Hairy black holes in scalar tensor theories

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Lisbon: One hundred years of strong gravity

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973. Beyond Horndeski seems also a possibility...
- contain or are limits of other modified gravity theories. $f(R)$, massive gravity etc.
- (Can) have insightful screening mechanisms (Chameleon, Vainshtein)
- Include terms that can screen classically a big cosmological constant (Fab 4 [CC, Copeland, Padilla and Saffin 2012])

Jordan-Brans-Dicke theory [review by Sotiriou 2014]

Simplest scalar tensor theory

$$S_{\rm BD} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\varphi R - \frac{\omega_0}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - m^2(\varphi-\varphi_0)^2\right) + S_m(g_{\mu\nu},\psi)$$

- ω_0 Brans Dicke coupling parameter fixing scalar strength
- $\phi = \phi_0$ constant gives GR solutions (with a cosmological constant) but spherically symmetric solutions are not unique!
- For spherical symmetry we find,

$$\gamma \equiv \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{2\omega_0 + 3 - exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}{2\omega_0 + 3 + exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}$$

- where $\gamma = 1 + (2.1 \pm 2.3)\times 10^{-5}$
- $\omega_0 > 40000$... Need a more complex version in order to screen the scalar mode locally

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973], [Deffayet et.al.]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = K(\phi, X),$$
$$L_3 = -G_3(\phi, X)\Box\phi,$$
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]
- Theory screens generically scalar mode locally by the Vainshtein mechanism.

Horndeski theory includes,

- R, $f(R)$ theories, Brans Dicke theory with arbitrary potential
- Scalar-tensor interaction terms: $G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, $P^{\mu\rho\nu\sigma}\nabla_\mu\nabla_\nu\phi\nabla_\rho\phi\nabla_\sigma\phi$, $V(\phi)\hat{G}$ (Fab 4)
- higher order Galileons : $\Box\phi(\nabla\phi)^2(DGP), (\nabla\phi)^4$ (ghost condensate)
- Higher order terms originate form KK reduction of Lovelock theory ([Van Acoleyen et.al. arXiv:1102.0487 [gr-qc]], [CC, Goutéraux and Kiritsis])
- Gallileons in flat spacetime have Gallilean symmetry [Nicolis et.al.: arXiv:0811.2197 [hep-th]]
- Horndeski theories appear at "decoupling limit" of DGP and massive gravity theories

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What about black holes in scalar-tensor theories?

Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...
For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

non minimally coupled scalars and static spacetimes [Babichev and CC], Gauss-Bonnet term [Sotiriou and Zhou] minimally coupled complex scalar and stationary spacetimes [Herdeiro and Radu]

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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi] = \int_{\mathcal{M}} \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right)\mathrm{d}^4x + S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
  The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

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- Static and spherically symmetric solution

$$\mathrm{d}s^2 = -\left(1-\frac{m}{r}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}}$$

- Geometry is that of an extremal RN.
  Problem:The scalar field is **unbounded** at $(r=m)$.
- Controversy on the stability [Bronnikov et al.-78, McFadden et al.-05]
  Not clear that the solution is a black hole.

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Scalar-tensor theories and black holes

- In scalar tensor theories "regular" black hole solutions are GR black holes with a constant scalar field
- Is it possible to have non-trivial and regular scalar-tensor black holes for an asymptotically flat or $\Lambda > 0$ space-time?
- How can we evade no-hair theorems?
- We will consider:
  - Higher order gravity theory
  - Translational symmetry for the scalar
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An integrability theorem and no-hair, [Babichev, CC and Hassaine]

Consider $L = L(g_{\mu\nu}, \nabla\phi, \nabla\nabla\phi) \subset L_H$,

- theory has shift symmetry in $\phi \to \phi + c$
  $\mathcal{E}_{(\phi)} = \nabla_\mu J^\mu = 0$, J^μ is a conserved current associated to the symmetry
- Suppose now a static and spherically symmetric spacetime,
  $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- and $\phi = qt + \psi(r)$.
  Galileon does not acquire the symmetries of spacetime. Are the EoM compatible?

Under these hypotheses:

$-qJ_r = \mathcal{E}_{tr} g^{rr}$ where $\mathcal{E}_{tr}$ is the tr-metric equation

No time derivatives present in the field equations

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Example theory

Consider the action,

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- Metric field equations read,

$$\zeta G_{\mu\nu} - \eta\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right) + g_{\mu\nu}\Lambda$$
$$+\frac{\beta}{2}\left((\nabla\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi\right.$$
$$\left.+g_{\mu\alpha}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0,$$

- Scalar field has translational invariance :$\phi \to \phi$+const.,
- Scalar field equation,

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\partial_\nu\phi.$$

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- but current is singular $J^2 = J^\mu J^\nu g_{\mu\nu} = (J^r)^2 g_{rr}$ unless $J^r = 0$ at the horizon...
  Generically $\phi = constant$ everywhere [Hui and Nicolis] and we have again the appearance of a no-hair theorem...
- But for a higher order theory $J^r = 0$ does not neccesarily imply $\phi = const.$

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Scalar field equation

- Hypotheses: $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- $\beta G^{rr} - \eta g^{rr} = 0$ and $\phi(t,r) = q\,t + \psi(r)$,
- Geometric constraint, $f = \frac{(\beta+\eta r^2)h}{\beta(rh)'}$, fixing spherically symmetric gauge.
- no scalar charge, current ok, $\phi' \neq 0$, and (tr)-eq satisfied
- Unknowns $\psi(r)$ and $h(r)$ and have two ODE's to solve, the (rr) and (tt). Hence hypotheses are consistent.
- The system is integrable for spherical symmetry boiling down to a single second order non-linear ODE for an arbitrary Shift symmetric theory!

