Accreting black holes as probes of strong-field gravity

Leonardo Gualtieri

Sapienza Università di Roma

Mostly based on:

A.Maselli, L.G., P.Pani, L.Stella, V.Ferrari, ApJ 801,1,2015
A. Maselli, P. Pani, L.G., V. Ferrari, in preparation

Testing strong-field gravity

I don't really need to explain here why we need to test strong-field gravity

One century after its formulation,
GR passed all observational and experimental tests with flying colours,
but these tests only probe the weak-field regime of gravity.

The strongest gravitational field probed in the solar system is that at the surface of sun (light deflection), for which gravitational redshift and spacetime curvature are

$$\frac{GM_\odot}{R_\odot c^2} \sim 10^{-6} \qquad \frac{GM_\odot}{R_\odot^3 c^2} \sim 10^{-28}\, cm^{-2}$$

The most accurate test in solar system is probably that of the post-Newtonian parameter γ from Cassini spacecraft, with an accuracy of $\sim 10^{-5}$.

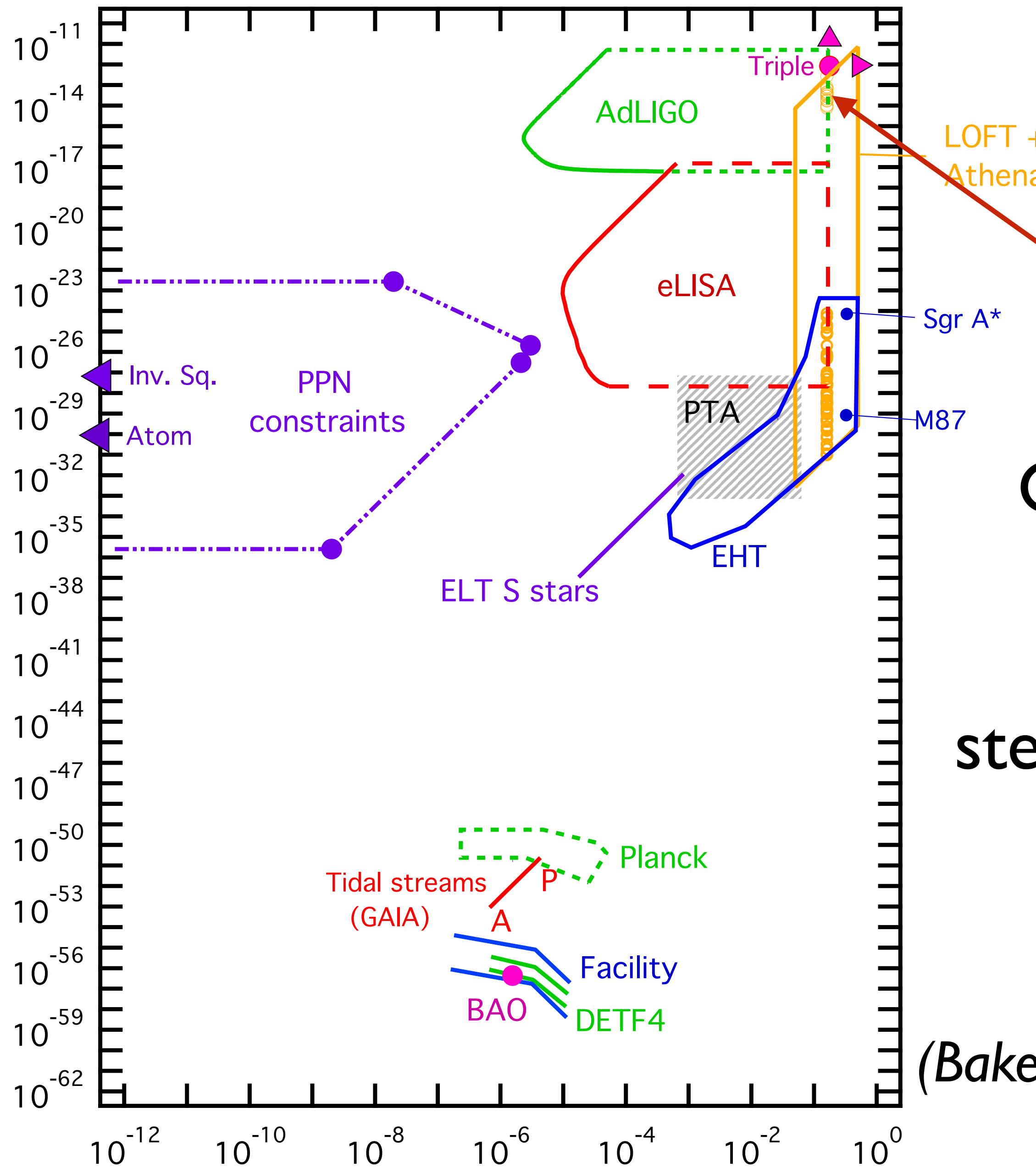
Binary pulsar tests involve masses $M_{NS} \sim M_\odot$ and distances similar to $R_\odot$

therefore they generally test the same regime as solar system test.

