A rotating hairy AdS(3) black hole with only one Killing vector field

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One Hundred Years of
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Based on work with Norihiro Iizuka and Kengo Maeda

arXiv: 1505.00394

- A brief review of AdS instability

- Attempt to construct a less symmetric AdS BH

Is AdS stable?

Positive-Energy Theorem:

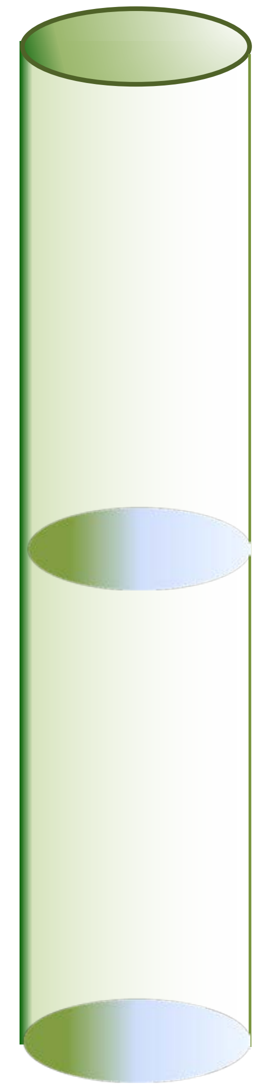
If the matter satisfies certain energy condition for all regular, asymptotically AdS initial data

$$E \geq 0$$

And only for exact AdS spacetime $E = 0$

AdS is a *ground state*

AdS should be stable(?)



c.f. Minkowski spacetime is a ground state for all regular, asymptotically flat spacetimes, and it is stable!

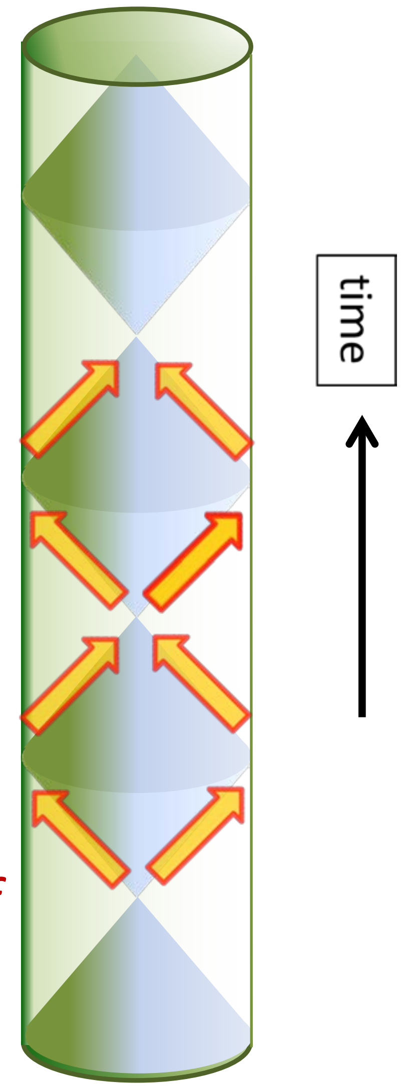
Dynamics in AdS

- Waves can reach AdS-infinity, bounce-off and return into the bulk within finite (coordinate) time

- AdS is like a confined box, whose conformal boundary acts just like a mirror.

- Under reflection boundary condition, No energy dissipation.

- Physical mechanism responsible for the *stability of Minkowski* spacetime is the *dissipation by dispersion*; the energy of perturbations radiates away to infinity. This is not the case for AdS



As a mathematical problem

- AdS is non-globally hyperbolic

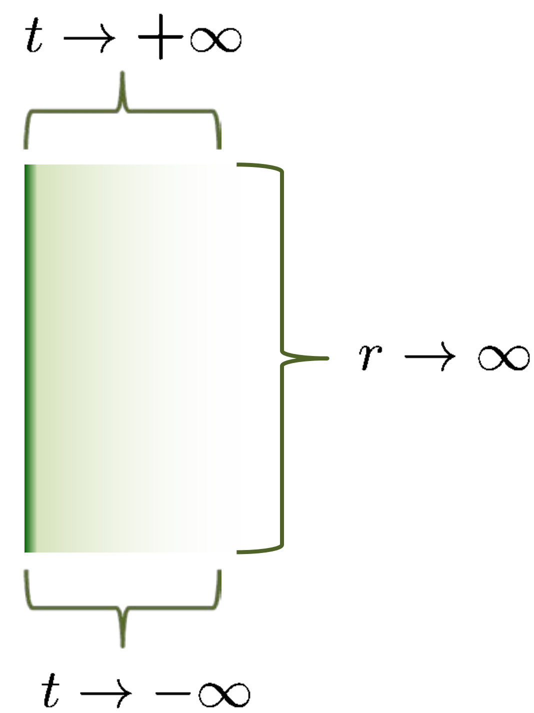
  Is initial-boundary value problem well-posed?

  yes! (Friedrich 95 AI-Wald04)

- AdS is rigid (M. Anderson 06)

Under AdS boundary conditions that asymptotic timelike-future, past, and spatial infinity be exact AdS, the inside must also be exact AdS

Any finite excitation might explore all configurations (including black holes)



- Based on the linear perturbation analysis

  Conjecture: Pure AdS is dynamically unstable

  (Dafermos-Holzegel 06)

  Conjecture: All asymptotically AdS spacetimes are dynamically unstable (Holzegel-Smulevici)

- AdS boundary acts like a confining box, hence AdS is like a closed Universe for the fields inside. It should be singular according to Hawking-Penrose's singularity theorem

  (Dias-Horowitz-Santos)

  ⇒ Negative answer to this singularity theorem (AI-Maeda 12)

Instabilities in AdS

AdS–infinity plays a crucial role.

