One hundred years of strong gravity @ Instituto Superior Técnico on 12th, June, 2015

On the gravitational collapse in confined geometries

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

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Hirotada Okawa

HO, V. Cardoso, P. Pani, Phys. Rev. **D 89** 041502(2014), arXiv:1311.1235[gr-qc],
HO, V. Cardoso, P. Pani, Phys. Rev. **D 90** 104032(2014), arXiv:1409.0533.[gr-qc],
HO, J. Lopes, V. Cardoso, arXiv:1504.05203.

Outline of my talk

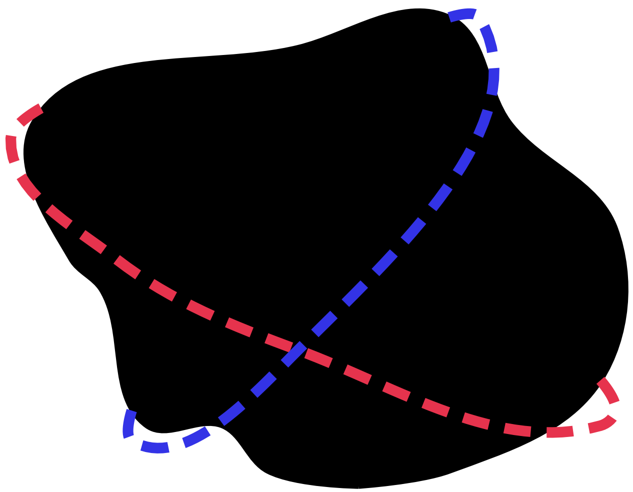
- **Introduction**
  - Gravitational collapse
  - Gravitational collapse in Anti de Sitter(AdS)
    →**A. Ishibashi's talk on yesterday**
- **Methods and Results**
  - Massive scalar field collapse in asymptotic flat spacetime
  - Scalar field collapse in AdS
- **Summary and Discussion**

Introduction

Stability against Gravitational Collapse

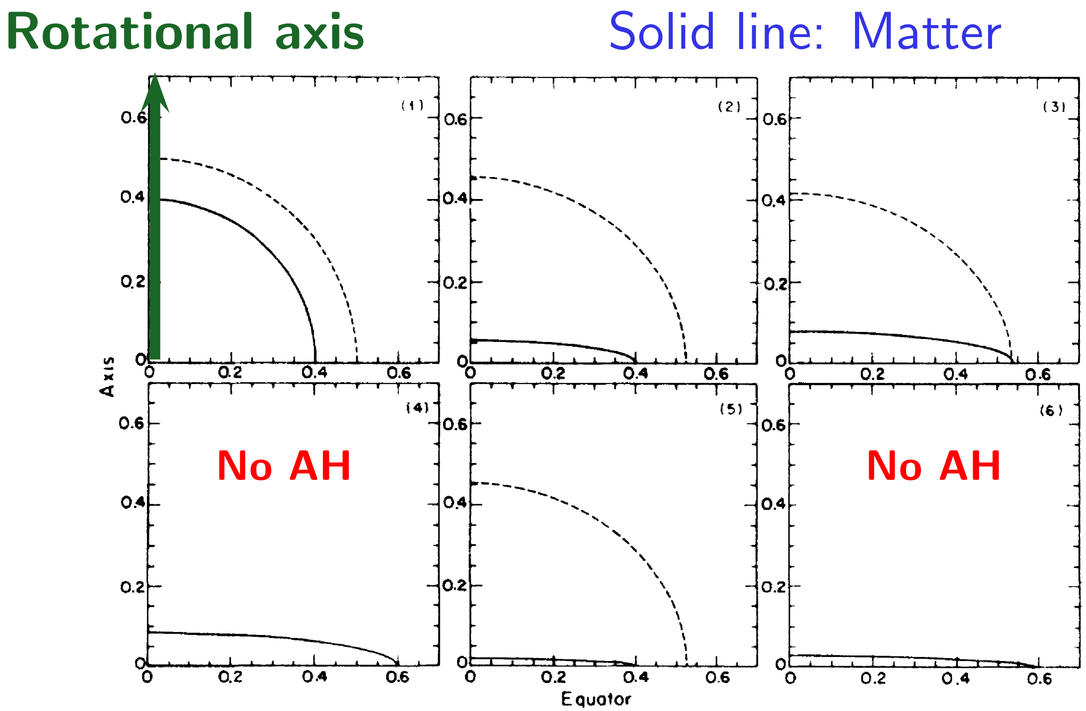
- Perturbations in the Minkowski background simply decay.
  (Christodoulou '86, Christodoulou&Klainermann '93)
- **Hoop conjecture**(Thorne '72)

