Binary black-hole spin precession: a tale of three timescales

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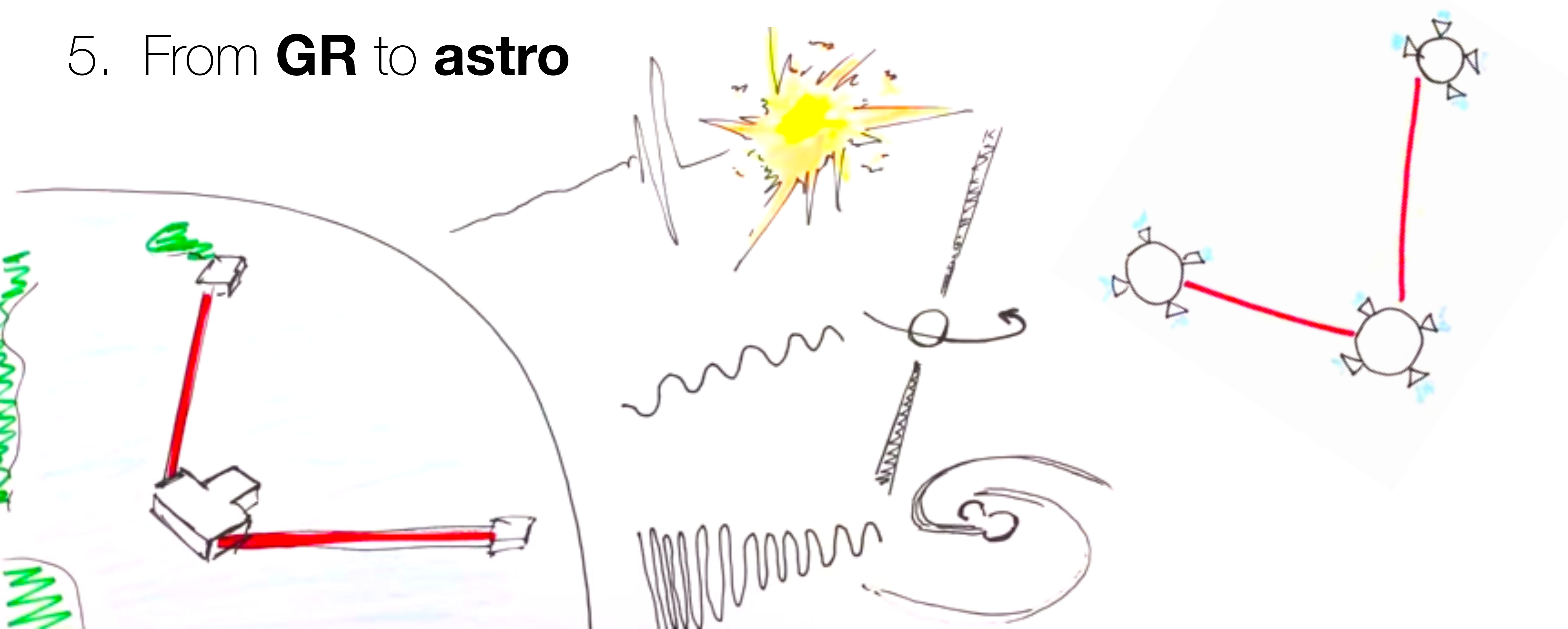
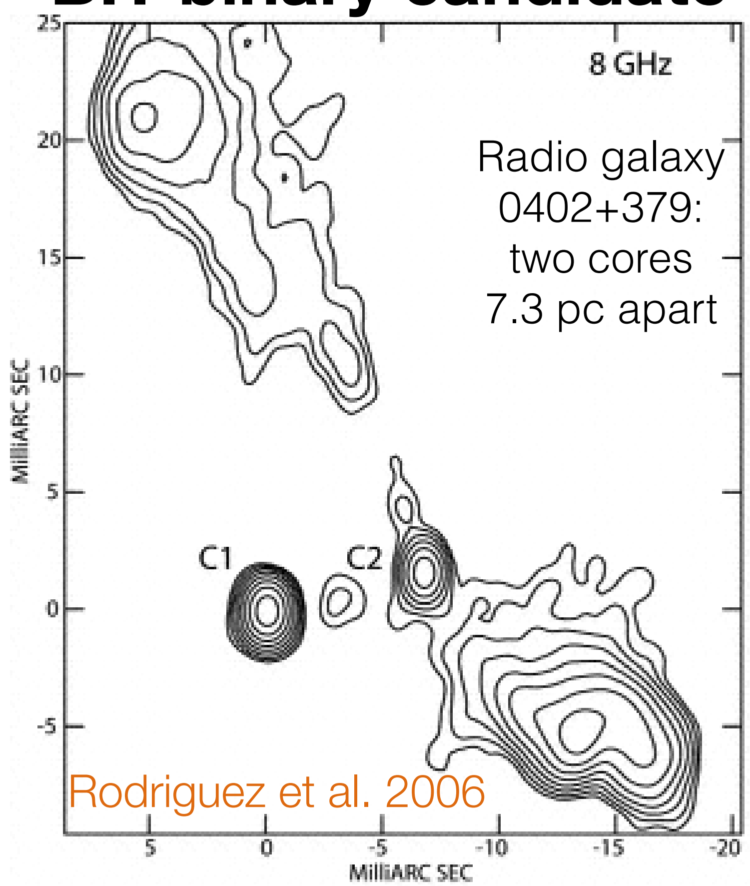
Outline

1. A timescale **hierarchy**
2. Analytical **precession**
3. Quasi-adiabatic **inspiral**
4. Morphological **phase transitions**
5. From **GR** to **astro**



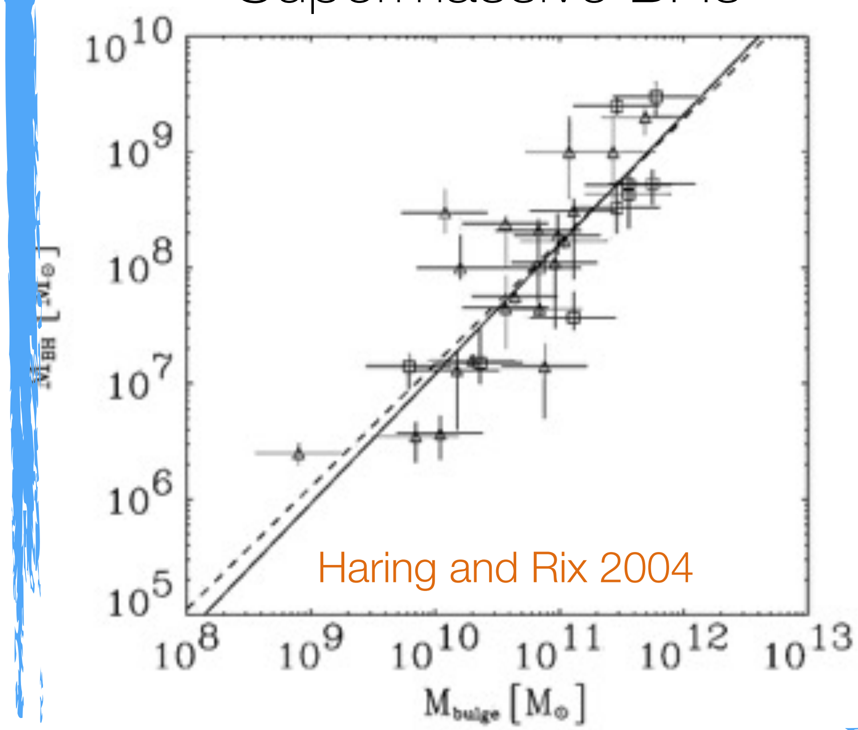
Spinning BH binaries?

