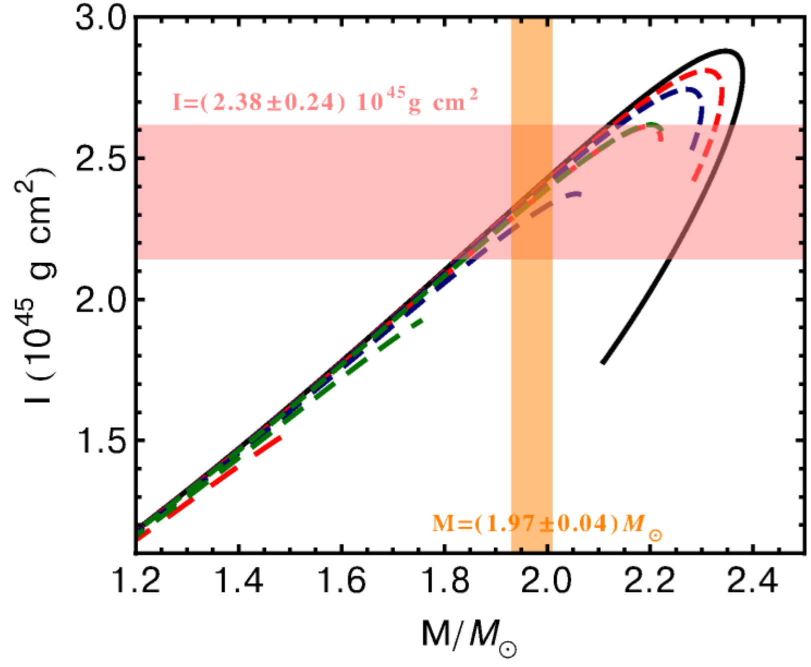


EOS APR

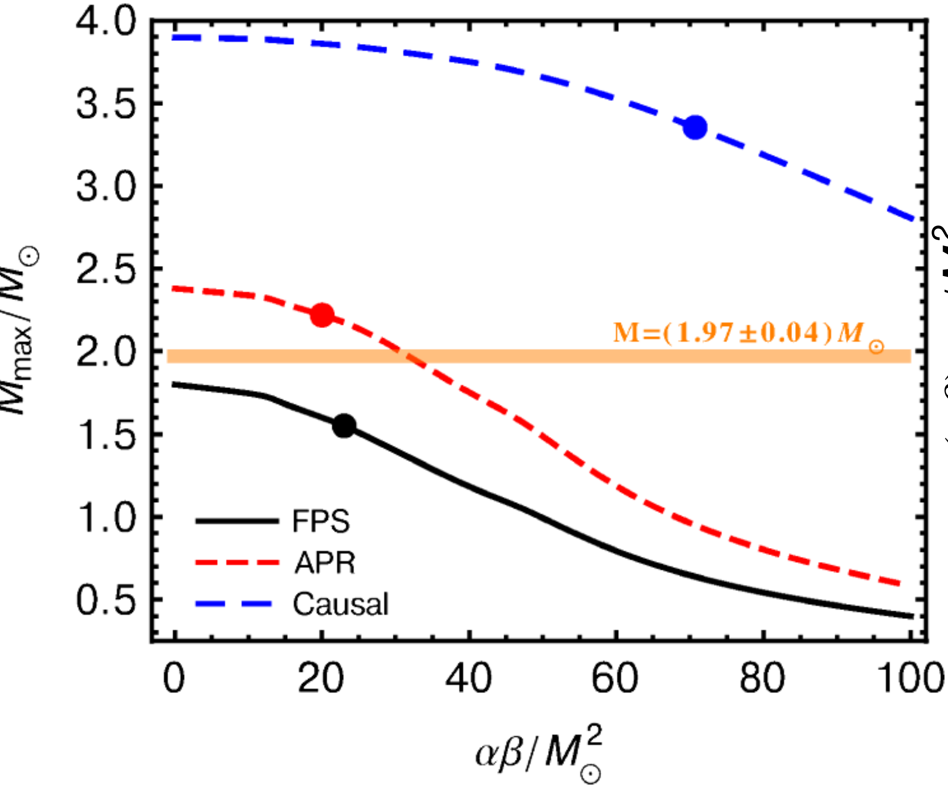
$dM_{max}/d(\alpha\beta)<0$

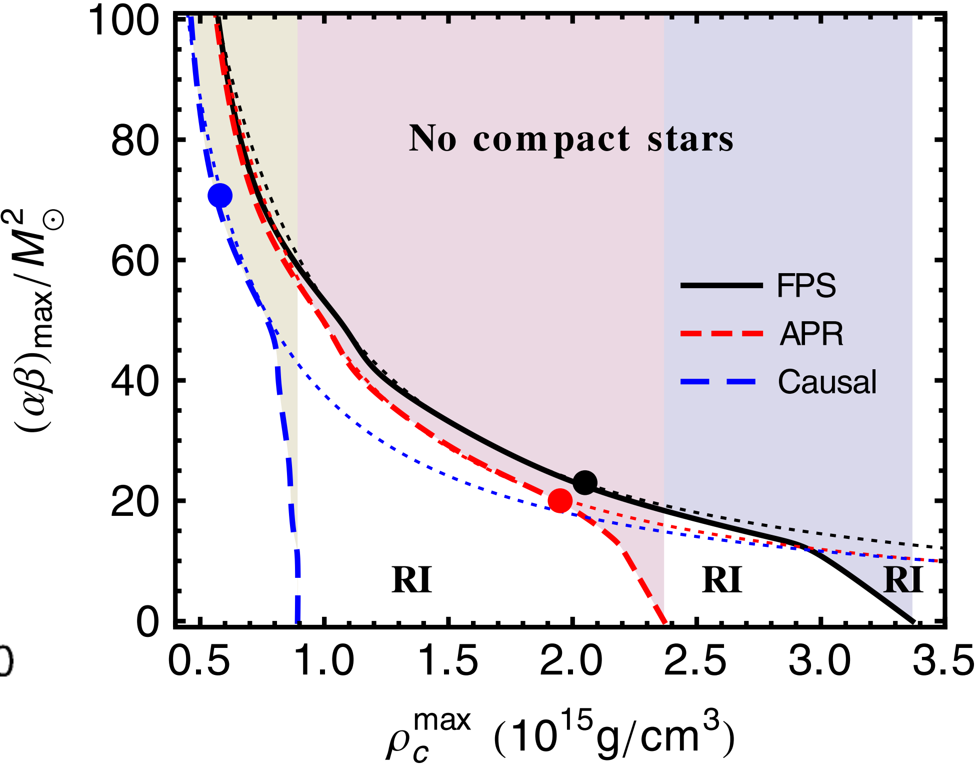
No stars for $\rho>\rho_c$





Constraints on Einstein-dilaton-Gauss-Bonnet couplings