(with exceptions: in some cases strong-field phenomena may affect
the motion of binary pulsar, e.g. spontaneous scalarization;
but testing these very specific phenomena is not the same thing as
testing the strong-field regime of gravity)

Testing strong-field gravity

Parameter space of strong gravity



Curvature

$$\xi = \frac{GM}{r^3 c^2}$$

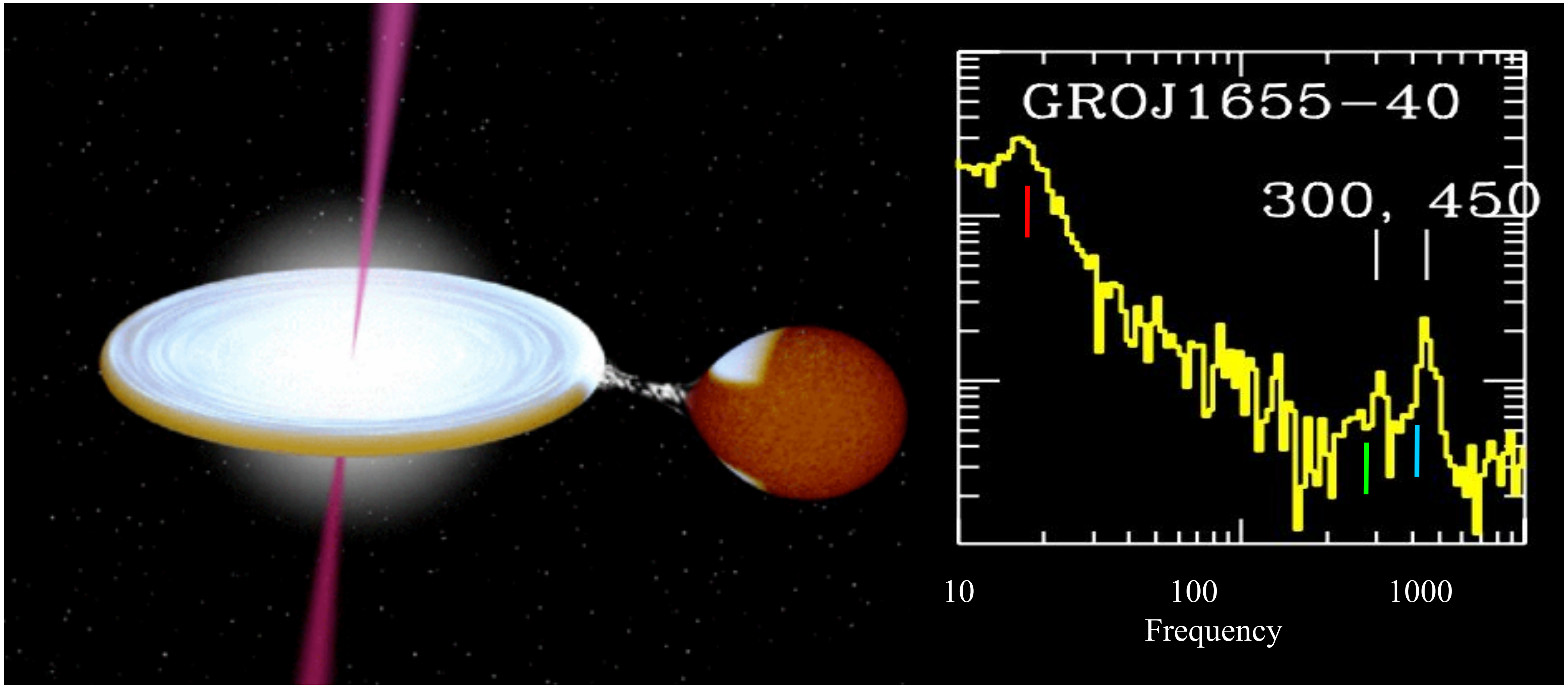
Potential $\epsilon = \frac{GM}{rc^2}$

(Baker et al., '15)

Curvature and potential both very large near horizon of stellar-mass ($M \sim 10 M_\odot$) BHs!

GW and EM observations allow for complementary tests!

Accreting black holes



Picture by Rob Hynes, Cambridge

Quasi-periodic oscillations (QPOs) observed in the X-ray flux from accreting BHs are associated with phenomena occurring near the BH horizon. Therefore, they are potentially a probe of strong gravity!

Presently, the accuracy of QPO measurements is not enough to test gravity: frequencies known at $\sim 1\sigma$ with 1-2% relative errors (for RXTE)

Accreting black holes

Future very large-area X-ray detector: LOFT
(proposed as an ESA M-class mission, ~~now under evaluation~~)

rejected from M4 evaluation few days ago!