Most convincing BH-binary candidate

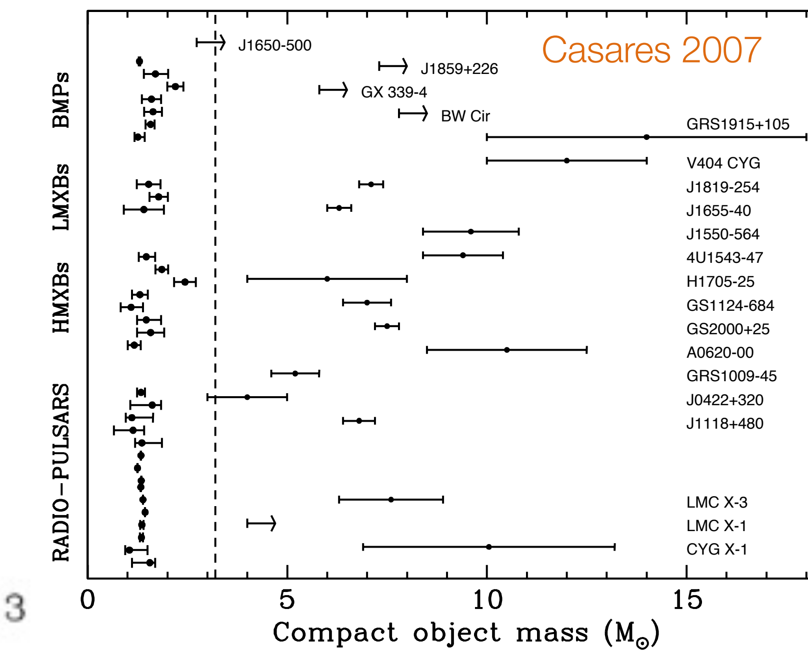


Mass measurements

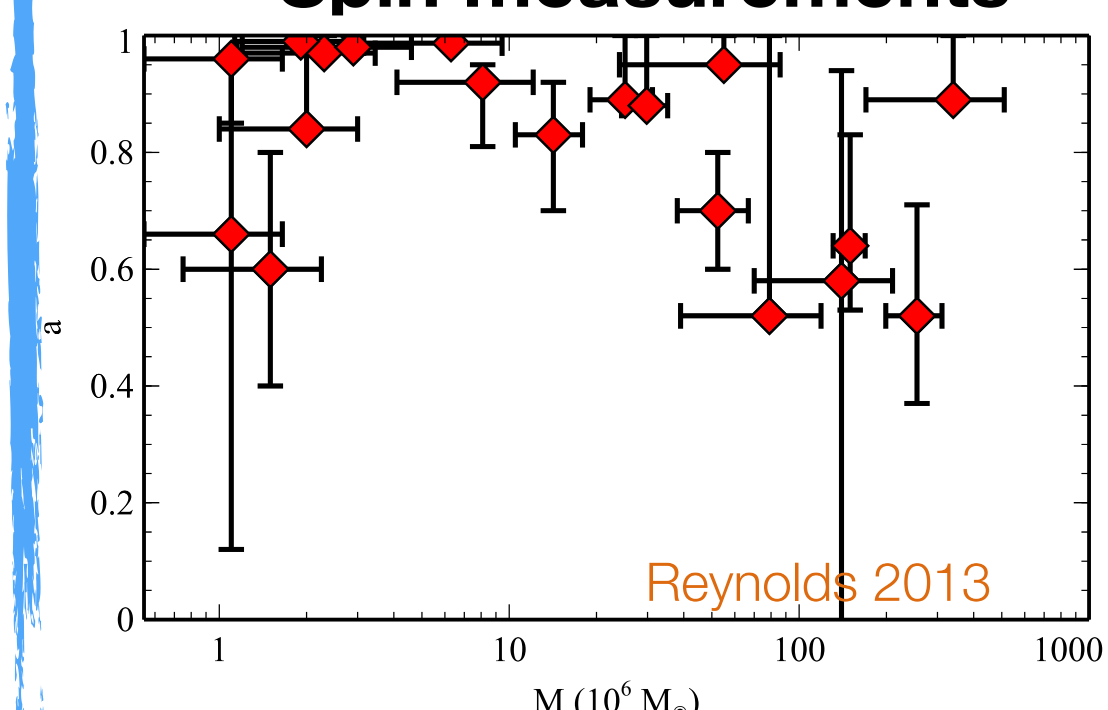
Supermassive BHs



Stellar-mass BHs



Spin measurements



Three dynamics, and three times

1. **Orbital** motion → $t_{\rm orb} \propto (r/r_g)^{3/2}$
   Kepler's third law
2. Spin & orbital-plane **precession** → $t_{\rm pre} \propto (r/r_g)^{5/2}$
   Apostolatos et al 1994
3. GW emission and **inspiral** → $t_{\rm RR} \propto (r/r_g)^{4}$
   Quadrupole formula
   Peters & Matthews 1963

if **(Post-)Newtonian** $r \gg r_g = GM/c^2$: timescale hierarchy

Orbit **<<** Precession **<<** Inspiral

BH binary **multi-timescale** analysis:

1. Solve the dynamics (hopefully analytically) on the shorter time
2. Quasi-adiabatic evolution ("average") on the longer time

Common practice in binary dynamics

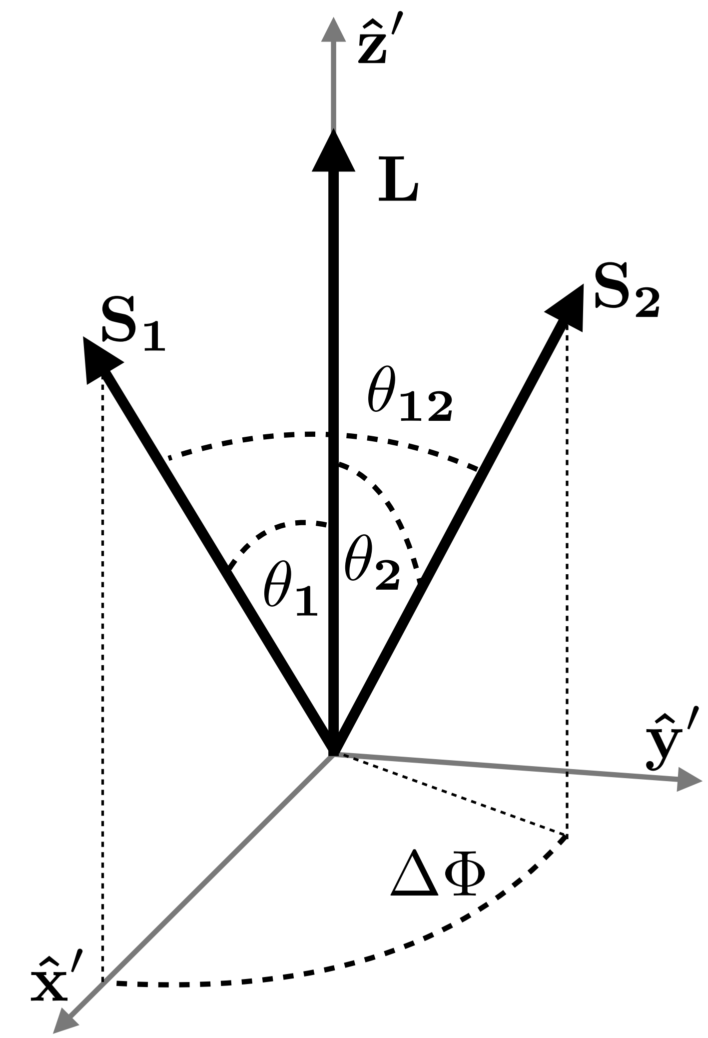
- periastron precession
- osculating orbital elements
- variation of constants

Usual **Post-Newtonian** dynamics

Orbit << Precession << Inspiral

Variables

- Total mass (units) $M = m_1 + m_2 = 1$
- Mass ratio $q = m_2/m_1 \leq 1$
- Three momenta, **9** components $\mathbf{L}, \mathbf{S_1}, \mathbf{S_2}$



Orbit-average PN evolutionary equations

e.g. Kidder 1995

- Spin precession $\dot{\mathbf{S}}_i = \mathbf{\Omega}_i \times \mathbf{S}_i$
- Momentum conservation $\dot{\hat{\mathbf{L}}} = -(\dot{\mathbf{S_1}} + \dot{\mathbf{S_2}})/\mathbf{L}$
- Radiation reaction $\dot{r} = \mathrm{PN\,approximant}$

$$L = q\sqrt{rM^3}/(1+q)^2$$

Constraints

- Spin magnitudes are constants $S_i = \chi_i m_i^2$
- Take a smart frame (3 constraints)

Double-spin precession is (usually) a **4D** problem: $r,\ \theta_1,\ \theta_2,\ \Delta\Phi$

A closer look...

