High energy particle collision and collisional Penrose process near a Kerr black hole

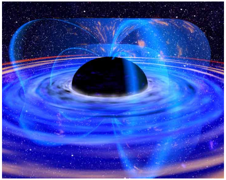
Tomohiro Harada

Department of Physics, Rikkyo University

12/06/2015 One Hundred Years of STRONG GRAVITY @ IST, Lisbon

Outline

Rotating BHs as particle accelerators

 as 

- Bañados, Silk, West 2009: Kerr BHs act as particle accelerators. (Piran, Shaham, Katz 1975)
  The CM energy of colliding particles can be unboundedly high near the horizon.
- Not only microscopic particles but also macroscopic objects: compact BHs and stars are accelerated by SMBHs.

Outline

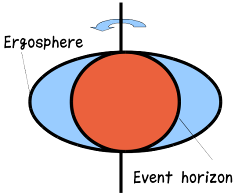
Kerr BHs

- Kerr metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

where $\rho^2 = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$.

- Nondimensional spin: $a_* = a/M$
- Horizon: $r_H = r_+ = M + \sqrt{M^2 - a^2}$
- Ergosphere: $r_E = M + \sqrt{M^2 - a^2\cos^2\theta}$
- Angular velocity: $\Omega_H = a/(r_H^2 + a^2)$
- Extremal: $a_* = 1$, where $r_H = M$, $r_E = M(1+\sin^2\theta)$, and $\Omega_H = 1/(2M)$



Geodesic motion in the equatorial plane

- Conserved quantities: $E = -p_t = -\xi^a p_a$, $L = p_\phi = \psi^a p_a$, where p^a is the four-momentum with $m^2 = -p_a p^a$.
- The geodesic eqs are reduced to a 1D potential problem

$$\frac{1}{2}\dot{r}^2 + V(r) = 0,$$

$$V(r) = -\frac{m^2 M}{r} + \frac{L^2 - a^2(E^2 - m^2)}{2r^2} - \frac{M(L - aE)^2}{r^3} - \frac{1}{2}(E^2 - m^2),$$

where the dot is the derivative w.r.t. the affine parameter.

- The condition $\dot{t} > 0$ near the horizon is reduced to $E - \Omega_H L \geq 0$.
- We call particles with $E - \Omega_H L = 0$ *critical particles*.

CM energy of colliding particles

- CM energy: the total energy of two particles at the same spacetime point observed in the centre-of-mass frame

$$p^a_{\rm tot} = p^a_1 + p^a_2, \quad E^2_{\rm cm} = -p^a_{\rm tot} p_{{\rm tot}\,a}.$$

- For the Kerr BH in the equatorial plane

$$\begin{aligned} E^2_{\rm cm} &= m_1^2 + m_2^2 \\ &\quad + \frac{2}{r^2}\left[\frac{P_1 P_2 - \sigma_1\sigma_2\sqrt{R_1}\sqrt{R_2}}{\Delta} - (L_1 - aE_1)(L_2 - aE_2)\right], \\ P(r) &= (r^2 + a^2)E - L, \\ R(r) &= P^2(r) - \Delta(r)[m^2 r^2 + (L - aE)^2]. \end{aligned}$$

CM energy for near-horizon collision

- $E_{\rm cm}$ in the limit to $r \to r_H$ for noncritical particles
  - "Head-on": $\sigma_1\sigma_2 = -1$, "Side": $\sigma_1\sigma_2 = 0$

$$E_{\rm cm} \propto \frac{1}{\sqrt{\Delta}} \propto \begin{cases} (r-r_H)^{-1/2} & (|a|<M) \\ (r-r_H)^{-1} & (|a|=M). \end{cases}$$

  - "Rear-end" collision (most likely to occur): $\sigma_1\sigma_2 = 1$

$$\begin{aligned} E_{\rm cm}^2 &= m_1^2 + m_2^2 - 2\frac{(L_1 - aE_1)(L_2 - aE_2)}{r_H^2} \\ &\quad + \frac{m_1^2 r_H^2 + (L_1 - aE_1)^2}{r_H^2}\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (1 \leftrightarrow 2). \end{aligned}$$

  Finite except in the limit $E_i - \Omega_H L_i \to 0$ (critical condition).