  BHs can be produced when sufficient energy is confined by sufficiently small region.



Hoop length $\mathcal{C} \lesssim 2\pi R_s$

Dashed line: Horizon
Solid line: Matter



Cf. Nakamura, Shapiro, Teukolsky '88

Critical Collapse in asymptotic flat spacetime

- **Metric ansatz**:

$$ds^2 = -\alpha^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2 d\Omega^2.$$

- **Klein-Gordon eq.**: $\Box\phi = 0$

(massless, $\Phi = \phi'$, $\Pi = a\dot{\phi}/\alpha$)

$$\dot{\Phi} = \left(\frac{\alpha}{a}\Pi\right)', \quad \dot{\Pi} = \frac{1}{r^2}\left(r^2\frac{\alpha}{a}\Phi\right)',$$

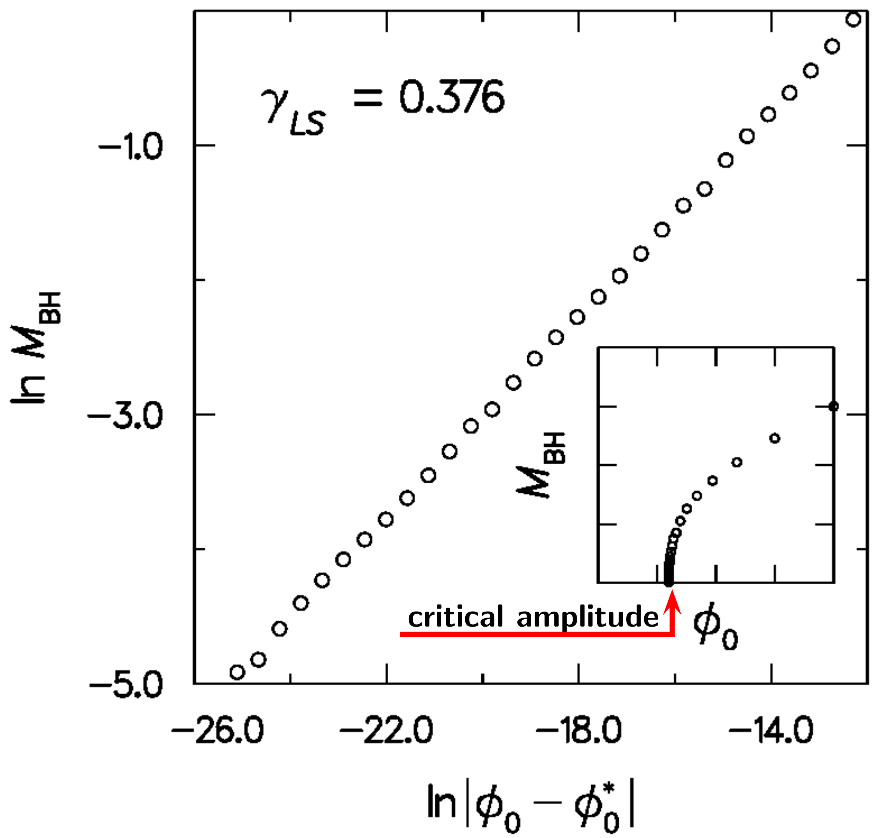
- Initial data: Gaussian wavepackets

- **Einstein eqs.**: $R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$0 = \frac{\alpha'}{\alpha} - \frac{a'}{a} + \frac{1-a^2}{r},$$

$$0 = \frac{a'}{a} + \frac{a^2-1}{2r} - 2\pi r(\Pi^2 + \Phi^2).$$

Choptuik '93



- Zero mass BH forms at the critical point.