Orbit << Precession << Inspiral

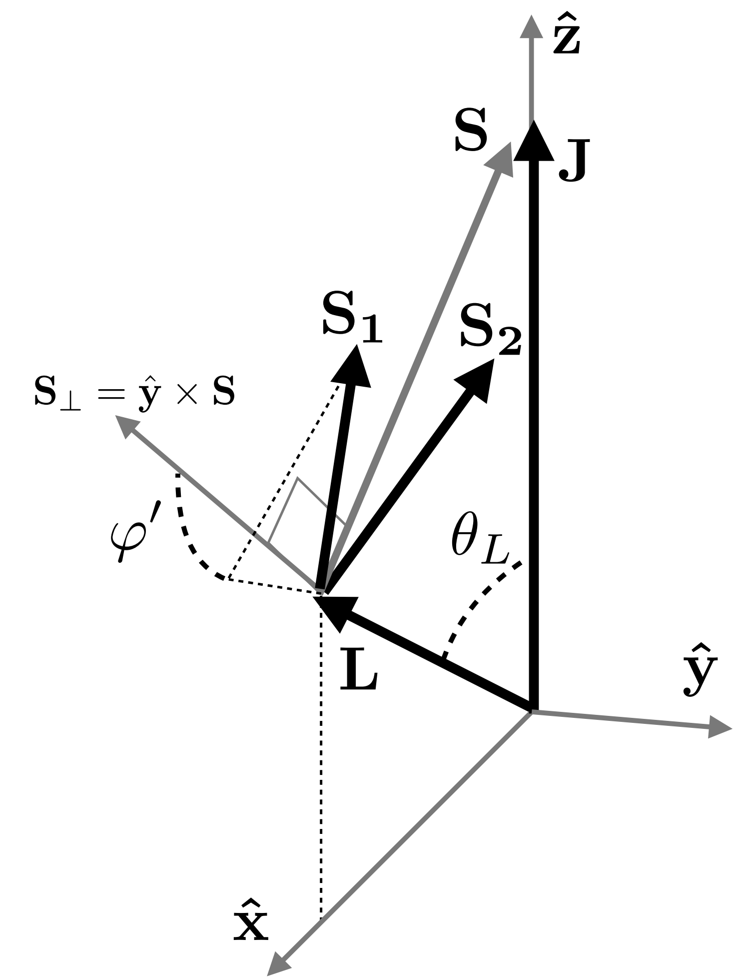
4D is too much. Can we further exploit the **timescale hierarchy**?

Let's **freeze** GW emission

- Separation r varies on t_{RR}
- Also $J = |\mathbf{L} + \mathbf{S_1} + \mathbf{S_2}|$ vary on t_{RR}
- Effective spin is constant (at least) at 2PN!

Damour 2001; Racine 2008

$$\xi \equiv M^{-2}[(1+q)\mathbf{S}_1 + (1+q^{-1})\mathbf{S}_2] \cdot \hat{\mathbf{L}}$$

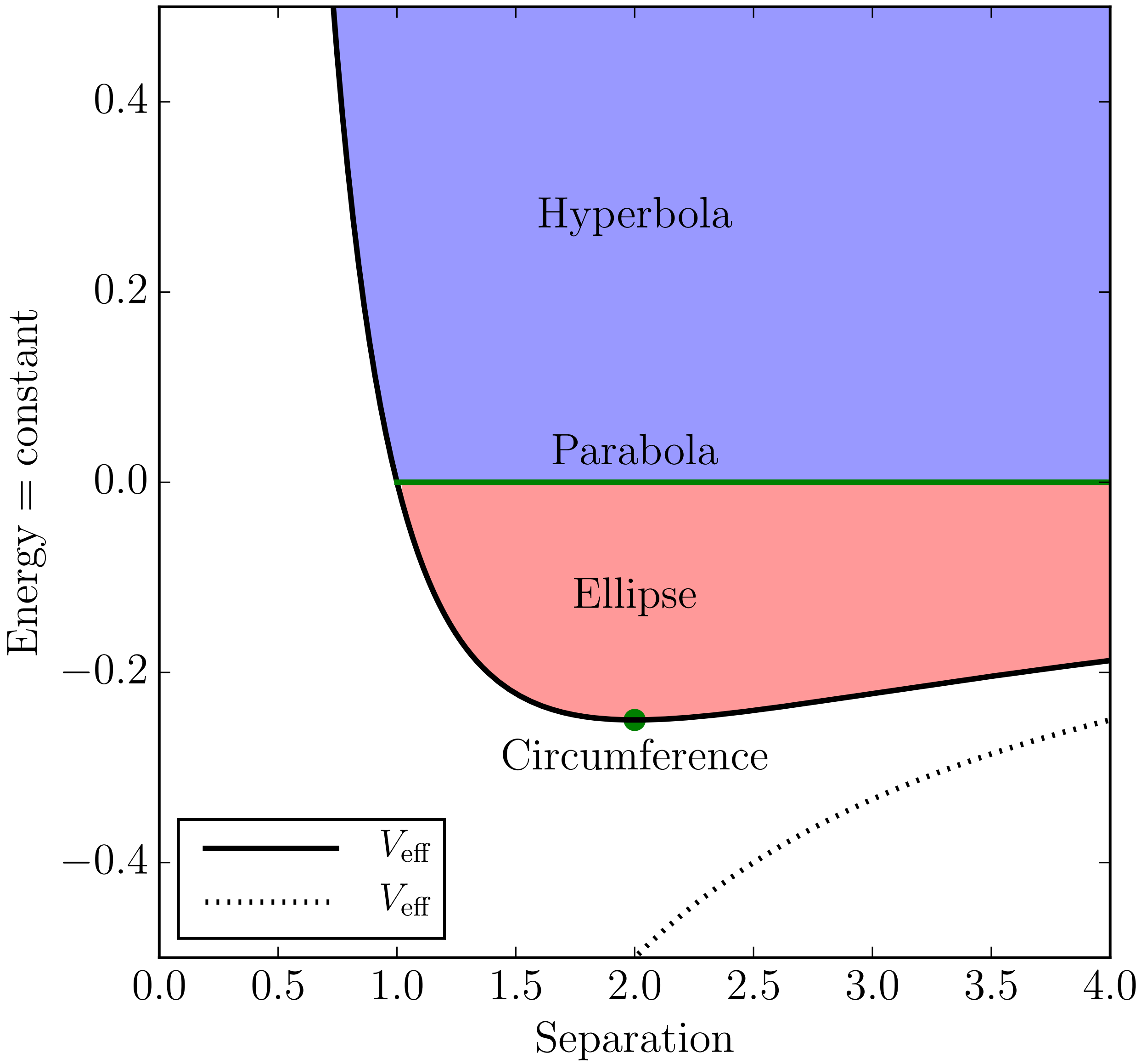


Upshot

- This construction **can be done** explicitly
- The solution is **fully analytical**
- Need a smarter frame…
- Chosen parameter is $S = |\mathbf{S_1} + \mathbf{S_2}|$

Double-spin precession is (actually) a **1D** problem!