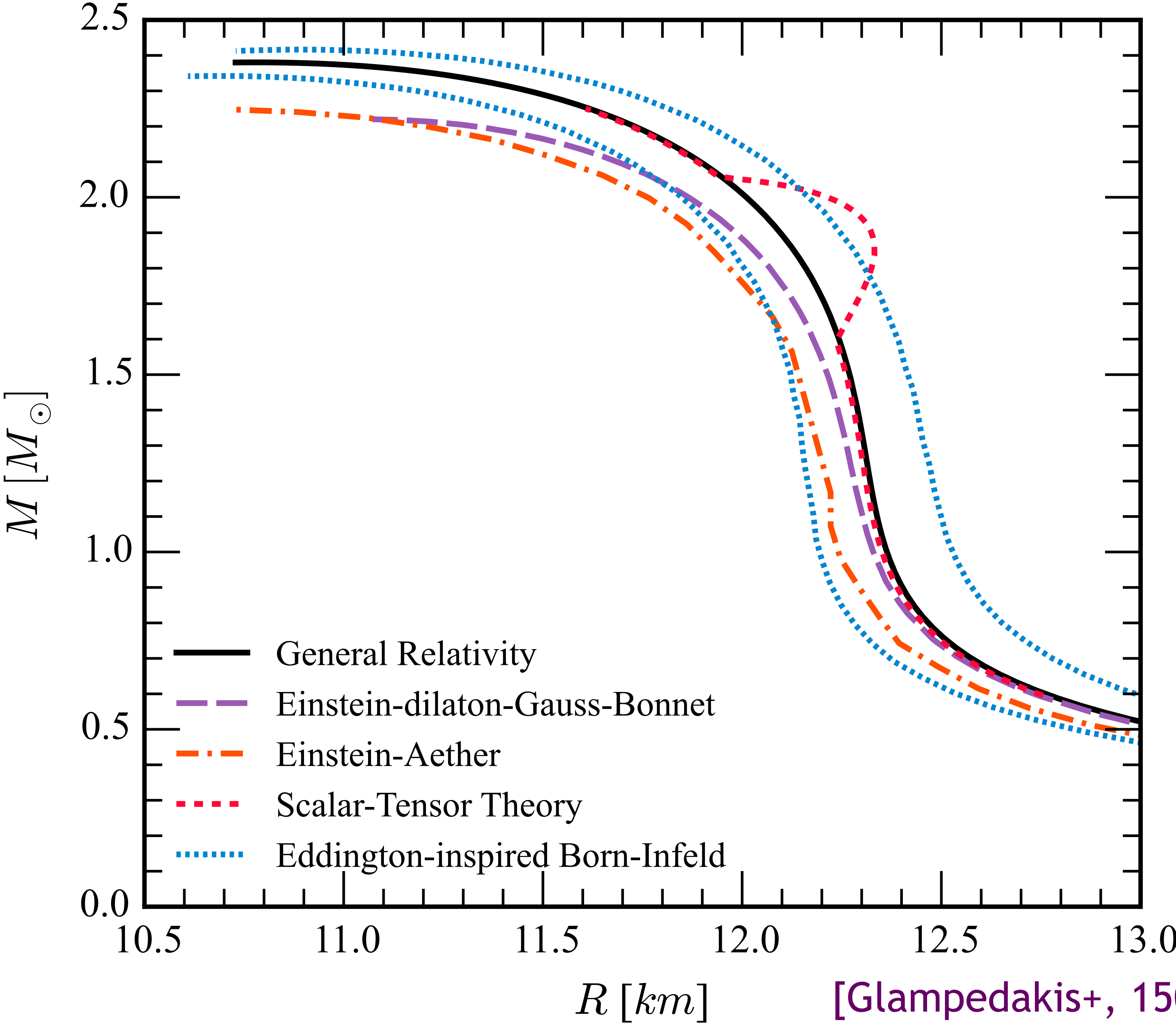
EOS	$M_{max} \gtrsim 1.4M_\odot$	$M_{max} \gtrsim 1.7M_\odot$	$M_{max} \gtrsim 1.93M_\odot$
FPS	$\alpha\beta \lesssim 30.1M_\odot^2$	$\alpha\beta \lesssim 13.9M_\odot^2$	no models
APR	$\alpha\beta \lesssim 50.3M_\odot^2$	$\alpha\beta \lesssim 41.9M_\odot^2$	$\alpha\beta \lesssim 33.6M_\odot^2$

$$\beta = \sqrt{2} \qquad \alpha \lesssim 23.8M_\odot^2 \qquad\qquad \frac{\alpha}{M_\odot^2} \lesssim 70\left[\frac{M_{BH}}{10M_\odot}\right]^2$$

Best bound on α already comes from NSs, not BHs!