With its ~10m^2 effective area,
it will be able to measure QPO frequencies with very high precision,
up to ~15 times that of RXTE.

This, together with gravitational wave detectors,
could allow tests of GR in the strong-field regime

Accreting black holes

We shall consider one of the main QPO model,
the relativistic precession model (RPM) *(Stella, Vietri, '98, '99)*

In this model, a blob of matter composing the disk moves on a circular, equatorial orbit hardly interacting with the surrounding matter.

The orbit is a geodesic of the BH spacetime (i.e., assuming GR, the Kerr solution) and the X-ray emission is modulated by the azimuthal (ν_φ), periastron (ν_{per}) and nodal (ν_{nod}) frequencies where

$$\nu_{per} \equiv \nu_\varphi - \nu_r \qquad \nu_{nod} \equiv \nu_\varphi - \nu_\vartheta$$

ν_r and ν_ϑ are the epicyclic frequencies of the geodesic, i.e., proper oscillations modes for small displacements δr, $\delta\vartheta$ from the circular, equatorial geodesic.

Azimuthal and epicyclic frequencies carry the imprint of the spacetime metric. If they are emitted near the inner region of the disk (i.e., near the ISCO) they carry the imprint of the strong-field region of the BH spacetime.

Accreting black holes

This model was used, for instance, in *Motta et al., MNRAS, '14a,'14b*
where the observations from the BH binaries XTE J1655-40, XTE J1550-164 from which a triplet of QPO, emitted at a certain r_{em}, was observed, were used to extract the mass M and the angular momenta $J=M^2a^*$ of the BHs.

Under the assumptions:

(i) The RPM describes accurately the QPOs
(ii) GR is the correct theory of gravity, thus the metric is the Kerr solution

The azimuthal and epicyclic frequencies are:

$$\nu_\varphi^{\rm GR} = \frac{1}{2\pi}\frac{M^{1/2}}{r^{3/2}+a^\star M^{3/2}}\,,$$

$$\nu_r^{\rm GR} = \nu_\varphi^{\rm GR}\left(1-\frac{6M}{r}+8a^\star\frac{M^{3/2}}{r^{3/2}}-3a^{\star 2}\frac{M^2}{r^2}\right)^{1/2}$$

$$\nu_\theta^{\rm GR} = \nu_\varphi^{\rm GR}\left(1-4a^\star\frac{M^{3/2}}{r^{3/2}}+3a^{\star 2}\frac{M^2}{r^2}\right)^{1/2}\,.$$

Using these expressions, the observed values of the QPO triplet yield the values of the three quantities M, a^*, r_{em}.

This is not a test of GR, but…

Accreting black holes

LOFT is expected to measure more QPO triplets from the same BH binary

ν^i_φ , ν^i_{per} , ν^i_{nod} $i=1,2,\ldots,n$

We would then measure 3n quantities,
with (in GR) n+2 unknowns M, a^*, r^i_{em}

The redundancy (occurring as n>1) would allow to test GR!

Testing GR with astrophysical observations

How can we test GR with astrophysical observations?
(see e.g. *Berti et al., arXiv:1412.3473, submitted to CQG*)

One could devise a phenomenological parametrization of the BH spacetime (bottom-up approach) but present parametrizations are impractical.

We shall instead follow a top-down approach:
consider modifications of GR,
possibly inspired by fundamental physics considerations,
and work out predictions, to be tested against observations.

Einstein-Dilaton Gauss-Bonnet theory

We tested GR against EDGB gravity:

Gauss-Bonnet invariant $\mathcal{R}^2_{\rm GB} = R_{\alpha\beta\delta\gamma}R^{\alpha\beta\delta\gamma} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$

is included in the action, coupled with a scalar field ϕ

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha e^\phi}{4}\mathcal{R}^2_{\rm GB}\right)$$

- The most natural way to modify the strong-field/high-curvature regime of gravity is to include in the action a quadratic term in the curvature
- The Gauss-Bonnet invariant is a total derivative, thus it has to be coupled to ϕ
- This is the only combination giving 2$^{\rm nd}$ order field equations, avoiding instability
- Scalar-tensor theories without quadratic terms in the curvature do not affect generally the strong-field/high-curvature regime; stationary BHs in these theories satisfy no-hair theorems, i.e., are the same as in GR
- Both features are *string-inspired*: the scalar field can be seen as the string *dilaton;* Gauss-Bonnet can be seen as the first term in an expansion in all curvature invariants and their powers, as suggested by low-energy effective string theory
- Implicit assumption: there is a new scale >> Planck scale: $\alpha^{1/2} \sim$ km