On the shoulders of giants



Kepler's two-body problem

What you do:

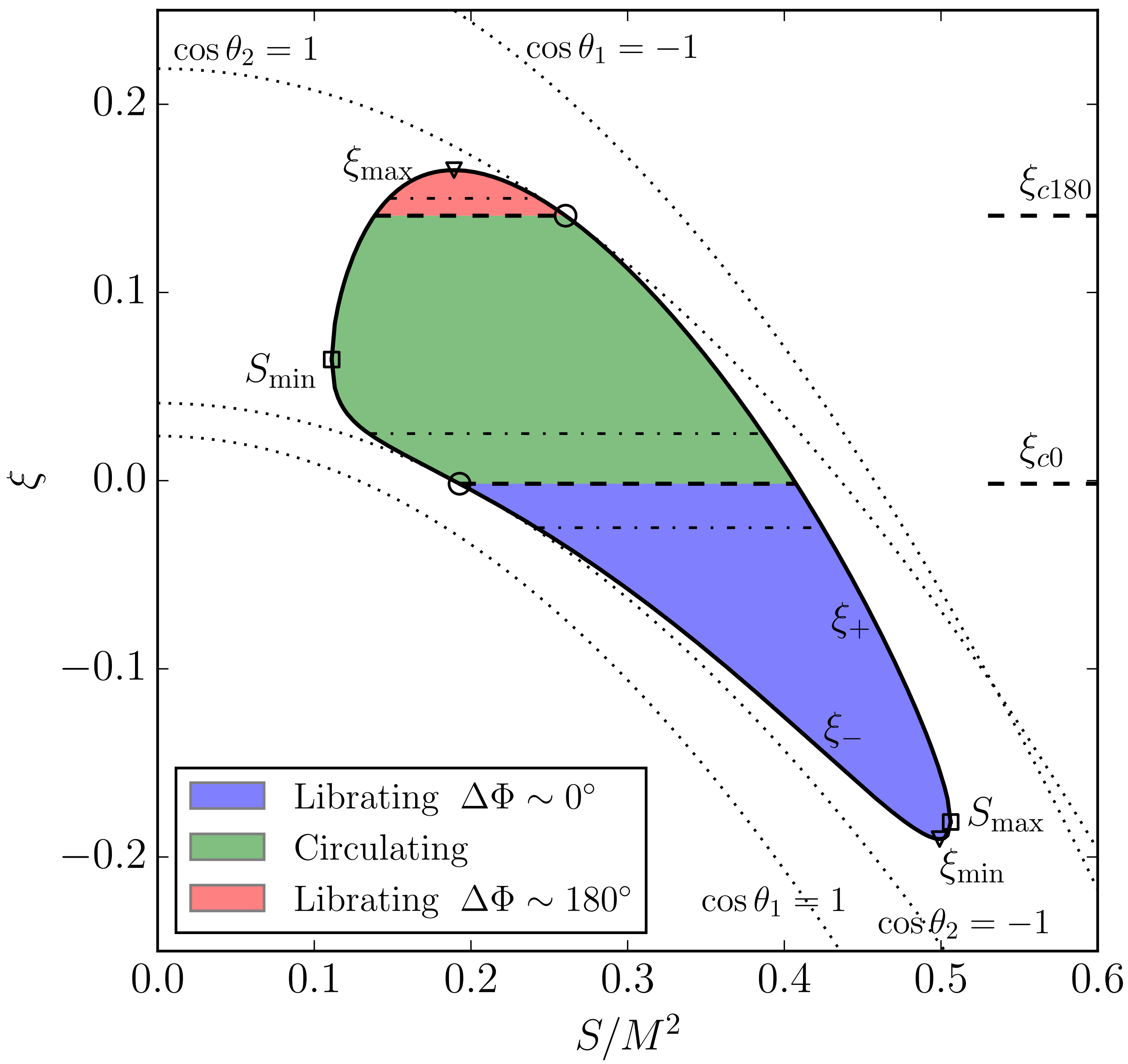
- One effective particle: 3D
- 3D to 2D problem:
  L is a **constant** of motion!
- Energy is **constant:** 2D to 1D?
- Effective potential

What you get:

- A lot of understanding
- Solutions are Kepler's orbits
- **Phases**: bound, unbound

Integrating GMm/r^2 to get a bunch of points along an orbit or…
knowing that that curve is an ellipse!

Effective potentials for spin precession



What you do:

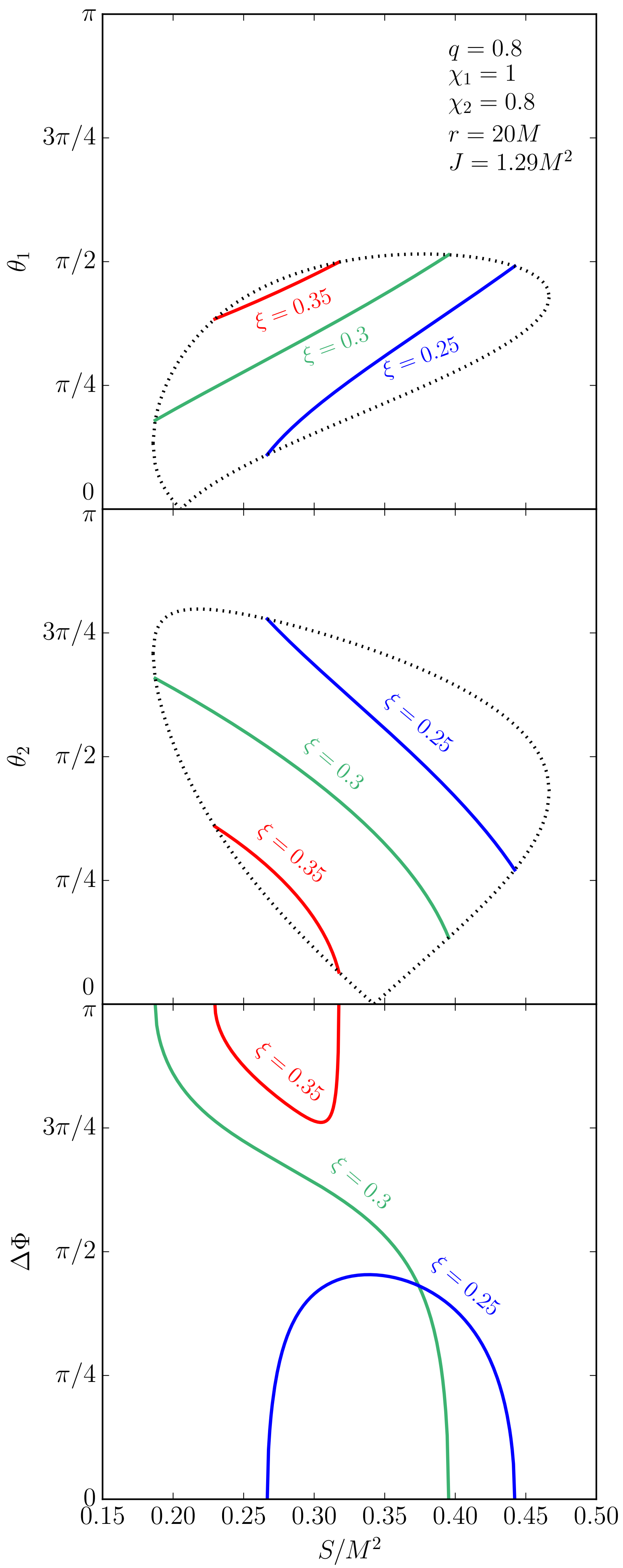
- Start from 4D problem
- 4D to 2D problem: GW are frozen, r and J are **constant**,
- Further constant of motion, **effective spin:** 2D to 1D
- Effective potentials for BH binary spin precession

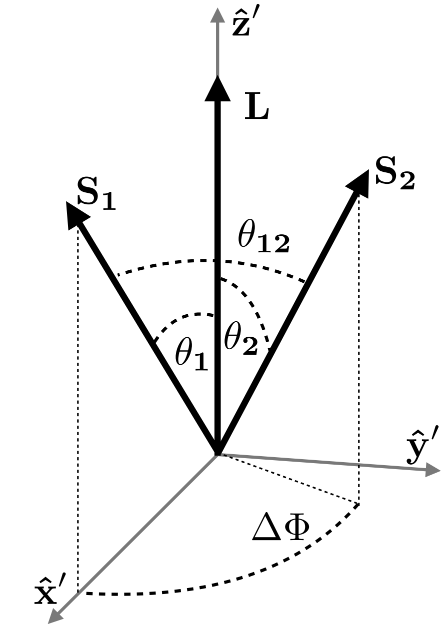
What you get:

- Analytical solutions
- **Phases**: circulating, librating
- A lot of understanding

Integrating the PN eq. to get a bunch of points on a precession cone or… **knowing** the shape of that cone!