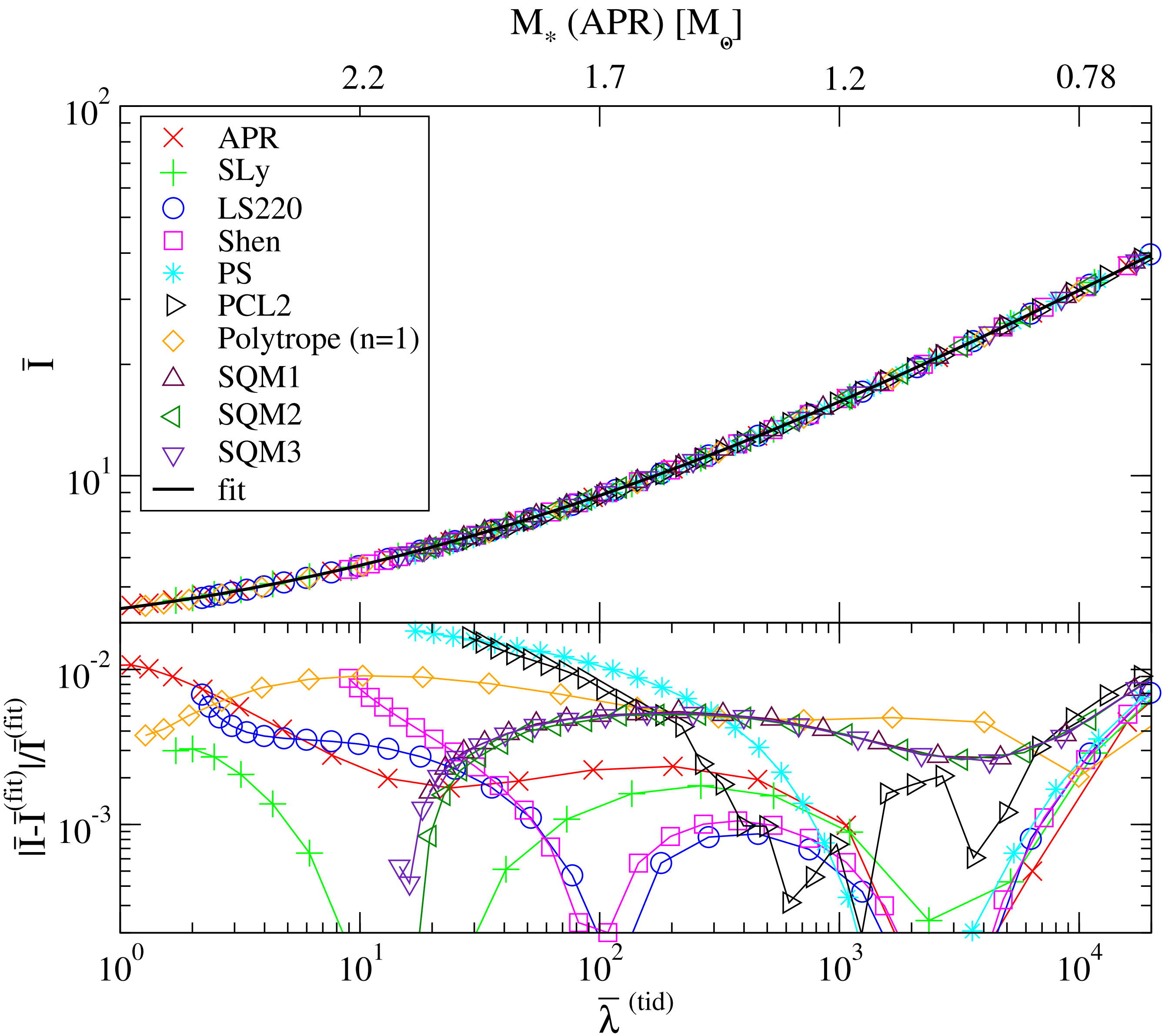
EOS/gravity and gravity theory degeneracies

EOS/gravity theory degeneracy



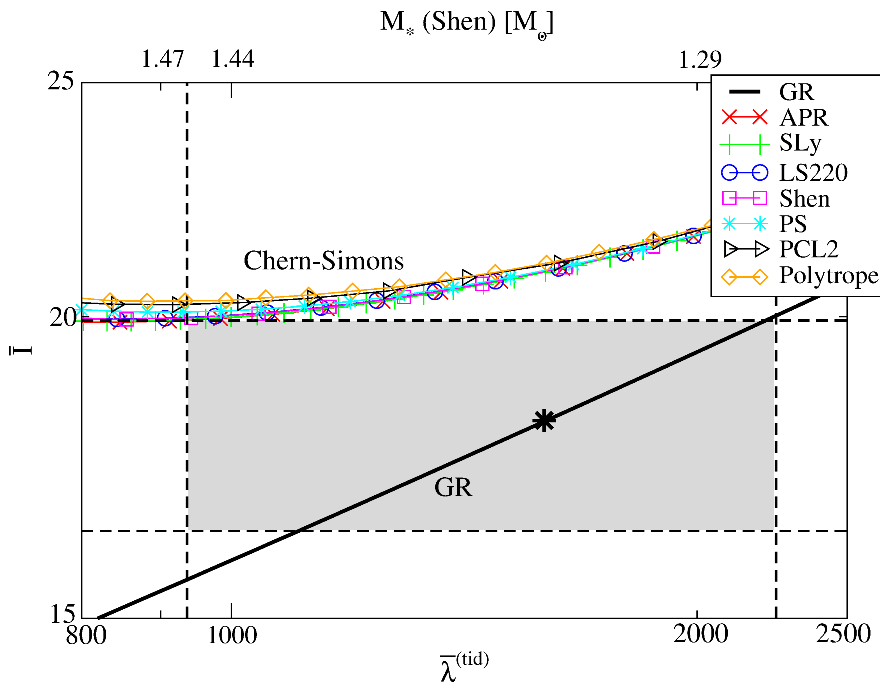
[Glampedakis+, 1504.02455]