- Boundary conditions (AdS itself)
- BF-bound / Condensation (AdS-BHs)
- Weakly turbulent instability (AdS itself)
- Superradiant Instability (Rotating BHs in AdS)

What is a possible endpoint of such instabilities?

BF-bound and instability

BF bound

Scalar field equation in AdS (in Poincare chart)

$$-\frac{\partial^2}{\partial t^2}\Phi = H\Phi \qquad H := -\frac{\partial^2}{\partial z^2} + \frac{\gamma^2 - 1/4}{z^2}$$

Schwarz-inequality $\frac{1}{4}\int dz \frac{\Phi^2}{z^2} \leq \int dz \left(\frac{\partial \Phi}{\partial z}\right)^2$

⇒ $0 \leq \int_0^\infty dz \Phi \left(-\frac{\partial^2}{\partial z^2} + \frac{-1/4}{z^2}\right)\Phi$ ($\gamma = 0$ case)

⇒ $0 \leq (\Phi, H\Phi)$

$\gamma^2 < 0$ Negative spectrum : unbounded below

$\gamma = 0$ corresponds to BF-bound $m_{\mathrm{BF}}^2 = -\frac{(D-1)^2}{4\ell^2}$

$0 \leq \gamma^2 < 1$ Spectrum depends on the choice of boundary conditions

$1 < \gamma^2$ Positive spectrum : only Dirichle boundary conditions allowed

- Hartnoll-Herzog-Horowitz 08
  holographic superconductors

- Dias-Monteiro-Reall-Santos-10 :
  Scalar field condensation instability in AdS

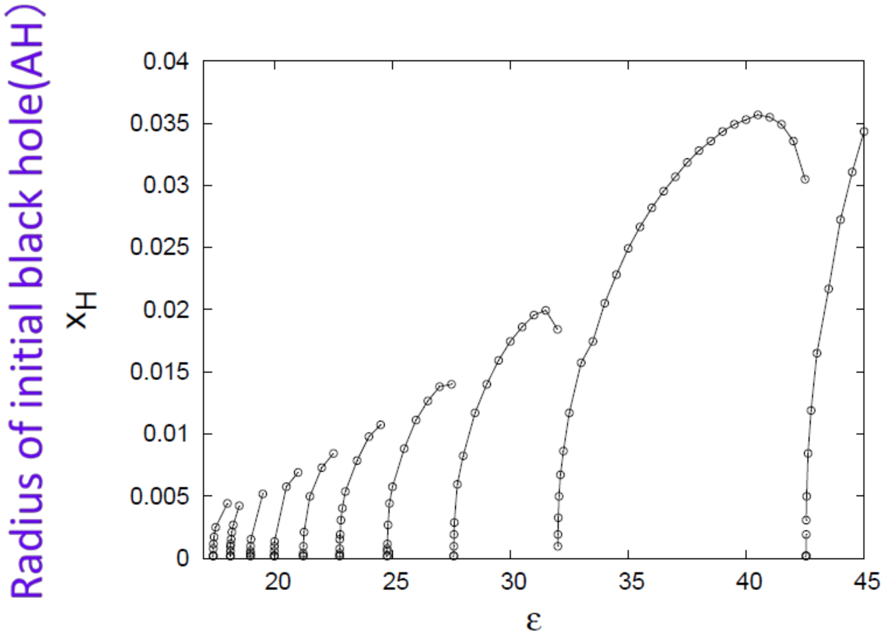
- Durkee-Reall 11
  Instability of Extremal Mypers-Perry BHs
  Near-horizon geometry contains AdS(2) as a subspace
  $\Rightarrow$ violation of near-horizon BF-bound for axisymmetric perturbation
  $\Rightarrow$ instability