Spin morphologies





How do **solutions** look like?

Spin tilts $\theta_1,\ \theta_2$

- Kind of boring… **monotonic**
- **Bounded** by the effective potentials

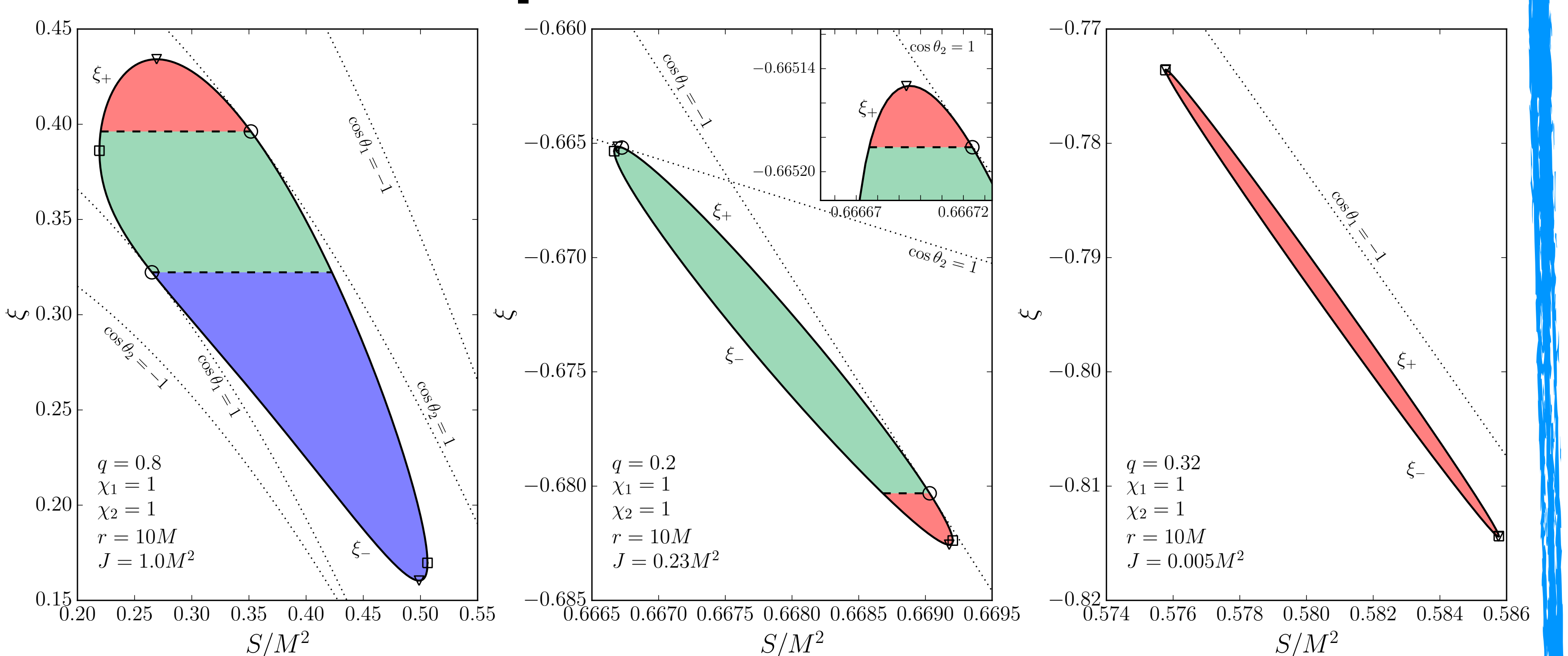
Azimuthal projections $\Delta\Phi$

- Three different **morphologies**
- Boundaries if aligned

- Librating $\Delta\Phi \sim 0$
- Circulating
- Librating $\Delta\Phi \sim \pi$

DG et al. 2015

Complete classification



Averaging the average

Orbit $\ll$ Precession $\ll$ Inspiral

Let's **turn on** GW emission

- Quasi-adiabatic approach
- Relate solutions at different separations
- Only r (or L) and J vary on $t_{\rm RR}$. Can we relate them?

Usual **orbit** average

$$\langle X\rangle_{\rm orb} = \frac{\int d\psi\; X\; dt/d\psi}{\int d\psi\; dt/d\psi}$$

Some parameters for the dynamics (here ψ is Kepler's true anomaly)

Orbital period

New **precession** average

$$\langle X\rangle_{\rm pre} = \frac{\int dS\; \langle X\rangle_{\rm orb}\; dt/dS}{\int dS\; dt/dS}$$