$$M_{BH} \propto |\phi_0 - \phi_0^*|^\gamma$$

Critical Collapse in AdS

Metric ansatz: $r \to \tan x$

$$ds^2 = \frac{\ell^2}{\cos^2 x}\left(-Ae^{-2\delta}dt^2 + A^{-1}dx^2 + \sin^2 x d\Omega^2\right)$$

Klein-Gordon eq.: $\Box\phi = 0$
($\Phi \equiv \phi'$, $\Pi \equiv A^{-1}e^{\delta}\dot{\phi}$.)

$$\dot{\Phi} = \left(Ae^{-\delta}\Pi\right)', \quad \dot{\Pi} = \frac{1}{\tan^2 x}\left(\tan^2 x A e^{-\delta}\Phi\right)'.$$

Einstein eqs.: $R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \mathbf{\Lambda} g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$$A' = \frac{1 + 2\sin^2 x}{\sin x \cos x}(1 - A) - \sin x \cos x A\left(\Phi^2 + \Pi^2\right),$$

$$\delta' = -\sin x \cos x(\Pi^2 + \Phi^2).$$

Critical Collapse in AdS

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$$A' = \frac{1+2\sin^2 x}{\sin x \cos x}(1-A) - \sin x \cos x A\left(\Phi^2 + \Pi^2\right),$$
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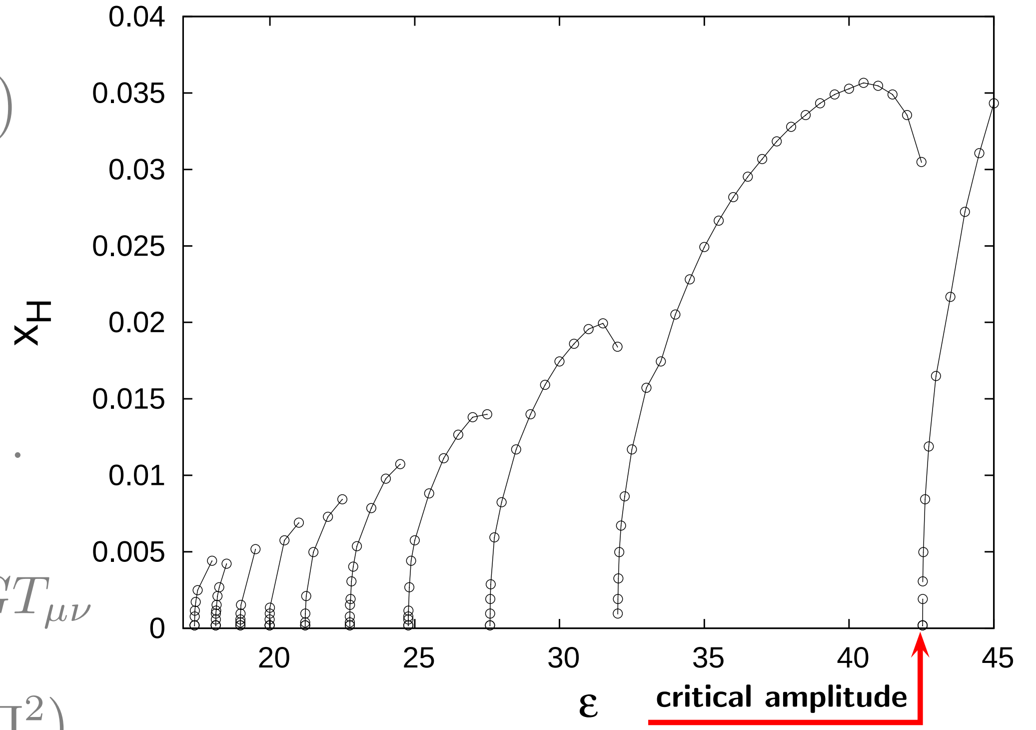
Bizoń&Rostworowski('11)



- Anti-de-Sitter spacetime can be regarded as a confined geometry.
- Waves with smaller amlitude than the critical amplitude can be reflected by the AdS boundary(spatial infinity), and eventually collapse to a BH.