  -- proof Hollands-AI 14

Turbulent instability

Turbulent instability

The energy cascades from low frequency to high frequency



Initial small perturbations grow by repeating bounce-off by AdS infinity

Black hole forms even starting from arbitrarily small initial perturbations

Initial data amplitude and a sequence of critical amplitude

$$x_H(\varepsilon) \sim (\varepsilon - \varepsilon_n)^{\gamma}$$

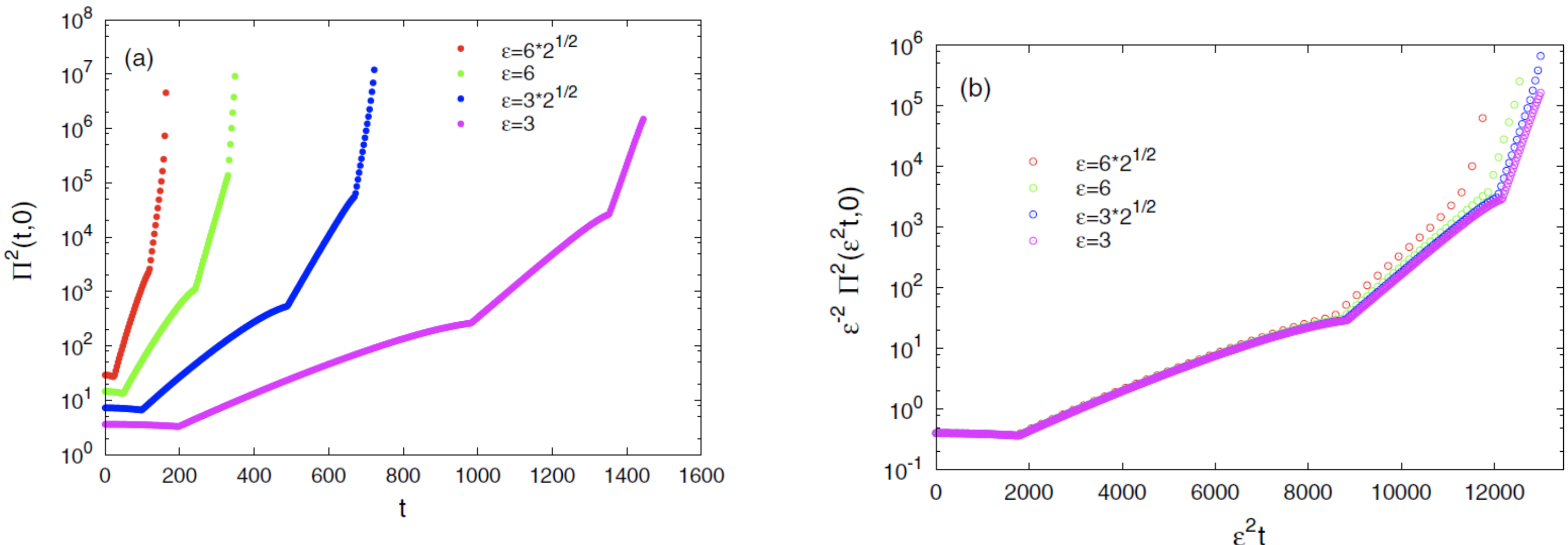
Spherically symmetric, mass-less scalar field

Bizon, Rostworowski 2011 Jalmuzna, Rostworowski, Bizon 2011

Vacuum, gravitational waves Dias, Horowitz, Santos 2011

Initial data: $\Phi(0,x) = 0\,, \quad \Pi(0,x) = \varepsilon \exp\left(-\frac{4\tan^2 x}{\pi^2\sigma^2}\right)$

$\Pi = A^{-1} e^{\delta} \dot{\phi}$

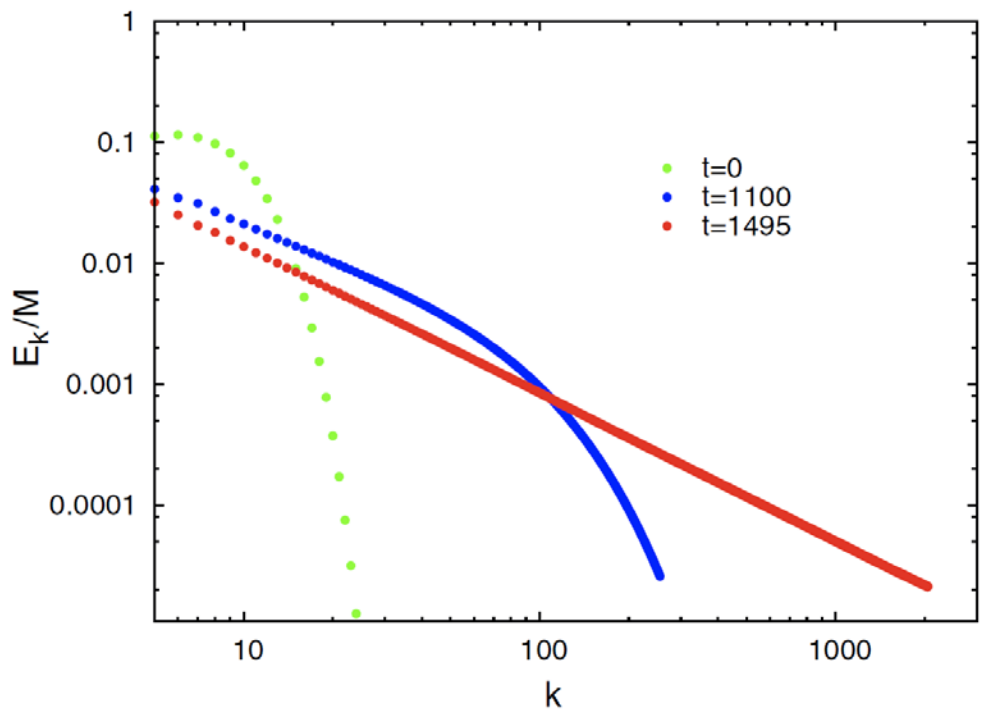


Time

AdS (d>3) appears to be unstable against the formation of a black hole for a large class of arbitrarily small perturbations

The secular terms are progenitors of higher-order mode mixing that shifts the energy spectrum to higher frequencies

k-mode Energy: $E_k := \Pi_k^2 + \omega_k^{-2}\Phi_k^2$



Energy shifts from low-to-higher frequencies

Is AdS generically singular ?

No!