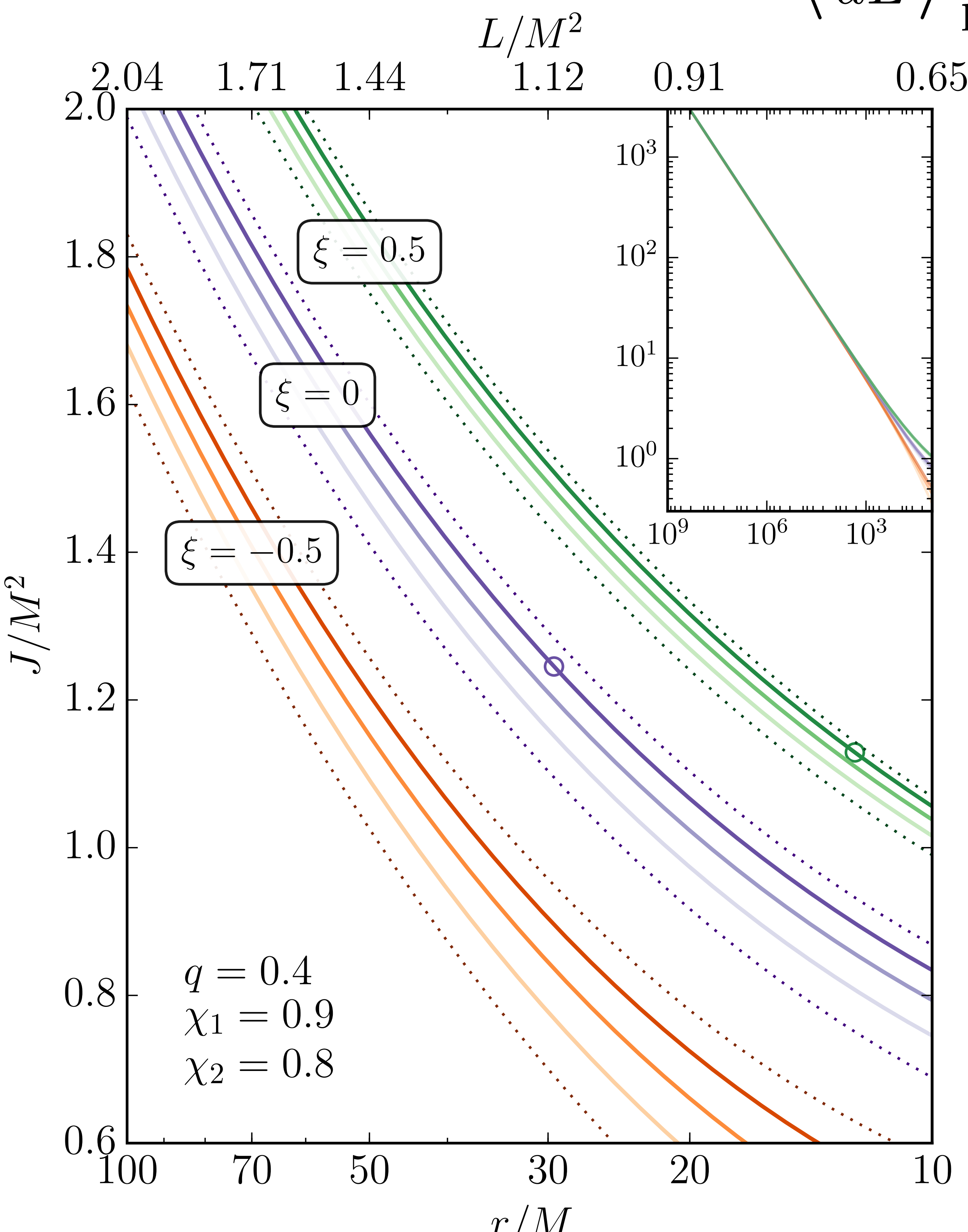
Dynamics is now parametrized by S

Precession period

A **new** Post-Newtonian approach

Result is very simple!
Details: **DG** et al. 2015

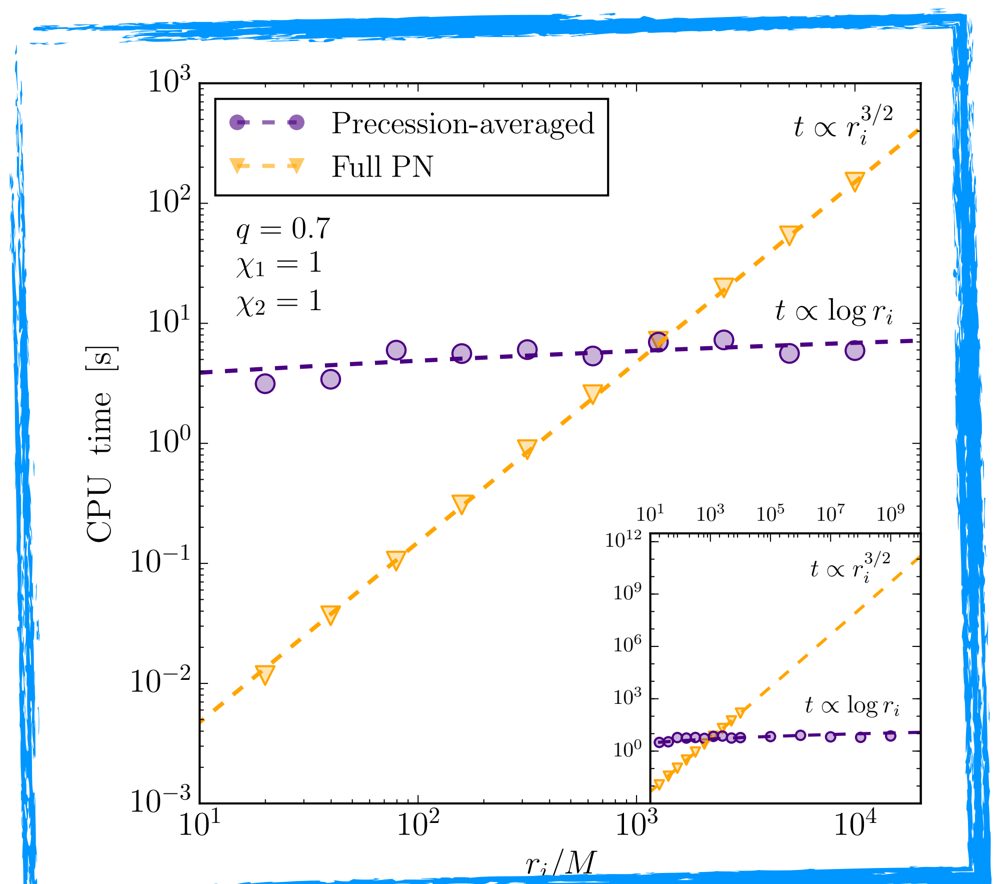
$$\left\langle \frac{dJ}{dL} \right\rangle_{\rm pre} = \frac{1}{2LJ}\left(J^2 + L^2 - \langle S^2 \rangle_{\rm pre}\right)$$



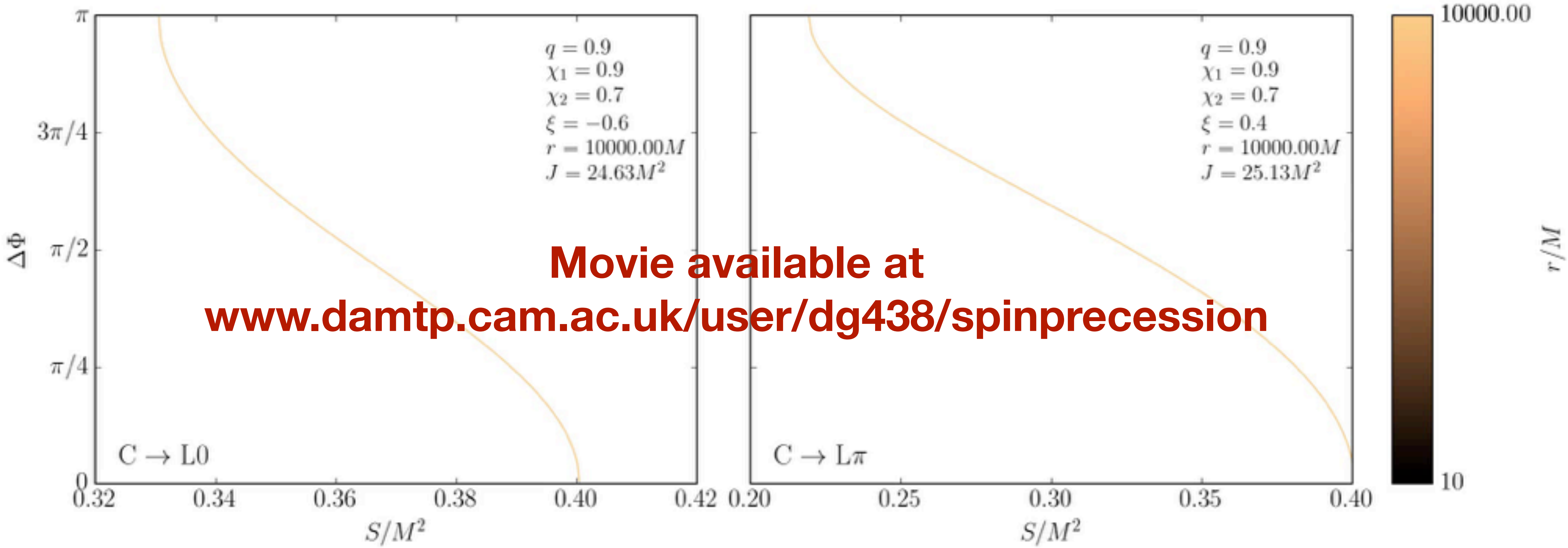
How do **solutions** look like?

- PN evolution is reduced to solving **one single ODE**
- Computationally, very very **easy**
- Domain can be compactified and integrations carried over from $r/M = \infty$

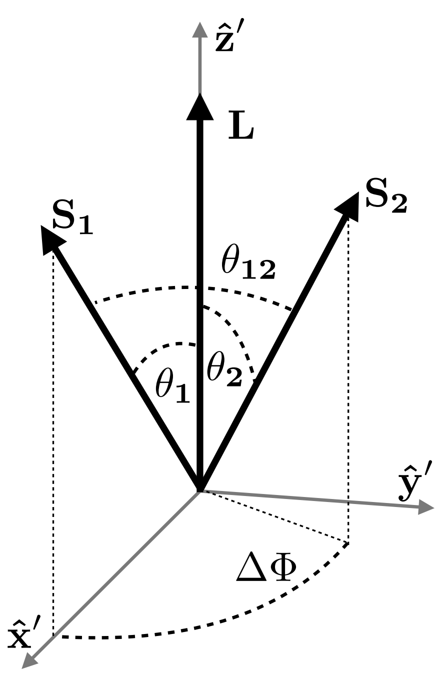
!