Einstein-Dilaton Gauss-Bonnet theory

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Field equations:

$$G_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi - \alpha\mathcal{K}_{\mu\nu}\,,$$
$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}\partial^\mu\phi) = -\frac{\alpha}{4}e^\phi\mathcal{R}^2_{\rm GB}\,,$$

where $\mathcal{K}_{\mu\nu} = \frac{g_{\mu\rho}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\rho}}{8}\epsilon^{k\lambda\alpha\beta}\nabla_\gamma\left(\epsilon^{\rho\gamma\mu\nu}R_{\mu\nu\alpha\beta}\partial_k e^\phi\right)$

Since field equations are 2nd order, EDGB gravity can be considered either as an effective theory ($\alpha/M^2 << 1$) or as a non-effective theory (α finite)

Einstein-Dilaton Gauss-Bonnet theory

Solution of the EDGB field equations
describing stationary, slowly rotating BHs
(Kanti et al. '96; Pani & Cardoso '09)

only exists for

$$0 < \frac{\alpha}{M^2} \lesssim 0.691$$

We considered this entire range (=> EDGB gravity as a non-effective theory)

Note that α is dimensionful!
The dimensionless parameter is $\zeta=\alpha/M^2$ => larger effect for smaller BHs (i.e., larger curvature)

Best observational bound: $\alpha \lesssim 47 M_\odot^2$
(*Yagi, PRD '14*) from orbital decay of LMXRBs
is weaker than theoretical bound when $M \lesssim 8.2 M_\odot$

Spacetime metric (at first order in the rotation rate):

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - 2r^2 \sin^2\theta\omega(r)dtd\varphi\,.$$

$f(r)$, $g(r)$, $\omega(r)$ solution of a system of ordinary differential equations in r

Testing gravity with accreting BHs

A.Maselli, L.G., P.Pani, L.Stella, V.Ferrari, ApJ 801,1,2015

We computed the azimuthal and epicyclic frequencies, and then ν_φ , ν_{per} , ν_{nod} for a stationary, slowly rotating EDGB BH, as functions of M,a*,r

Assuming as fiducial values those of *XTE 1655-40* (for which M, a* were extracted within GR): $M=5.3\ M_\odot$, $0\leq a^*\leq 0.1$, $r=1.1\ r_{ISCO}$,

we found that the relative differences $$\epsilon_i = \frac{\nu_i^{\rm EDGB} - \nu_i^{\rm GR}}{\nu_i^{\rm GR}} \qquad i = \varphi, r, \theta, {\rm nod}\,, {\rm per}$$

reach values as high as ~1-4%

This much larger than the expected sensitivity of LOFT, which should measure QPO frequencies with a fraction of % error!

However, in order to find out whether it would actually be possible to test gravity with LOFT, we had to perform a more refined analysis.

Testing gravity with accreting BHs

Let us assume that two different QPO triplets are measured with LOFT expected sensitivity.
We performed the following analysis:

- We chose "true" (supposed to be unknown by the observer) values of M, a^*, α
- Using EDGB equations, we generated two sets of frequencies

$$\nu_{ref1}=(\nu_\varphi, \nu_{per}, \nu_{nod})_1 \,, \quad \nu_{ref2}=(\nu_\varphi, \nu_{per}, \nu_{nod})_2 \,,$$

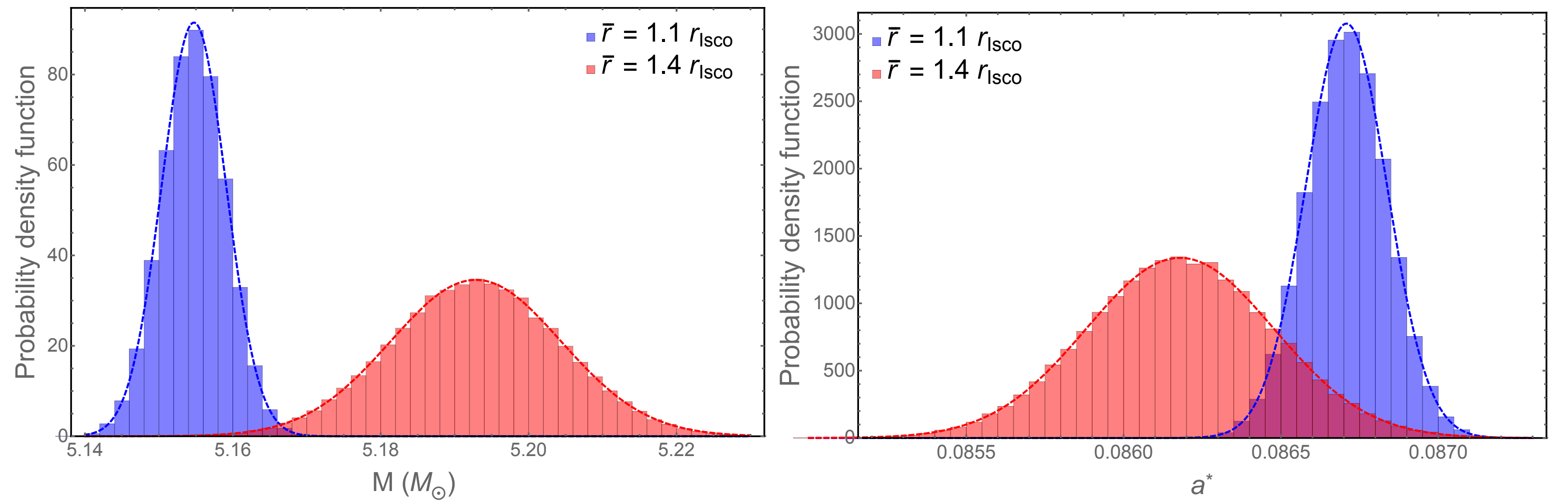
corresponding to two values of r/r_{ISCO}.
These are the measured triplets, with LOFT uncertainties $(\sigma_\varphi, \sigma_{per}, \sigma_{nod})$.