Stability islands of initial data in a sea of turbulent instability

Dias-Horowitz-Marolf-Santos 2012

Buchel-Liebling-Lehner 2013

Maliborski-Rostworowski 2013

e.g. geons, boson stars in AdS ,

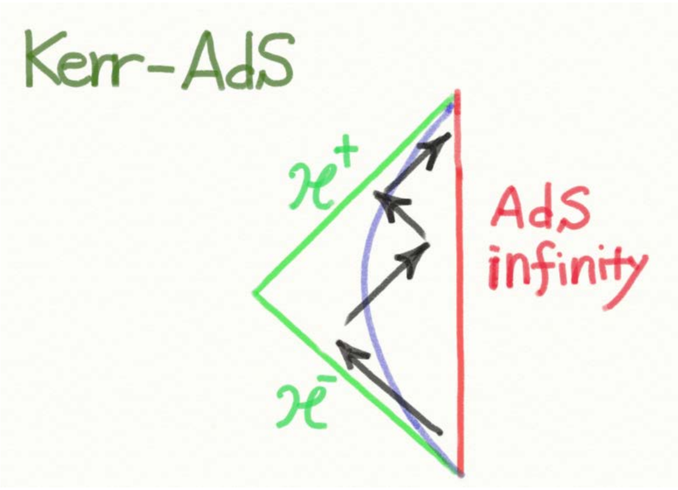
time-periodic regular solutions

-- indicating there exist generic initial data
in strongly coupled CFT that never thermalize

Superradiant instability

Stability in AdS

- Superradiant instability for rotating BHs



Hawking-Reall 99, Cardoso-Dias 04
Cardoso-Dias-Yoshida 06,
Kodama 07
Uchikata-Yoshida-Futamase 09
Kodama-Konoplya-Zhidenko 09

$$E := -\int_S dS n^a \chi^b T_{ab} \quad \chi^a\text{: co-rotate Killing vector}$$

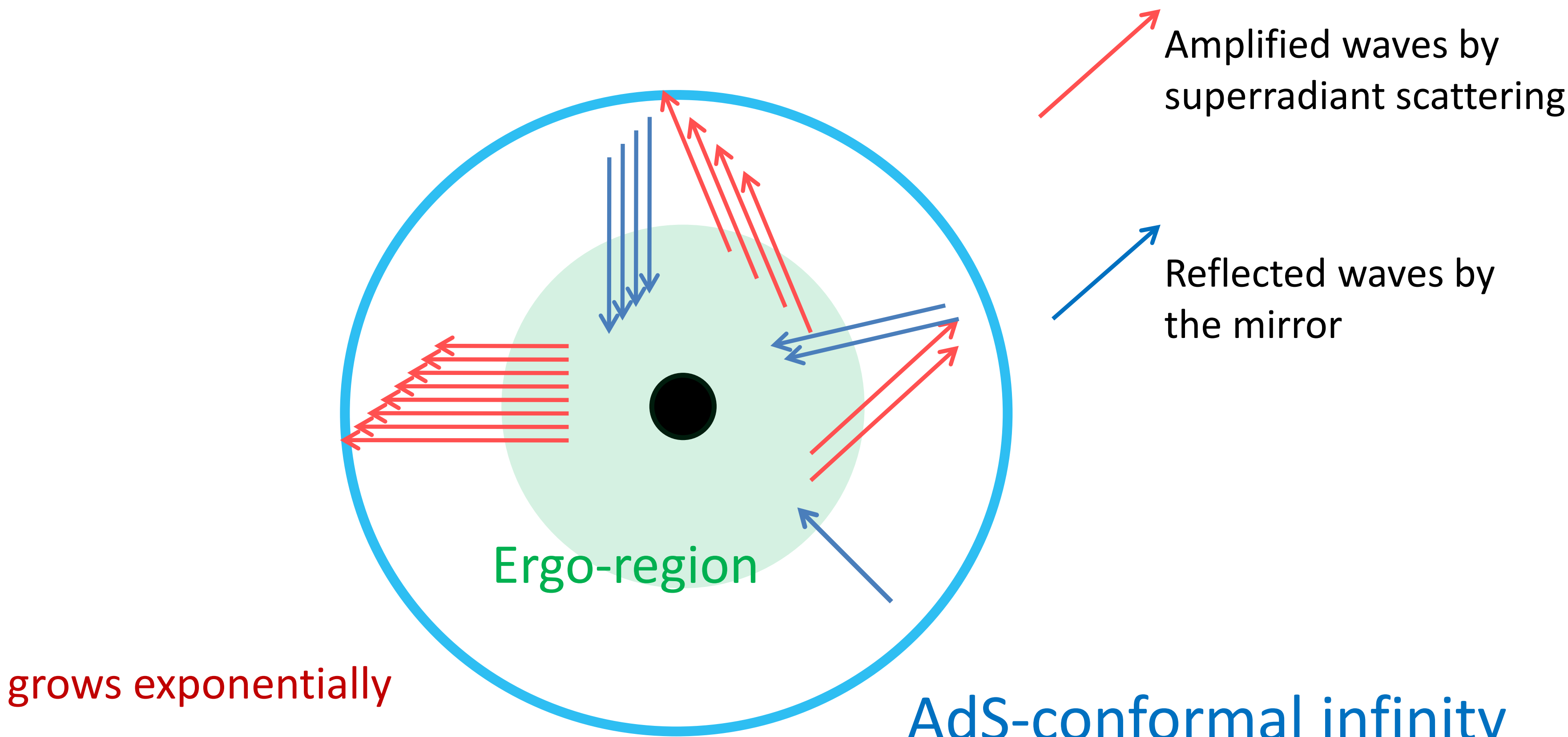
Note: χ^a can be non-spacelike if $a^2 \leqslant r_H^4/\ell^2 \quad \Longrightarrow \quad E \geqslant 0$