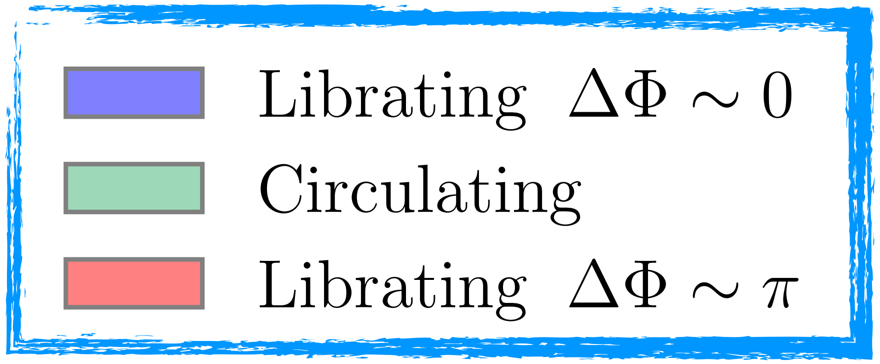
Morphological **phase transitions**



Movie available at
www.damtp.cam.ac.uk/user/dg438/spinprecession



There's a lot of interesting dynamis **going on** here!



Phase transitions are…

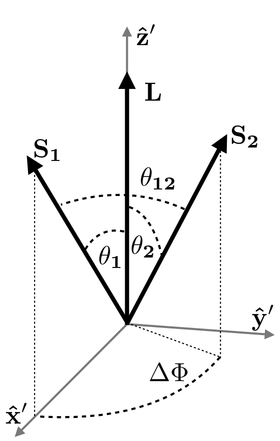
- **Discontinuities** in the evolution of $\Delta\Phi$
- At large separations, **circulation** $\Delta\Phi \in [-\pi, \pi]$
- At small separations: **libration** $\Delta\Phi \sim 0$ or $\Delta\Phi \sim \pi$

The role of **alignment**

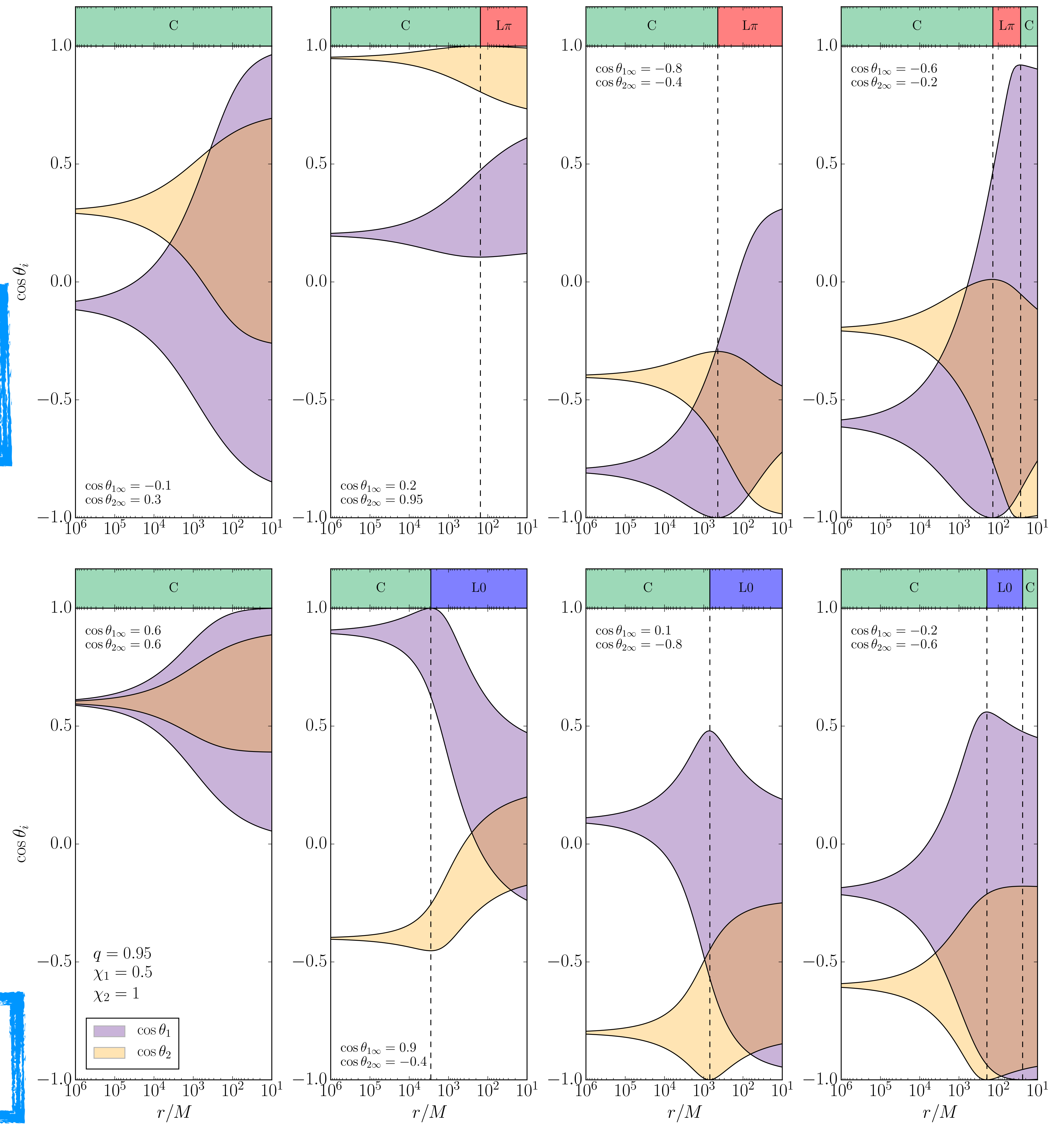
Evolution of the tilt angles on the inspiral time…

(allowed range of the evolution on the precessional time)

- The **range** grows fatter
- "**Bounce**" at the alignment configuration…
- … and sharp **transition** towards another morphology