- We imagined that an ingenuous observer interpret these data as generated by a Kerr BH, finding the corresponding (M_1,a^*_1,r_1) , (M_2,a^*_2,r_2).
If $\alpha=0$ we expect $M_1=M_2$, $a^*_1=a^*_2$, but if $\alpha\neq0$, we expect $M_1\neq M_2$, $a^*_1\neq a^*_2$

- To quantify this discrepancy, we used a Monte Carlo approach, generating Gaussian distribution of triplets with standard deviation $(\sigma_\varphi, \sigma_{per}, \sigma_{nod})$.

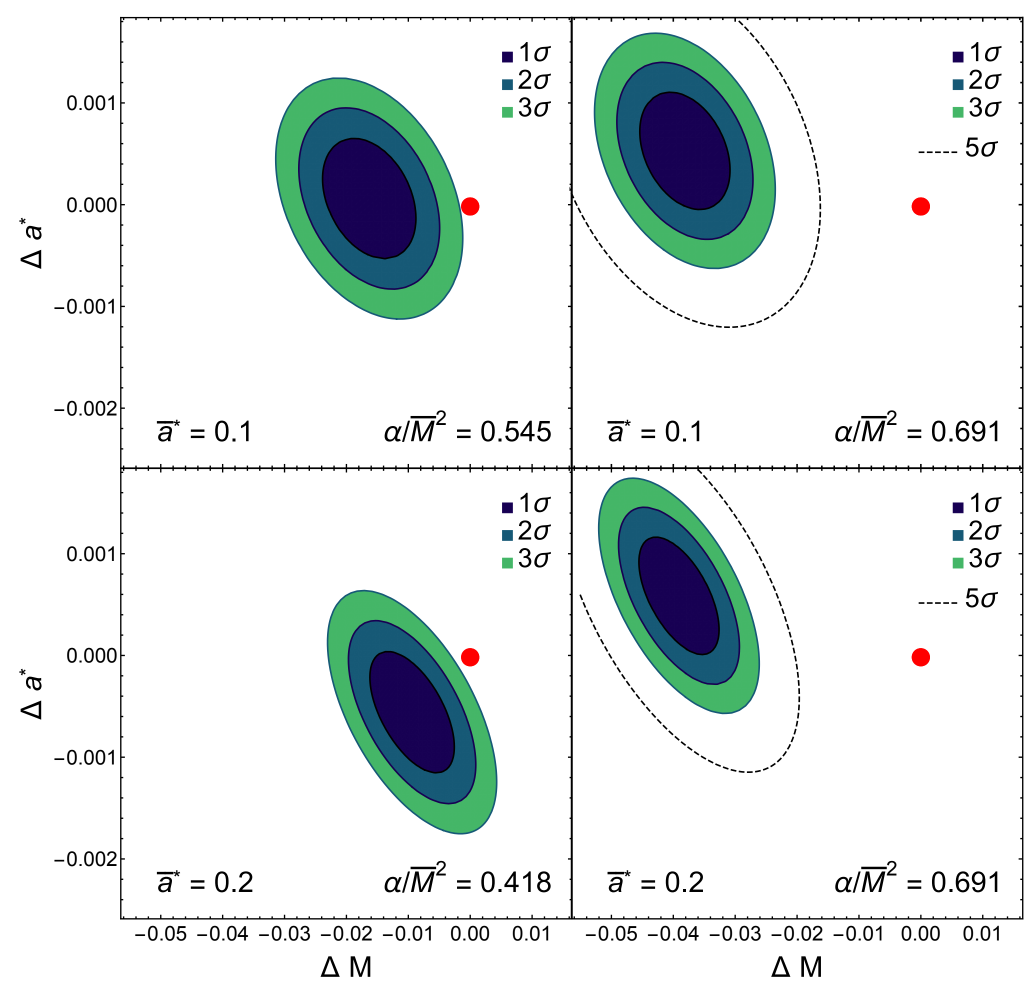
- The corresponding variables M_1,M_2, a^*_1,a^*_2 have also Gaussian distributions

Testing gravity with accreting BHs



We also computed the χ^2 variable for

$\Delta M = M_1 - M_2 \quad \Delta a^* = a^*_1 - a^*_2$:



Testing gravity with accreting BHs

Adopting a 3σ threshold, for a BH spinning as fast as $a^*=0.2$
we could detect discrepancies between GR and EDGB gravity for
$0.418<\zeta<0.619$ $(\zeta=\alpha/M^2)$
If no discrepancies are detected, we would rule out this range of α.