Stability depends also on the boundary conditions.

e.g. Static case: AdS-Schwarzschild Kodama & AI 04

Superradiant instability of Kerr-AdS

- Amplified scattered waves bounce back and forth between the mirror and the BH



End point of superradiant instability

- Kunduri-Lucietti-Reall 06 :
  Grav. insatbility of High-D Kerr (MP)-AdS

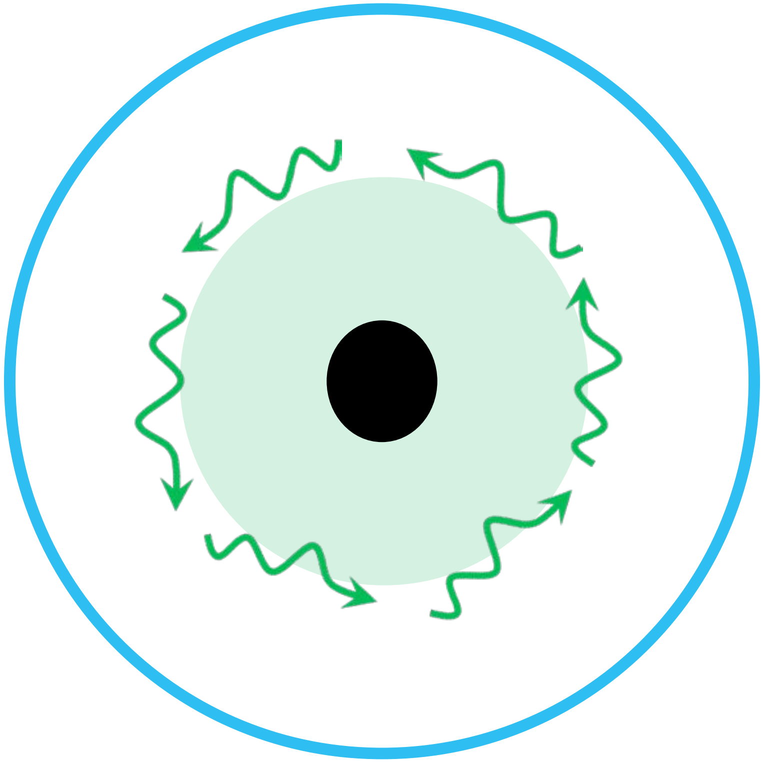
  -- suggest: non-axisymmetric BH that is
  stationary only wrt Killing of bounded norm

- Cardoso-Dias-Lemos-Yoshida 04, 06:
  Scalar / Grav. instability of Kerr-AdS

  -- suggest: slightly oblate and coexist in equilibrium with some outside radiation.

  AdS-boundary is the Einstein universe rotating with the speed of light velocity.

Axisymmetry must be broken in AdS !?



$$\Omega \leq 1/\ell$$

Scalar field/grav. waves condensation outside the BH

Black Hole with only one Killing field

- Dias-Horowitz-Santos 2011:

Complex scalar field violates axial-symmetry, hence the solution is invariant only under a single Killing field.

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left[R + \frac{12}{\ell^2} - 2\left|\nabla\vec{\Pi}\right|^2\right]$$

$$\vec{\Pi} = \Pi\, e^{-i\,\omega\,t + i\,\psi}\begin{bmatrix}\sin\left(\frac{\theta}{2}\right)\ e^{-i\frac{\phi}{2}} \\ \\ \cos\left(\frac{\theta}{2}\right)\ e^{i\frac{\phi}{2}}\end{bmatrix}$$

- Stotyn-Park-McGrath-Mann 2012 Stotyn-Chanona-Mann 2014:

Any (odd-) dimensions $D \geq 3$

- Herdeiro-Radu 2014:

  Asyptotically flat hairy BH with complex massive scalar field

- C. Charmousis' talk

  C. Herdeiro's talk

Black Hole with only one Killing field

Complex scalar field violates axial-symmetry, hence the solution is invariant only under a single Killing field.

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left[R + \frac{12}{\ell^2} - 2\left|\nabla\vec{\Pi}\right|^2\right]$$

$$\mathrm{d}s^2 = -fg\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f} + r^2\left[h\left(\mathrm{d}\psi + \frac{\cos\theta}{2}\mathrm{d}\phi - \Omega\mathrm{d}t\right)^2 + \frac{1}{4}\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$$

$$\vec{\Pi} = \Pi\, e^{-i\,\omega\, t + i\,\psi}\begin{bmatrix}\sin\left(\frac{\theta}{2}\right)\, e^{-i\frac{\phi}{2}} \\ \cos\left(\frac{\theta}{2}\right)\, e^{i\frac{\phi}{2}}\end{bmatrix}$$

Killing horizon : $K^a = t^a + \Omega_H \varphi^a$ t^a and φ^a are not Killing fields

The metric itself admits more than one Killing field

Our purpose

Attempt to construct a rotating AdS black hole whose metric admits only a single Killing field

Setup

- 3-dimensions w/ massive reall scalar field

$$L = R + \frac{2}{l^2} - 2(\nabla_\mu \phi \nabla^\mu \phi + m^2 \phi^2 + \eta \phi^4)$$

- Nonlinear perturbation from rotating BTZ

$$ds^2 = -\frac{(r_+^2 - r_-^2)^2 z}{l^2(1-z)(r_+^2 - r_-^2 z)} dt^2 + \frac{l^2}{4z(1-z)^2} dz^2 + \frac{r_+^2 - r_-^2 z}{1-z} \left(d\varphi - \frac{r_+ r_-(1-z)}{l(r_+^2 - r_-^2 z)} dt \right)^2 ,$$

$z = 1$ at AdS-infinity

Strategy

- Under general boundary conditions
  Identify the marginally stable solution.