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- Unknowns $\psi(r)$ and $h(r)$ and have two ODE's to solve, the (rr) and (tt). Hence hypotheses are consistent.
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Solving the remaining EoM

- From (rr)-component get ψ'

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta+\eta r^2)} \left(q^2 \beta(\beta+\eta r^2) h' - \frac{\zeta\eta + \beta\Lambda}{2} (h^2 r^2)' \right)^{1/2}.$$

- and finally (tt)-component gives $h(r)$ via,

$$h(r) = -\frac{\mu}{r} + \frac{1}{r}\int \frac{k(r)}{\beta+\eta r^2} dr,$$

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$$q^2\beta(\beta+\eta r^2)^2 - \left(2\zeta\beta + (2\zeta\eta - \lambda) r^2\right) k + C_0 k^{3/2} = 0,$$

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Fab 4 limit: $\Lambda = 0$, $\eta = 0$

- Consider $S = \int d^4x \sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- $G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = \nabla_\mu\left(G^{\mu\nu}\nabla_\nu\phi\right) = 0$
- in Eq of scalar $\beta G^{\mu\nu}$ $\rightarrow$ Einstein equation
- $G^{rr} = 0 \rightarrow f = \frac{h}{(rh)'}$ and $\phi(t,r) = q\,t + \psi(r)$
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Schwarzschild geometry with a non-trivial scalar field. But is the scalar regular on the horizon?

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Scalar-tensor Schwarzschild black hole

- $\phi_{\pm} = qt \pm q\mu \left[2\sqrt{\frac{r}{\mu}} + \log \frac{\sqrt{r}-\sqrt{\mu}}{\sqrt{r}+\sqrt{\mu}} \right] + \phi_0$
- Scalar looks singular for $r \to r_h$ but $t_h \to \infty$!
- Consider $v = t + \int (fh)^{-1/2} dr$ then $ds^2 = -hdv^2 + 2\sqrt{h/f}\, dvdr + r^2 d\Omega^2$ Regular chart for horizon, EF coordinates
- $\phi_+ = q \left[v - r + 2\sqrt{\mu r} - 2\mu \log \left(\sqrt{\frac{r}{\mu}} + 1 \right) \right] + \text{const}$
- Scalar regular at future black hole horizon!
- Metric is Schwarzschild, scalar is regular and non-trivial
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de Sitter black hole

- Consider $S=\int d^4x\sqrt{-g}\left[\zeta R-2\Lambda-\eta\left(\partial\phi\right)^2+\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$
- de Sitter asymptotics with $q^2=(\zeta\eta+\beta\Lambda)/(\beta\eta)$ and $C_0=(\zeta\eta-\beta\Lambda)\sqrt{\beta}/\eta$
- $f=h=1-\frac{\mu}{r}+\frac{\eta}{3\beta}r^2$ de Sitter Schwarzschild! with
- $\psi'=\pm\frac{q}{h}\sqrt{1-h}$ and $\phi(t,r)=q\,t+\psi(r)$
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- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- $q^2\eta=\Lambda-\Lambda_{eff}>0$
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- where Λ_{eff} is a geometric acceleration
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Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi]=\int_{\mathcal{M}}\sqrt{-g}\left(\frac{R}{16\pi G}-\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi-\frac{1}{12}R\phi^2\right)\mathrm{d}^4x+S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta}\mapsto\tilde{g}_{\alpha\beta}=\Omega^2 g_{\alpha\beta} \\ \phi\mapsto\tilde{\phi}=\Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
  The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

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BBMB completion [CC, Kolyvaris, Papantonopoulos and Tsoukalas]

- We would like to combine the above properties in order to obtain a hairy black hole.
- Consider the following action, $S(g_{\mu\nu}, \phi, \psi) = S_0 + S_1$ where
- $$S_0 = \int dx^4 \sqrt{-g} \left[\zeta R + \eta \left(-\frac{1}{2} (\partial \phi)^2 - \frac{1}{12} \phi^2 R \right) \right]$$
  and
- $$S_1 = \int dx^4 \sqrt{-g} \left(\beta G_{\mu\nu} \nabla^\mu \Psi \nabla^\nu \Psi - \gamma T^{BBMB}_{\mu\nu} \nabla^\mu \Psi \nabla^\nu \Psi \right) ,$$
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  $$T^{BBMB}_{\mu\nu} = \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{4} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi + \frac{1}{12} \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu + G_{\mu\nu} \right) \phi^2 .$$
- Scalar field equation of S_1 contains metric equation of S_0.
  $$\nabla_\mu J^\mu = 0 , \qquad J^\mu = \left(\beta G_{\mu\nu} - \gamma T^{BBMB}_{\mu\nu} \right) \nabla^\nu \Psi .$$