Accreting BHs are then a promising testbed for gravity,
since they can probe its strong-field/high-curvature regime!

Maselli et al. ApJ 2015 was just a preliminary study:

- Since BHs in EDGB gravity (with finite α/M^2) were only known at $O(a^*)$ we considered a^* up to 0.1-0.2, but actual BHs rotate more rapidly, and BHs with large a^* are more sensitive to strong gravity (smaller ISCO)
- This study assumed a particular QPO model, the RPM.
  Other models should be included in the analysis
  (e.g., the epicyclic resonance model of Kluzniak and Abramowicz) .

Not-so-slowly-rotating EDGB BHs

A. Maselli, P. Pani, L.G., V. Ferrari, in preparation

EDGB BHs at 2nd order in a* were already known in the ζ<<1 case

(Ayzenberg & Yunes, PRD 90, 044066 '14)

but we need to consider finite values of ζ (up to ~0.691).

Spherically symmetric background, O(a^{*2}) perturbations:

$$ds^2 = -e^{\Gamma}[1+2h(r,\theta)]dt^2 + e^{-\Lambda}[1+2m(r,\theta)]dr^2 + r^2[1+k(r,\theta)][d\theta^2 + \sin^2\theta(d\varphi - \hat{\omega}(r,\theta)dt)^2]$$

$$\begin{aligned}\hat{\omega} &= \omega(r)S_1(\theta)\,,\\ h &= h_0(r) + h_2(r)P_2(\cos\theta)\,,\\ m &= m_0(r) + m_2(r)P_2(\cos\theta)\,,\\ k &= k_0(r) + k_2(r)P_2(\cos\theta)\,,\\ \Phi(r) &= \phi(r) + \phi_0(r) + \phi_2(r)P_2(\cos\theta)\end{aligned}$$

Field equations expanded at O(a^{*2}):

$$\begin{aligned}&\mathcal{A}^1_{(0)}m_0' + \mathcal{A}^2_{(0)}m_0 + \mathcal{A}^3_{(0)}\omega'^2 + \mathcal{A}^4_{(0)} = 0\\ &\mathcal{B}^1_{(0)}h_0' + \mathcal{B}^2_{(0)}m_0 + \mathcal{B}^3_{(0)}\omega'^2 + \mathcal{B}^4_{(0)} = 0\\ &\phi_0'' + \mathcal{D}^1_{(0)}\phi_0' + \mathcal{D}^2_{(0)}\phi_0 + \mathcal{D}^3_{(0)} = 0\\ &\mathcal{A}^1_{(2)}h_2' + \mathcal{A}^2_{(2)}k_2' + \mathcal{A}^3_{(2)}h_2 + \mathcal{A}^4_{(2)}m_2 + \\ &\quad + \mathcal{A}^5_{(2)}\omega'^2 + \mathcal{A}^6_{(2)} = 0\,.\end{aligned}$$

$$\begin{aligned}&\mathcal{B}^1_{(2)}k_2' + \mathcal{B}^2_{(2)}k_2 + \mathcal{B}^3_{(2)}h_2 + \mathcal{B}^4_{(2)}m_2 + \\ &\quad + \mathcal{B}^5_{(2)}\omega'^2 + \mathcal{B}^6_{(2)} = 0\\ &\mathcal{C}^1_{(2)}h_2 + \mathcal{C}^2_{(2)}m_2 + \mathcal{C}^3_{(2)}\omega'^2 + \mathcal{C}^4_{(2)} = 0\\ &\phi_2'' + \mathcal{E}^1_{(2)}\phi_2' + \mathcal{E}^2_{(2)}\phi_2 + \mathcal{E}^3_{(2)} = 0\end{aligned}$$

Each of these coefficients is an expression in terms of background quantities

Not-so-slowly-rotating EDGB BHs

Field equations for finite ζ at 2nd order in a* are highly nonlinear, their numerical resolution is challenging.
We have solved them using a perturbative approach in ζ, up to $O(\zeta^7)$:
calling χ=a*, two-parameter perturbative expansion up to $O(\chi^2\zeta^7)$

The expression for metric coefficients too long to be shown here, but e.g.