- Perturbatively construct a rotating BH upto $O(\epsilon^4)$ of the scalar field amplitude ϵ

- The lumps of non-linearly perturbed geometry admits only one Killing vector field that corotates with the BH

- The entropy of our hairy BH is larger than the entropy of BTZ with the same mass and angular momentum

Behavior of scalar field near AdS-infinity

$$\phi = \mathrm{Re}[\Pi_1(z)]\cos(\omega t - k\varphi)$$

Under in-going boundary condition at the horizon

$$\Pi_1 \simeq \alpha(1-z)^{\frac{1-\sigma}{2}} + \beta(1-z)^{\frac{1+\sigma}{2}}$$

$$\sigma = \sqrt{1+m^2l^2}$$

BF-bound corresponds to $m^2l^2 = -1$

Horizon Killing field $K^a = \partial_t + \Omega\partial_\phi$ is everywhere causal $K_aK^a \leq 0$

No Superradiant instability in the standard sense
under Dirichlet boundary condition $\alpha = 0$

General boundary conditions and Instability in BTZ

For $-1 < m^2 l^2 < 0$ one can choose general boundary conditions

$$\alpha = \kappa^{-1} \beta$$

that correspond to adding a double-trace interaction

$$\sim (1/2\kappa^2) \int dx^2 \mathcal{O}^2$$

to the boundary theory: $\mathcal{O}$ is a operator dual to ϕ

Note: depending on the choice of boundary conditions,
AdS spacetime itself can be unstable…

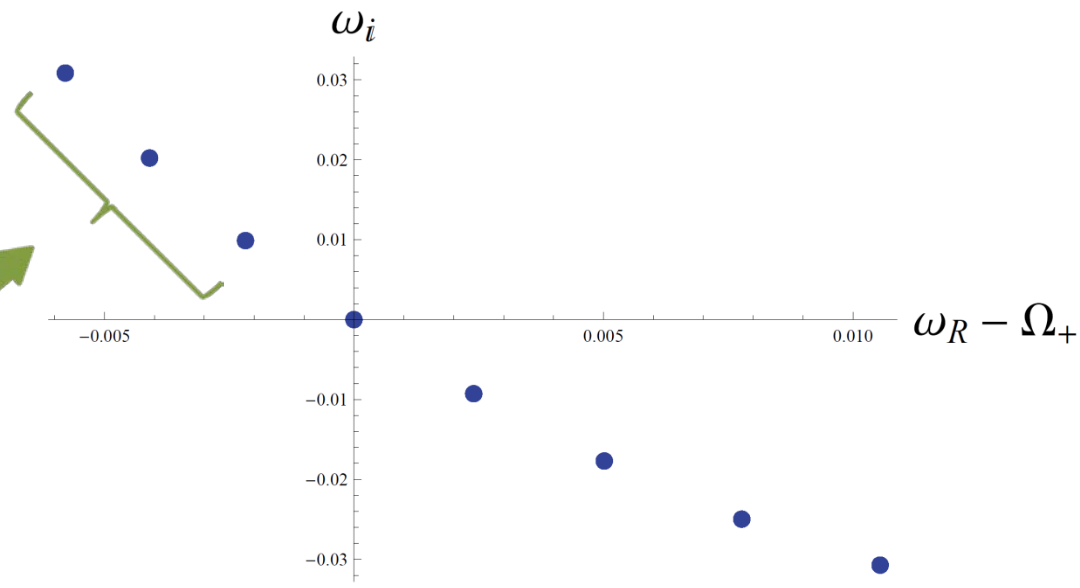
General boundary conditions and Instability in BTZ

For $-1 < m^2 l^2 < 0$ one can choose general boundary conditions

$$\alpha = \kappa^{-1} \beta$$

Our choice: $l = 1 = k$

$$\begin{cases} m^2 = -8/9l^2 \\ \kappa = -0.414 \end{cases}$$



Unstable modes:
$\omega_I > 0 \quad \omega_R < \Omega_+$

Marginal mode: $r_+ = 5 \quad r_- = 3 \quad \Omega_+ = 3/5$

Perturbation from the marginally stable solution

Having the marginally stable solution we expect there to exist a hairy BH dressed with a condensed scalar field

In the probe limit, scalar field depends only on $\Omega_+ t - \varphi$ and z

Metric ansatz:

$$ds^2 = -fe^{-2\delta}dt^2 + \frac{g'^2 dz^2}{4gf} + g(d\varphi - \Omega dt)^2$$

$$g(z) \quad f(y,z) \quad \delta(y,z) \quad \Omega(y,z) \qquad y := \omega_* t - \varphi$$