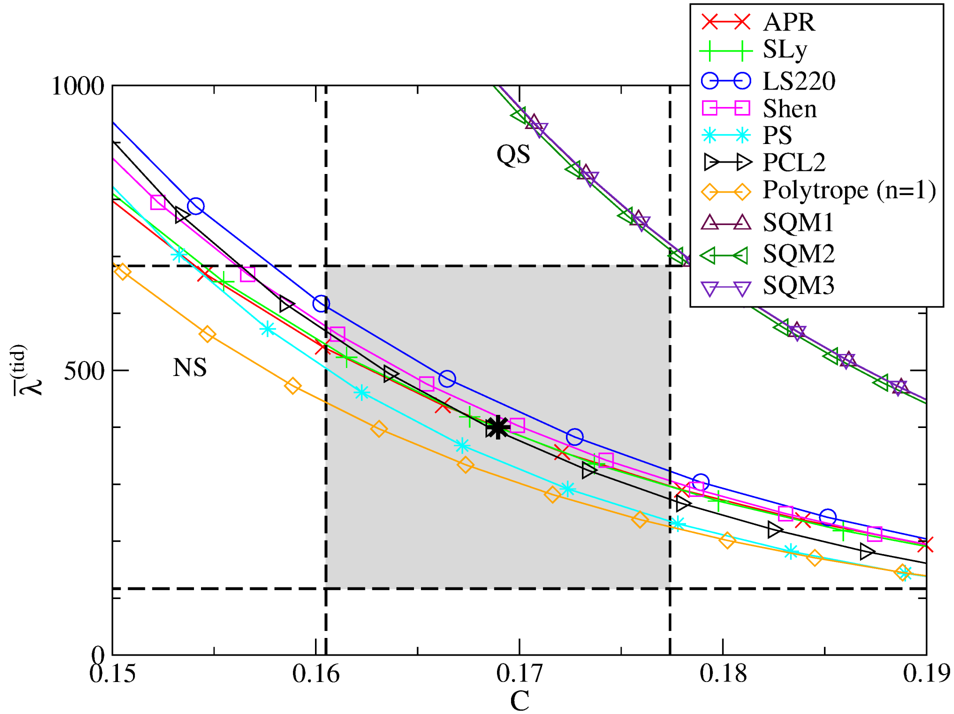
Are we testing the EOS or gravity? Universal relations



I-Love-Q and three-hair relations could help tell theories apart

Are we testing the EOS or gravity? Universal relations





Issues:

In most theories other than dynamical Chern-Simons (scalar-tensor, EdGB, EiBI) **universal relations same as in GR: see e.g. Mojica's talk**

R^2, Lorentz-violating theories: universal relations not studied

Massive gravity, general Horndeski: no studies of stellar structure at all!

All theories in one sweep? post-TOV

The post-TOV formalism

Glampedakis+ 1504.02455

The post-TOV formalism

Main idea: augment the TOV equations by adding 1PN and 2PN terms with arbitrary coefficients built out of the available fluid parameters:

$$p, \ \rho, \ \Pi, \ m, \ r \qquad\qquad \epsilon = \rho(1+\Pi)$$

1PN-order terms follow from the standard PPN expansion:

$$\Lambda_1 \sim \Pi, \ \frac{m}{r}, \ \frac{r^3 p}{m}$$

[Wagoner-Malone 74, Ciufolini-Ruffini 83]

Tightly constrained!