$$
\begin{aligned}
\frac{r_{\rm Isco}}{M} = \; & 6 - 4\sqrt{\frac{2}{3}}\chi - \frac{8\chi^2}{27} + \zeta^2(1.4\chi^2 - 12\chi - 10)10^{-2} + \zeta^3(2.7\chi^2 - 6.6\chi - 5.1)10^{-2} + \\
& + \zeta^4(3.9\chi^2 - 5.4\chi - 3.8)10^{-2} + \zeta^5(5.0\chi^2 - 4.9\chi - 3.3)10^{-2} + \\
& + \zeta^6(6.4\chi^2 - 4.8\chi - 3.3)10^{-2} + \zeta^7(8.0\chi^2 - 5.1\chi - 3.4)10^{-2} \; .
\end{aligned}
$$

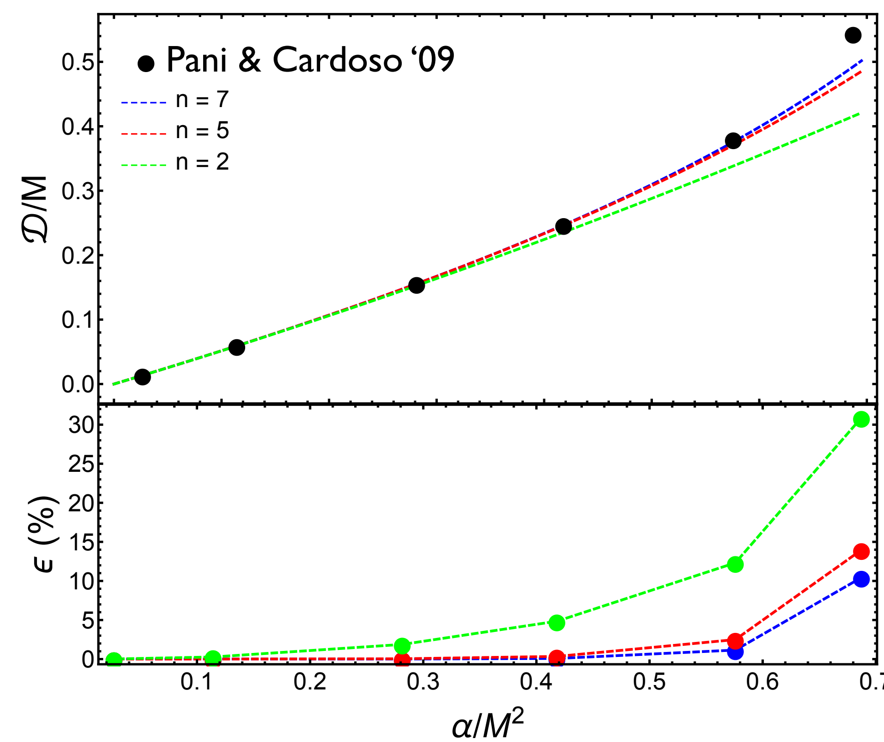
Azimuthal and epicyclic frequencies are given by similar expressions.

At $O(\chi^2\zeta^2)$, we reproduce the results of *Ayzenberg & Yunes '14*

Not-so-slowly-rotating EDGB BHs

To estimate the truncation errors due to neglecting $O(\zeta^7)$ terms,
we compared our solution at first order in the spin
with that of *Pani & Cardoso PRD 79, 084031 '09*, which is $O(\chi)$ and exact in ζ:

Dilatonic charge

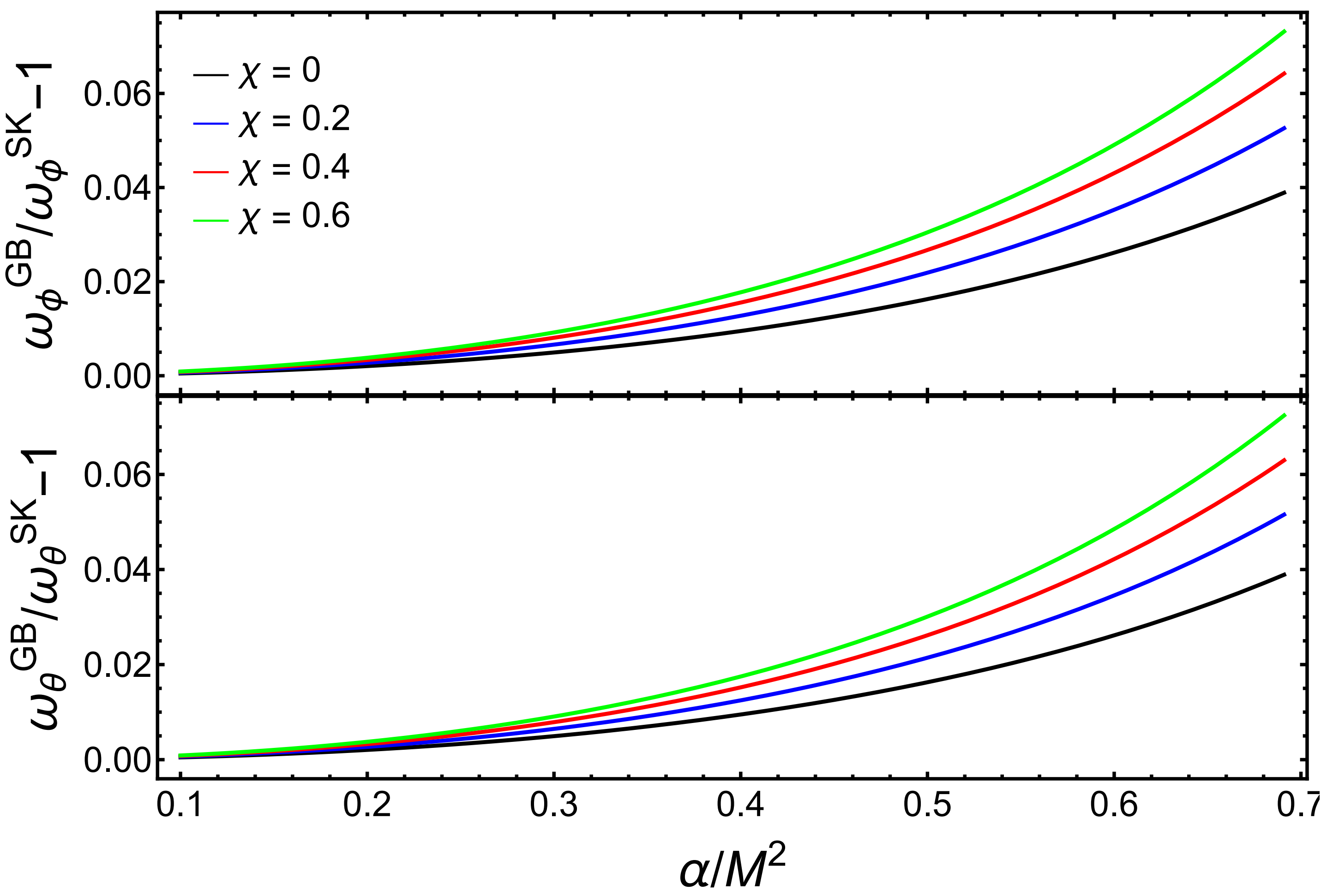


	χ	$\epsilon_{n=2}(\%)$	$\epsilon_{n=4}(\%)$	$\epsilon_{n=6}(\%)$	$\epsilon_{n=7}(\%)$
$r_{\rm h}/M$	0	5.90	4.45	3.72	3.48
$r_{\rm Isco}/M$	0	1.00	0.58	0.43	0.39
	0.05	1.11	0.65	0.49	0.44
	0.10	1.23	0.72	0.54	0.49
$M\omega_\varphi\|_{\rm Isco}$	0	1.36	0.79	0.59	0.53
	0.05	1.56	0.95	0.72	0.66
	0.10	1.88	1.22	0.98	0.91
$M\omega_\theta\|_{\rm Isco}$	0.05	1.53	0.92	0.69	0.63
	0.10	1.78	1.13	0.88	0.81

	χ	$\epsilon_{n=2}(\%)$	$\epsilon_{n=4}(\%)$	$\epsilon_{n=6}(\%)$	$\epsilon_{n=7}(\%)$
$r_{\rm h}/M$	0	1.33	0.52	0.24	0.17
$r_{\rm Isco}/M$	0	0.32	0.093	0.038	0.026
	0.05	0.37	0.12	0.055	0.042
	0.1	0.42	0.14	0.074	0.059
$M\omega_\varphi\|_{\rm Isco}$	0	0.44	0.13	0.053	0.036
	0.05	0.56	0.23	0.14	0.13
	0.1	0.80	0.44	0.35	0.33
$M\omega_\theta\|_{\rm Isco}$	0.05	0.54	0.20	0.12	0.10
	0.1	0.71	0.36	0.27	0.25

Not-so-slowly-rotating EDGB BHs

How does the spin affect the magnitude of the EDGB corrections?



As expected, EDGB corrections are larger for larger values of the spin parameter χ

Conclusions

- Accreting, stellar-mass BHs can be used to test the strong-field, high-curvature regime of gravity. The QPO of their X-ray flux are a sensitive probe of their inner region.

- We have considered the RPM of QPOs, in which they correspond to the azimuthal and epicyclic frequencies of circular geodesics near the ISCO, and EDGB gravity, probably the simpler best motivated GR deviation which modifies its strong-field regime

- To begin with, we have considered slowly rotating BHs. We found that a detection of QPOs with the expected sensitivity of the proposed ESA M-class mission LOFT would set strong constraints on the parameter space of this theory

- We are now extending our study to BHs at 2nd order in the rotation rate. At higher values of the rotation rate, circular stable geodesics can probe a region with higher curvature; thus, the EDGB deviations are stronger, and the QPO-based test is more effective