Perturbative expansion:

$$\begin{cases} \phi(y,z) = \epsilon\phi_1(y,z) + \epsilon^3\phi_2(y,z) + \cdots \\ F = F_0(y,z) + \epsilon^2 F_1(y,z) + \epsilon^4 F_2(y,z) + \cdots \\ \omega_* = \dfrac{3}{5l} + \epsilon^2\omega_1 + \cdots \end{cases}$$

$$F = f,\ g,\ \Omega,\ \delta$$

Boundary conditions

$$\lim_{z\to 1}\Omega = \lim_{z\to 1}\delta = 0$$ at AdS-infinity

$$\omega_* = \Omega\Big|_{z=0}$$ Regularity at the horizon

Expansion in $O(\epsilon^2)$

$$\phi_1 = \Pi_1(z)\cos(ky)$$

$$\begin{cases} f_1(y,z) = z(P(z)\cos(2ky) + Q(z)) + a_1 f^{(1)}_{\rm BTZ}(z) \\ \delta_1(y,z) = R(z)\cos(2ky) + S(z), \quad g_1(z) = a_1 g^{(1)}_{\rm BTZ}(z) \\ \Omega_1(y,z) = zT(z)\cos(2ky) + U(z) \end{cases}$$

$(P,\ R,\ T)$ decouple from $(Q,\ S,\ U)$ and given by $T,\ T',\ \Pi_1,$ and Π_1'

Master equation

$$zT'' + \left(4 + \frac{2k^2l^2}{r_+^2 - r_-^2} - \frac{2r_+^2}{r_+^2 - r_-^2 z}\right)T' - \frac{2r_-^2(r_+^2 - r_-^2 + k^2l^2)}{(r_+^2 - r_-^2)(r_+^2 - r_-^2 z)}T = S_T(\Pi_1, \Pi_1')$$

Zero mode solutions

$$h(z) = -\frac{k^2 r_-^2}{r_+^2}\int_0^z \frac{\Pi_1^2}{1-z}dz - \frac{4r_+^2}{l^2}\int_0^z (1-z)\Pi_1'^2 dz$$
$$+\frac{2r_+r_-}{l}[U(z)-U(0)] - \frac{2r_+^2(r_+^2-r_-^2)}{l^2}\left[\frac{S(z)}{r_+^2-r_-^2 z} - \frac{S(0)}{r_+^2}\right],$$

$$S = \frac{1}{r_+^2 - r_-^2}\int_z^1 \left[\frac{k^2 r_-^2 l^2}{2r_+^2}\Pi_1^2 + 2(1-z)(r_+^2 - r_-^2 z)\Pi_1'^2\right] dz,$$

$$U = \frac{r_+^4(z-1)B}{(r_+^2 - r_-^2 z)(r_+^2 - r_-^2)}$$
$$+\frac{z-1}{(r_+^2-r_-^2 z)(r_+^2-r_-^2)}\int_0^z (r_+^2 - r_-^2 z')^2 \mathcal{S}_U(z')dz',$$
$$+\frac{1}{r_+^2-r_-^2}\int_z^1 (r_+^2 - r_-^2 z')(z'-1)\mathcal{S}_U(z')dz',$$

$$\mathcal{S}_U = -\frac{r_- l k^2}{2r_+(1-z)(r_+^2 - r_-^2 z)}\Pi_1^2 - \frac{2r_+r_-(1-z)}{l(r_+^2 - r_-^2 z)}\Pi_1'^2,$$

$$h = zQ + \frac{2(r_+^2-r_-^2)z}{l^2}\Pi_1\Pi_1'$$

Higher order solutions

$$\phi_2(y,z) = \phi_{21}(z)\cos(ky) + \phi_{23}(z)\cos(3ky),$$
$$\mathcal{L}_k\phi_{21} = \mathcal{S}_{\phi 1}, \quad \mathcal{L}_{3k}\phi_{23} = \mathcal{S}_{\phi 3},$$