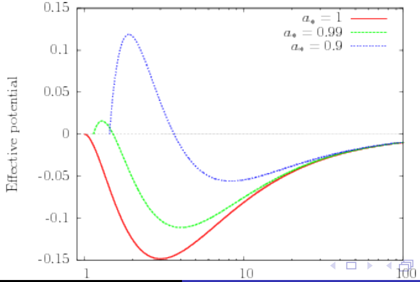
- Need special treatment for critical particles.

Motion of critical particles

- A massive particle which was at rest at infinity can reach the horizon if $l = L/(mM)$ satisfies

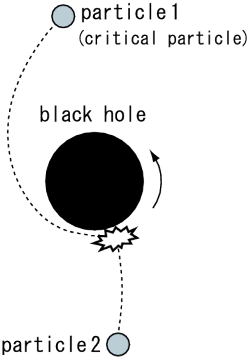
$$-2(1+\sqrt{1+a_*}) = l_L < l < l_R = 2(1+\sqrt{1-a_*}).$$

- $l_R \leq l_c$, where $l_R = l_c(=2)$ only for $a_* = 1$; a critical particle reaches the horizon only for this case after infinitely long proper time.



Banados-Silk-West process

- A particle with $l_L < l < l_R = l_c = 2$ can reach the horizon of an extremal Kerr BH from infinity.
- $E_{\rm cm}$ of particles 1 and 2 for the near-horizon collision diverges in the limit $l_1 \to 2$ (or $l_2 \to 2$). (Bañados, Silk, West 2009).
- For $l_1 = 2$ and $l_L < l_2 < 2$, $E_{\rm cm} \propto (r - r_H)^{-1/2}$ (Grib, Pavlov 2010).
- Necessary to finetune the angular momentum. Natural finetuning by the ISCO (Harada, Kimura 2011).



A physical explanation

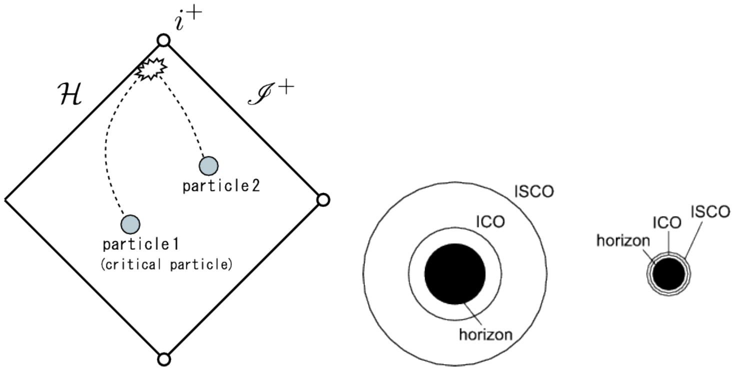


Figure: An infalling subcritical particle is accelerated to the light speed. If the observer can stay at a constant radius near the horizon, he or she will see the particle falling with almost the speed of light. (Harada, Kimura 2014, Zaslavskii 2011)

Outline

Collisional Penrose process

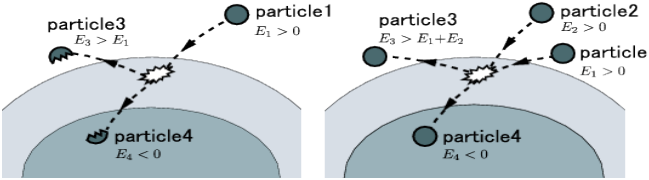


Figure: Left: Penrose process, Right: Collisional Penrose process

- A negative energy particle is possible in the ergoregion. This enables us to extract energy from the BH.
- The high energy collision may produce superheavy and/or superenergetic particles.
- Collisional Penrose process (Piran, Shaham & Katz 1975) $\eta = E_3/(E_1 + E_2)$ can be larger than unity.