2PN-order terms obtained by dimensional analysis:

$$\Lambda_2 \sim \Pi^\theta (r^2 p)^\alpha (r^2 \rho)^\beta \left(\frac{m}{r}\right)^{2-2\alpha-\beta-\theta}$$

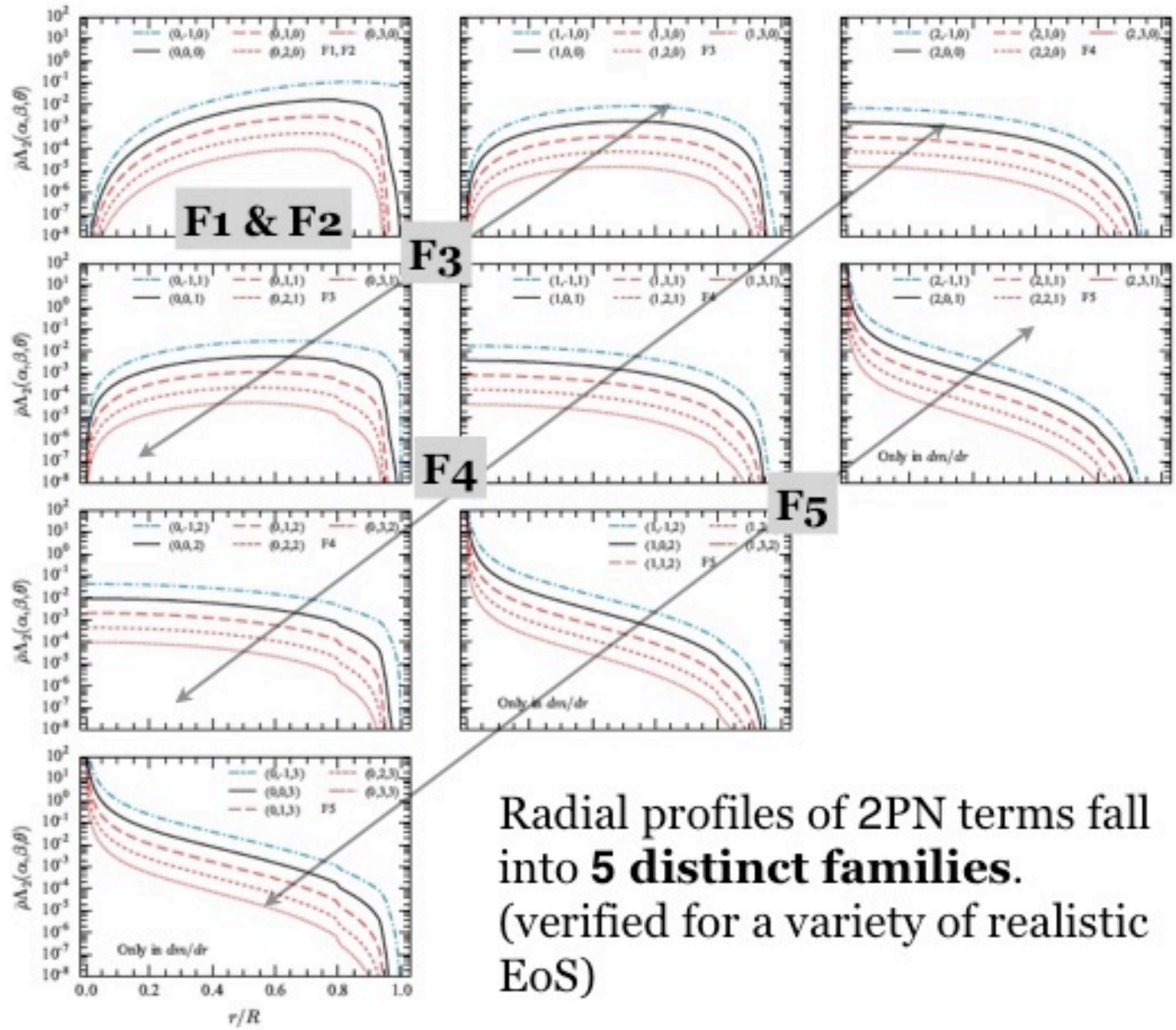
Regularity at surface +

Field equations linear in stress-energy tensor:

$$\begin{array}{rcl} & \beta & \geq -1 \\ 0 \leq & \theta & \leq 2 \text{ or } 3 \\ 0 \leq & \alpha & \leq 2-\theta \text{ or } 3-\theta \end{array}$$

Family ties!