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Black hole with primary hair

- Solve as before assuming linear time dependence for Ψ
- Scalar ϕ has an additional branch regular at the "horizon"
- A second solution reads,

$$h(r) = 1 - \frac{m}{r}, \qquad f(r) = (1 - \frac{m}{r})\left(1 - \frac{\gamma c_0^2}{12\beta r^2}\right)$$

$$\phi(r) = \frac{c_0}{r},$$

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- Scalar ϕ has an additional branch regular at the "horizon"

- A second solution reads,

$$h(r) = 1 - \frac{m}{r}, \qquad f(r) = (1 - \frac{m}{r})\left(1 - \frac{\gamma c_0^2}{12\beta r^2}\right)$$

$$\phi(r) = \frac{c_0}{r} \ ,$$

$$\psi = qv - q\int \frac{dr}{\sqrt{\left(1 - \frac{\gamma c_0^2}{12\beta r^2}\right)}(1 \mp \sqrt{\frac{m}{r}})} \ .$$

Conclusions

- Hairy black holes: non minimally coupled scalars and static spacetimes [Babichev and CC]
  minimally coupled complex scalar and stationary spacetimes [Herdeiro and Radu]: in both cases scalars have not the same symmetry as spacetime
- For a theory with Shift symmetry and higher order terms
- Scalar field with linear time dependence: EoM compatible. System is integrable
- Time dependence essential for regularity on the event horizon
- Higher order terms essential for novel branches of black holes
- Method can be applied in differing Gallileon context [Kobayashi and Tanahashi], in higher dimensions, including gauge fields.
- Is there a way to find observable for q? Is there a distinction possible?
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sectionAdding a $U(1)$ gauge field-EM

Adding electromagnetic charge: $U(1)$ gauge field

Following the same idea we can add an EM field

$$I[g_{\mu\nu},\phi,A_\mu]=\int\sqrt{-g}d^4x\ \Big[R-2\Lambda-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+\beta\,G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi-\eta\,(\partial\phi)^2-\gamma\,T_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi\Big]\,,$$

where we have defined

$$T^{EM}_{\mu\nu}:=\frac{1}{2}\left[F_{\mu\sigma}F_\nu{}^\sigma-\frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right].$$

The gauge field couples to Gallileon. We have a conserved current as before

$$\nabla_\mu J^\mu=\nabla_\mu\left[(\beta\,G^{\mu\nu}-\eta\,g^{\mu\nu}-\gamma\,T^{\mu\nu}_{EM})\,\nabla_\nu\phi\right]=0,$$

Adding "electromagnetic charge"

We consider spherical symmetric with a dyonic gauge field,

$$ds^2 = -h(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2(\theta) d\varphi^2\right), \qquad \phi(t,r) = \psi(r) + q\,t, \qquad A_\mu dx^\mu = A(r)dt - P\cos(\theta)d\varphi$$

We define an auxiliary function S,

$$S(r) = \frac{\beta(rh(r))' + \frac{\gamma}{4}r^2F^2}{\eta r^2 + \beta - \frac{\gamma P^2}{4r^2}}$$

where F is the electric field strength and the EOM reduce to,

$$\beta\left[q^2\beta - \frac{r^2}{4\beta}(\gamma-\beta)F^2\right] - S(r)\left[(\eta - \beta\Lambda)\,r^2 + 2\,\beta - \frac{1}{4r^2}P^2(\beta+\gamma)\right] + C_0 S(r)^{3/2}\left[\eta r^2 + \beta - \frac{\gamma P^2}{4r^2}\right] = 0,$$

$$\sqrt{\frac{f}{h}}\,r^2\,F\,\left[1 + \frac{\gamma}{2}\left(f\,(\psi')^2 - \frac{q^2}{h}\right)\right] = Q$$

Example: Self tuning RN solution

$$h(r) = 1 - \frac{\mu}{r} + \frac{\eta\, r^2}{3\,\beta} + \frac{\gamma\,\left(Q^2 + P^2\right)}{4\,\beta\, r^2},$$

$$(\psi'(r))^2 = \frac{1 - f(r)}{f(r)^2} q^2,$$

$$F_{tr} = F(r) = \frac{Q}{r^2}, \quad F_{\theta\varphi} = C(\theta) = P\,\sin(\theta).$$

The coupling constants, the constants of integration and q are related as

$$P^2 \beta\,(\Lambda\gamma + \eta) = Q^2 \eta\,(\gamma - \beta), \qquad q^2 = \frac{\eta + \Lambda\beta}{\beta\,\eta}, \qquad C_0 = \frac{1}{\eta}(\eta - \beta\Lambda).$$