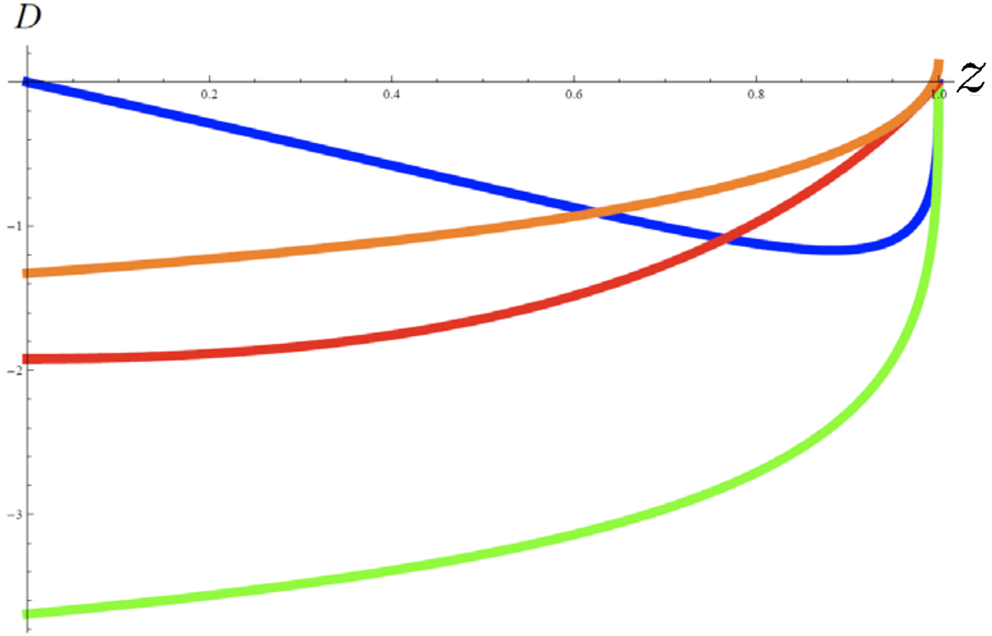


FIG. 2: $D = \phi_{21}$ (solid, blue), $D = \phi_{23}/2$ (dashed, green), and $D = \Omega_{20}$ (dotted, red), $D = 10^{-3} \cdot h_{20}$ (dotdashed, orange) for the parameter choice $r_+ = 5$, $r_- = 3$, $k = 1$, $l = 1$, and $\eta = 1$.

Effective holographic energy

$$T_{ij} = K_{ij} - Kh_{ij} - \frac{1}{l}h_{ij}$$
$$- \sqrt{z}(1-z)\{(1-\sigma)(1-z)^{-\sigma} + 2\kappa\}\,\tilde{\alpha}^2\,\frac{h_{ij}}{l}$$

$$\phi \simeq \tilde{\alpha}(t,\varphi)(1-z)^{\frac{1}{3}} + \kappa\tilde{\alpha}(t,\varphi)(1-z)^{\frac{2}{3}}$$

Energy and Angular momentum

$$E^{(2)}_{\rm hair} := \overline{T^{(2)}}_{tt} \simeq -\frac{h(1)}{2l}\epsilon^2 + \frac{r_+^2 + r_-^2}{r_+ l^3}\epsilon^2 a_1,$$
$$J^{(2)}_{\rm hair} := -\overline{T^{(2)}_{t\varphi}} = -\epsilon^2\left(\frac{r_+^2 - r_-^2}{l}U'(1) - \frac{2a_1 r_-}{l^2}\right).$$

First law of the hairy BH

$$E^{(2)}_{\mathrm{hair}} = TS^{(2)}_{\mathrm{hair}} + \Omega_+ J^{(2)}_{\mathrm{hair}} \qquad T := \frac{r_+^2 - r_-^2}{2\pi r_+ l^2}$$

Comparison with BTZ BH in $O(\epsilon^4)$

Expand the horizon radius : $R_+ = r_+ + a_1\epsilon^2 + a_2\epsilon^4 + \cdots$

Set : $S_{\mathrm{BTZ}} = S_{\mathrm{hair}} \quad E^{(2)}_{\mathrm{BTZ}} = E^{(2)}_{\mathrm{hair}} \quad J^{(4)}_{\mathrm{BTZ}} = J^{(4)}_{\mathrm{hair}}$

$$E^{(4)}_{\mathrm{BTZ}} = \frac{l^3 (E^{(2)}_{\mathrm{hair}})^2}{2r_-^2} - \frac{\epsilon^2 a_1 (r_+^2 + r_-^2)}{r_+ r_-^2} E^{(2)}_{\mathrm{hair}} + \frac{r_- J^{(4)}_{\mathrm{hair}}}{l r_+} + \frac{\epsilon^4 a_1^2}{2l^3}\left(3 + \frac{r_+^2}{r_-^2}\right) + \frac{\epsilon^4 a_2 (r_+^2 - r_-^2)}{r_+ l^3}$$

Evaluation:

$$\Delta^{(4)} E := E^{(4)}_{\mathrm{hair}} - E^{(4)}_{\mathrm{BTZ}} \simeq -5.8 \times 10^2 \cdot \epsilon^4 < 0$$

Evaluation: $\Delta^{(4)}E := E^{(4)}_{\rm hair} - E^{(4)}_{\rm BTZ}$

$$\simeq -5.8 \times 10^2 \cdot \epsilon^4 < 0$$

This in turn implies that if we increase $E^{(4)}_{\rm hair}$

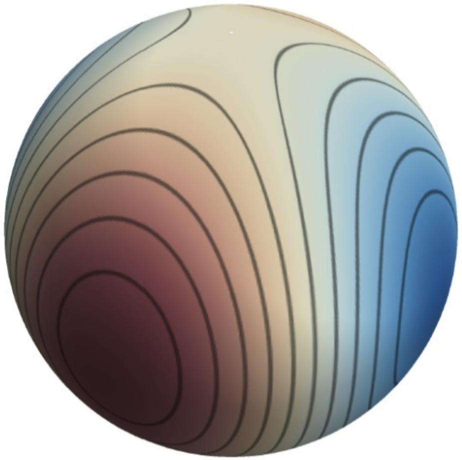
so that it becomes equal to $E^{(4)}_{\rm BTZ}$

then we have $\boxed{S^{(4)}_{\rm hair} > S^{(4)}_{\rm BTZ}}$

- Dias-Santos-Way 2015

Black resonators: only one Killing field

-- connects Superradiant instability of Kerr-AdS to Turbulent instability



Summary and discussion

- We have constructed a hairy AdS_3 black hole whose metric possesses only one Killing field.

- The entropy of the hairy BH is larger than that of BTZ within our perturbative analysis
  ⇒ possible end point? Stable?

- The solution does not dissipate
  ⇒ similar to time-periodic solution of Maliborski-Rostworowski with a BH added.