Conservation laws and escape to infinity

- $E_1 + E_2 = E_3 + E_4, \quad L_1 + L_2 = L_3 + L_4$
- $p_1^r + p_2^r = p_3^r + p_4^r$
- Can the ejecta espape to infinity?

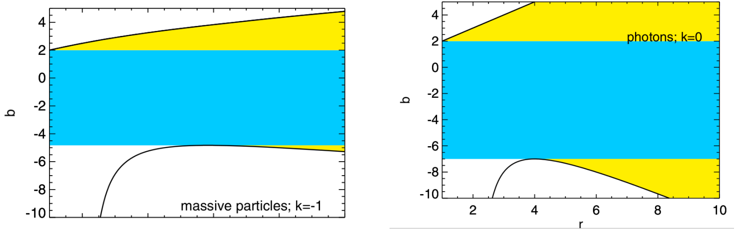
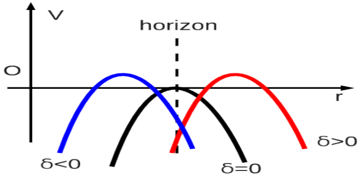


Figure: Turning points for an extremal Kerr black hole for the impact parameter b (Taken from Schnittmann 2014)

Near-critical particle near the horizon

- A sequence of particles: $L = 2ME(1+\delta)$, where $\delta = \delta_{(1)}\epsilon + O(\epsilon^2)$, at $r = M/(1-\epsilon)$.
- $\dot{t} > 0$ at $r = M/(1-\epsilon)$ yields $\delta < \epsilon + O(\epsilon^2)$.
- Turning points of the potential

$$r_{t,\pm}(e) = M\left(1 + \frac{2e}{2e \mp \sqrt{e^2+1}}\delta_{(1)}\epsilon\right) + O(\epsilon^2), \quad \text{where} \quad e = E/m.$$



- To escape to infinity from $r = M/(1-\epsilon)$, we need
  - (a) $e \geq 1$, $\delta_{(1)} < 0$ and $\sigma = 1$
  - (b) $e \geq 1$, $\delta_{(1)} > 0$ and $r \geq r_{t,+}(e)$, i.e., $\delta_{(1)} \leq \delta_{(1)\max}(e)$

Collision and reaction near the horizon

- Let us consider the process $1+2\to 3+4$ at $r=M/(1-\epsilon)$, 1=critical, 2=subcritical, 3=escape, 4=negative energy
- Let $L_3=2ME_3(1+\delta)$, where $\delta=\delta_{(1)}\epsilon+O(\epsilon^2)$
- The condition $\dot{t}>0$ and the conservation of energy and angular momentum are taken into account.
- Expand p_i^r in terms of ϵ. $p_1^r+p_2^r=p_3^r+p_4^r$ in $O(\epsilon)$ gives the equation for E_3. Assuming $\sigma_1=\sigma_2=\sigma_4=-1$,

$$\left(2E_1-\sqrt{3E_1^2-m_1^2}\right)+2E_3(\delta_{(1)}-1)=\sigma_3\sqrt{E_3^2(3-8\delta_{(1)}+4\delta_{(1)}^2)-m_3^2}.$$

Upper limits of the energy emission

- Assuming $E_1 \geq m_1$, we can derive upper limits on E_3.
  - (i) $\sigma_3 = 1$: $E_3 < E_1$, i.e., no energy extraction.
  - (ii) $\sigma_3 = -1$ and $r \geq r_{t,+}(e)$: E_3 takes a maximum $(2-\sqrt{2})/(2-\sqrt{3})E_1 = 2.186E_1$ for $m_3 = 0$ and $\delta_{(1)} = +0$.
- $\eta_{\max}$: 1.372 (Inverse Compton), 1.093 (Pair annihilation), 1.466 (Any reaction)

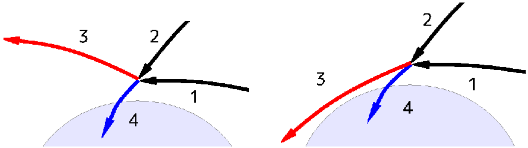


Figure: Left: without bounce ($\sigma_3 = 1$), Right: with bounce ($\sigma_3 = -1$).