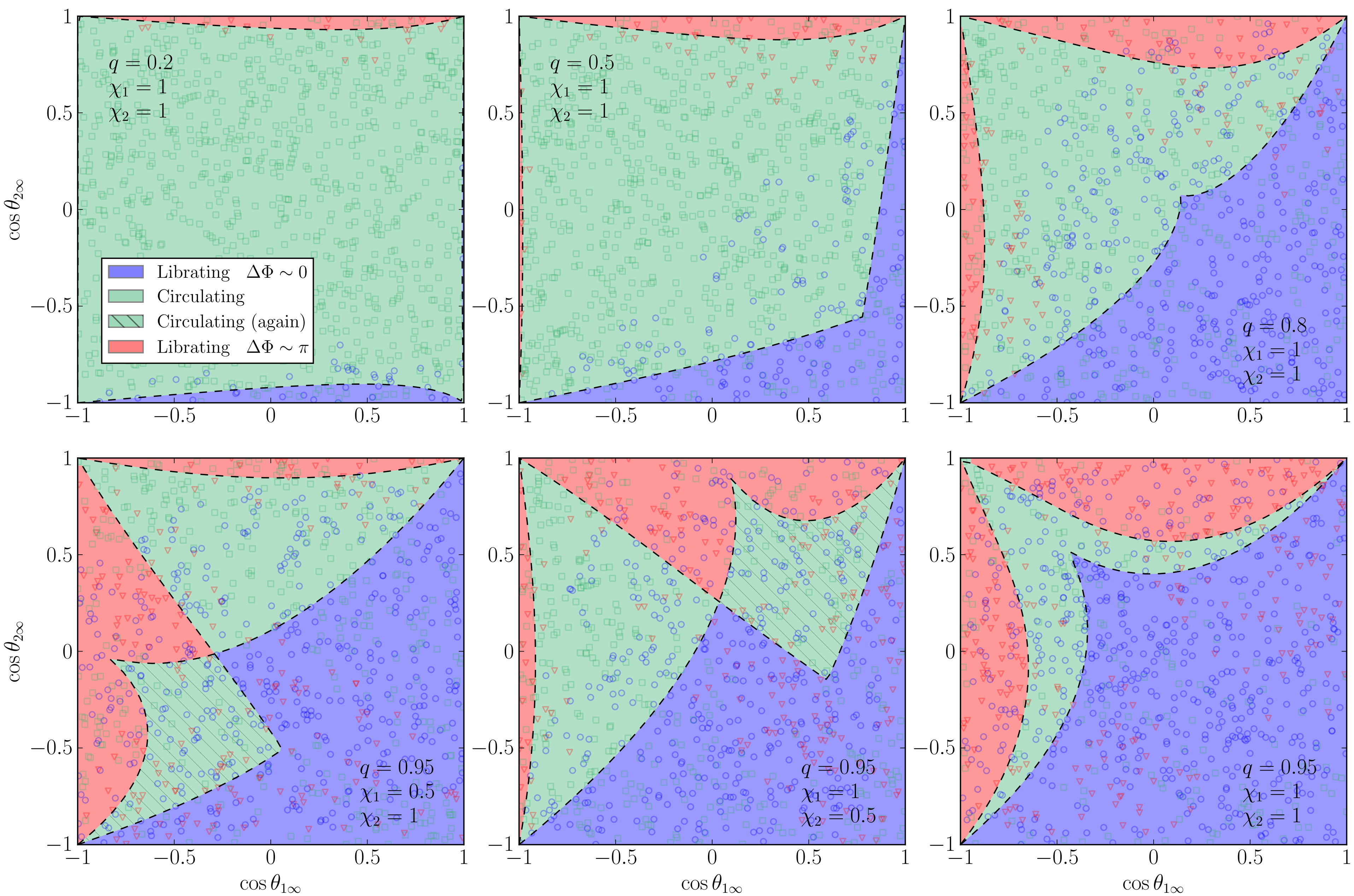


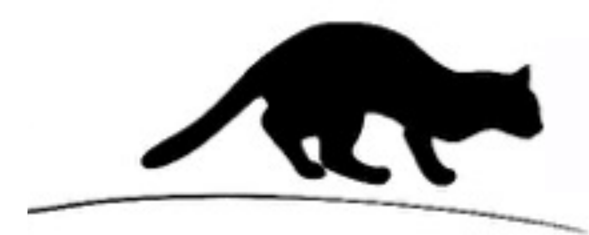
Librating $\Delta\Phi \sim 0$
Circulating
Librating $\Delta\Phi \sim \pi$



A predictive statement

- The morphology **is a feature** of spin precession that does <u>not</u> vary on the precessional time!
- No **theoretical uncertainties** related to the precessional phase
- The final spin orientations are scattered around, but…
- ….**each** blue, green, red dots **comes from** the blue, green, red region!





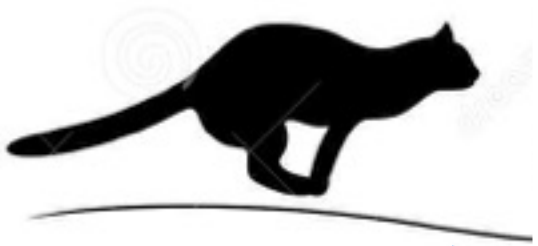
Initial means astrophysics

From the final morphology, one estimates the **initial spin** orientation: **how BH form!**

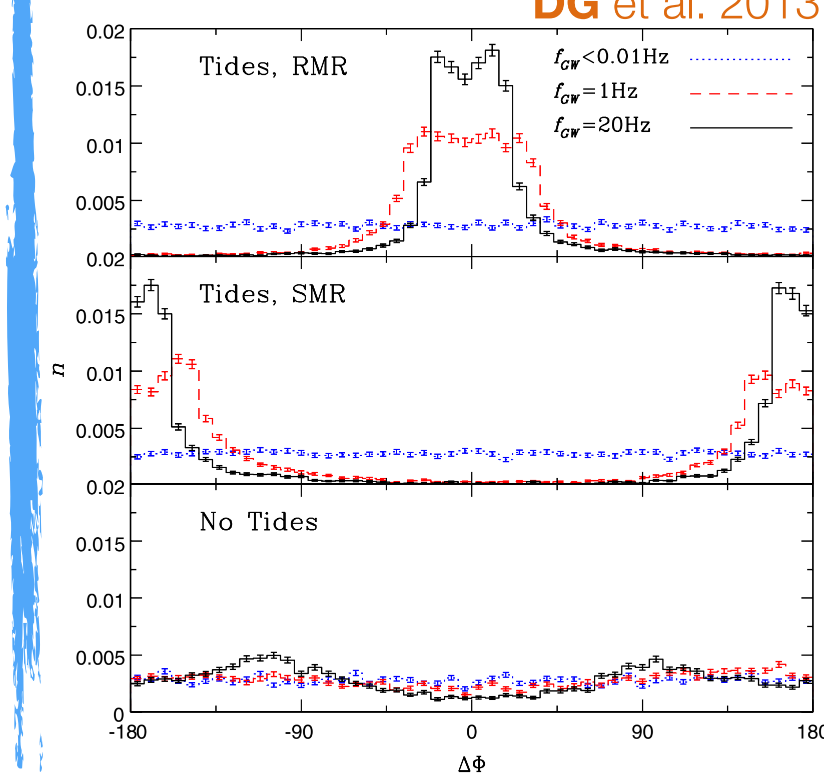
A multi-timescale perspective

Usual PN approach:
orbit-averaged equations.
You don't follow each BH along its orbit, but consider the orbits "as a whole"

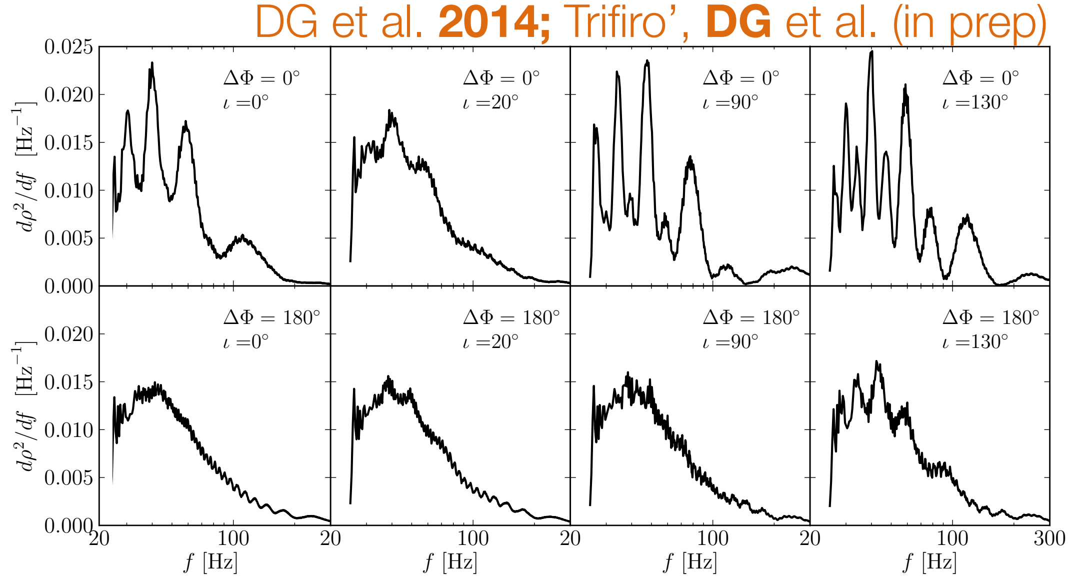
Our new PN approach:
precession-averaged equations.
You don't even track each spin along its precession cone, but consider the cones "as a whole"



Binary-star astrophysics
DG et al. 2013



GW parameter estimation
DG et al. 2014; Trifiro', DG et al. (in prep)



New, fast, waveform PN approximant?
Chatziioannou, Klein et al (in prep)

Summary

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and movies online
davidegerosa.com/spinprecession