Family	2PN term	(α,β,θ)
F1	$m^3/(r^5\rho)$	$(0,-1,0)$
F2	$(m/r)^2$	$(0,0,0)$
F2	$rm\rho$	$(0,1,0)$
F3	$mp/(r\rho)$	$(1,-1,0)$
F3	r^2p	$(1,0,0)$
F3	$\Pi m^2/(r^4\rho)$	$(0,-1,1)$
F3	$\Pi m/r$	$(0,0,1)$
F3	$r^2\Pi\rho$	$(0,1,1)$
F4	$r^3p^2/(\rho m)$	$(2,-1,0)$
F4	$r^6p^2/(m^2)$	$(2,0,0)$
F4	$\Pi p/\rho$	$(1,-1,1)$
F4	$\Pi r^3p/m$	$(1,0,1)$
F4	$\Pi^2 m/(r^3\rho)$	$(0,-1,2)$
F4	Π^2	$(0,0,2)$
F5	$\Pi r^4p^2/(\rho m^2)$	$(2,-1,1)$
F5	$\Pi r^7p^2/m^3$	$(2,0,1)$
F5	$\Pi^2 rp/m\rho$	$(1,-1,2)$
F5	$\Pi^2 r^4 p/m^2$	$(1,0,2)$
F5	$\Pi^3/(r^2\rho)$	$(0,-1,3)$
F5	$\Pi^3 r/m$	$(0,0,3)$



Radial profiles of 2PN terms fall into **5 distinct families**. (verified for a variety of realistic EoS)

Final "post-TOV" equations

$$\boxed{\begin{aligned} \frac{dp}{dr} &= \left(\frac{dp}{dr}\right)_{\rm GR} - \frac{\rho m}{r^2}\left(\mathcal{P}_1 + \mathcal{P}_2\right) \\ \frac{dm}{dr} &= \left(\frac{dm}{dr}\right)_{\rm GR} + 4\pi r^2 \rho \left(\mathcal{M}_1 + \mathcal{M}_2\right) \end{aligned}}$$