Revision of the energy efficiency

- $1 + 2 \to 3 + 4$, 1=critical, 2=subcrit., 3=escape, 4=negative
- If $p_1^r < 0$, $p_2^r < 0$ and $p_3^r > 0$, then $\eta_{\max} < 1$.
- If $p_1^r < 0$, $p_2^r < 0$ and $p_3^r < 0$, then $\eta_{\max} \simeq 1.4$. (Bejger et al. 2012, Harada, Nemoto, Miyamoto 2012)
- If $p_1^r > 0$, $p_2^r < 0$ and $p_3^r < 0$, then $\eta_{\max} \simeq 14$. Possible if 1 is slightly supercritical (Schnittmann 2014).
- Analytic approach: Ogasawara, Harada, Miyamoto, in prep.

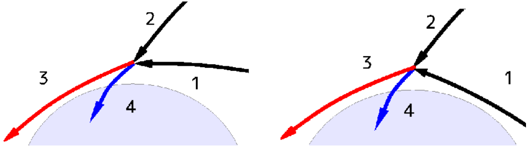


Figure: Left: Pre-Schnittmann, Right: Schnittmann.

Super-Penrose process

- $1 + 2 \to 3 + 4$, 1=ingoing subcritical, 2=outgoing subcritical, 3=escape, 4=negative
- Head-on collision in the ergoregion. $\eta_{\max} = \infty$. (Berti, Brito, Cardoso 2015)

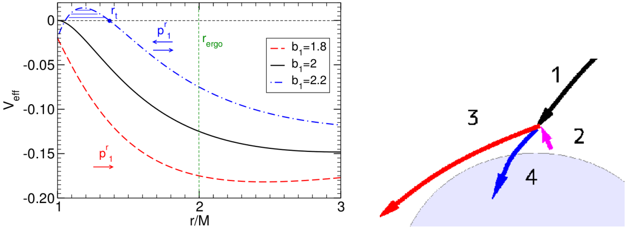


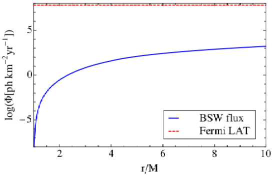
Figure: Left: Outgoing subcritical particle (Taken from Berti, Brito, Cardoso 2015), Right: Super-Penrose process

Physical significance of the super-Penrose process

- Where does the outgoing subcritical particle come from? One can assume a preceding process near the horizon: $X + Y \to 2 + Z$, where particles X and Y fall from infinity.
- What does $\eta = E_3/(E_1 + E_2) \to \infty$ mean? If we consider the preceding process, $\eta_{\rm tot} := E_3/(E_X + E_Y + E_1) \simeq 14$ at most. (Leiderschneider, Piran 2015, Berti, Brito, Cardoso 2015)
- Interesting in astrophysical situations, such as in the presence of accretion disks and/or magnetic fields.

Implications to observation

- Near-extremal BH spins are “measured” ($a_* > 0.98$, see McClintock et al. 2011).
- The observational effects of the BSW process (Bañados et al. 2011, Williams 2011, Gariel, Santos, Silk 2014 etc)
- But the BSW flux from dark matter annihilation is too low for the Fermi satellite detection (McWilliams 2013).



- The observational implications of the variant processes are yet to be clear.

Outline

Summary

- **High energy particle collision near a near-extremal Kerr BH is robust as it is founded on the basic properties of geodesic orbits.**
- **The upper limit of the energy efficiency of the collisional Penrose process has been revised from $\simeq 1$ to $\simeq 1.4$, $\simeq 14$, and even ∞ for the variant processes.**
- **These processes are potentially interesting in astrophysics, although their observational implications are yet to be clear.**