$$\mathcal{P}_1 = \delta_1 \frac{m}{r} + 4\pi \delta_2 \frac{r^3 p}{m}$$

$$|\delta_i| \ll 1 \quad \to \quad |\mathcal{P}_1|, \; |\mathcal{M}_1| \ll 1$$

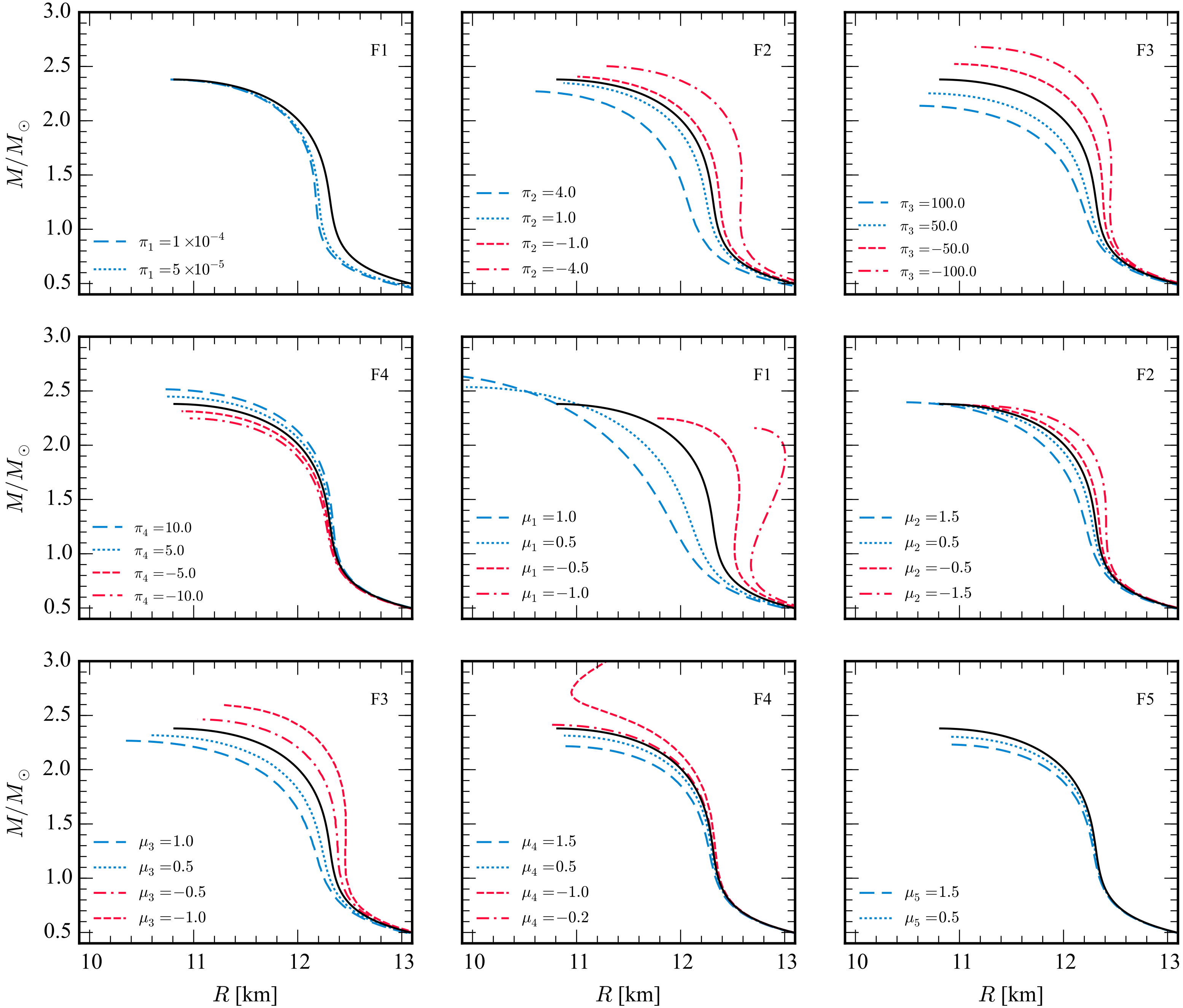
$$\mathcal{M}_1 = \delta_3 \frac{m}{r} + \delta_4 \, \Pi$$

$$\mathcal{P}_2 = \pi_1 \frac{m^3}{r^5 \rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 \, r^2 p + \pi_4 \frac{\Pi p}{\rho}$$

$$\mathcal{M}_2 = \mu_1 \frac{m^3}{r^5 \rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 \, r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \, \Pi^3 \frac{r}{m}$$

[Glampedakis+, 1504.02455]

Gravity-theory degeneracy and the “post-TOV” expansion



Post-TOV effective metric

The post-TOV equations have an effective GR-like formulation:

$$\nabla_\nu T^{\mu\nu}_{\rm eff} = 0, \qquad T^{\mu\nu}_{\rm eff} = (\epsilon_{\rm eff} + p) u^\mu u^\nu + p g^{\mu\nu}$$

$$\frac{dp}{dr} = -\frac{1}{2}(\epsilon_{\rm eff} + p)\frac{d\nu}{dr} \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon_{\rm eff}$$

with a gravity-modified effective EOS

$$p = p(\epsilon_{\rm eff}), \quad \epsilon_{\rm eff} = \epsilon + \rho \mathcal{M}_2$$

and an effective interior metric

$$g_{\mu\nu} = {\rm diag}[\, e^{\nu(r)}, (1 - 2m(r)/r)^{-1}, r^2, r^2 \sin^2\theta \,]$$

[Glampedakis+, 1504.02455]

Summary

Neutron stars: possibly best astrophysical laboratory for strong gravity:

- **large curvatures**
- tests of **gravity/matter coupling**

"Theory of theories": terms of order R and R^2 – only few well motivated

- scalar-tensor
  spontaneous scalarization unlikely, observables must be close to GR
  (**but** anisotropy? dynamical scalarization?)
- tensor-multi-scalar
  multiscalarization? AdLIGO signatures invisible to binary pulsars?
- EdGB
  strong theory constraints from a single 2 solar mass measurement!

Issues: EOS/gravity degeneracy, gravity theory degeneracy

A theory-agnostic **post-TOV formalism**

to do: M(R) curves, redshift, cooling, surface emission from bursters
Slow rotation, universal relations, break degeneracies?