Weak turbulence in AdS

- Pertubative expansion

$$\phi = \sum_{j=0}^{\infty} \phi_{2j+1}\epsilon^{2j+1}, \quad A = 1 - \sum_{j=1}^{\infty} A_{2j}\epsilon^{2j},$$

$$\delta = \sum_{j=1}^{\infty} \delta_{2j}\epsilon^{2j}.$$

- Linear order

$$\ddot{\phi}_1 + L\phi_1 = 0, \quad L = -\frac{1}{\tan^2 x}\partial_x(\tan^2 x\partial_x),$$

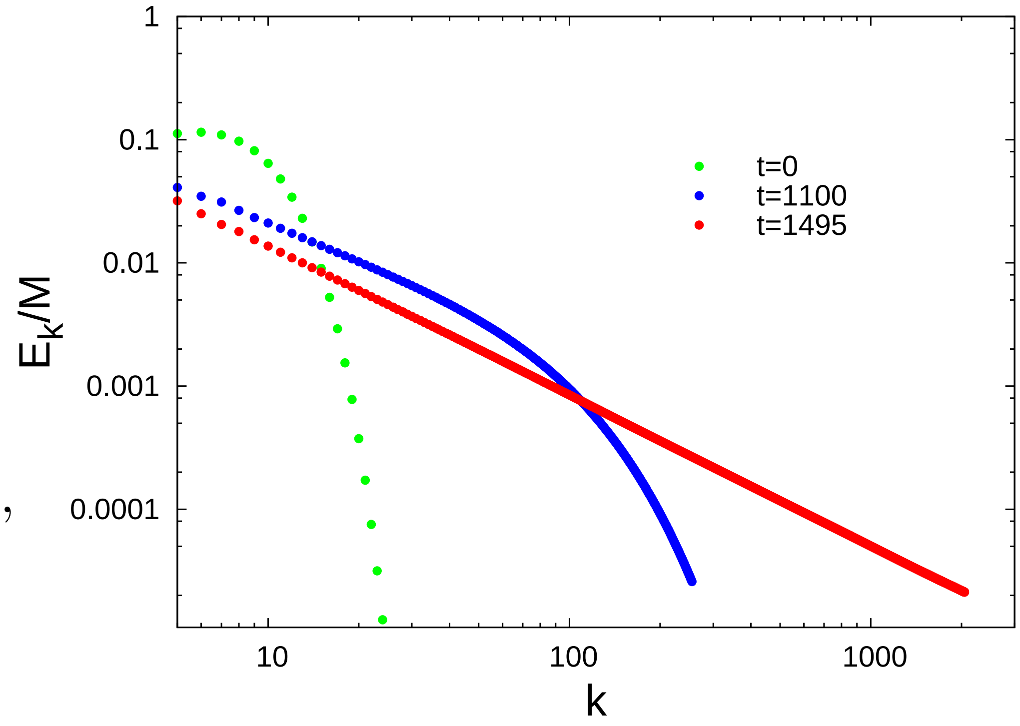
- Stable solution
(Breitenlohner&Freedman '82, Ishibashi&Wald '04)

$$\phi_1(t,x) = \sum_{j=0}^{\infty} a_j \cos(\omega_j t + \beta_j)e_j(x)$$

- Eigenvector(Eigenvalue $\omega_j = 3 + 2j$)

$$e_j(x) = d_j \cos^3 x\, {}_2F_1(-j, 3+j, 3/2; \sin^2 x)$$

Bizoń('13)



- Next order

From Einstein eqs.

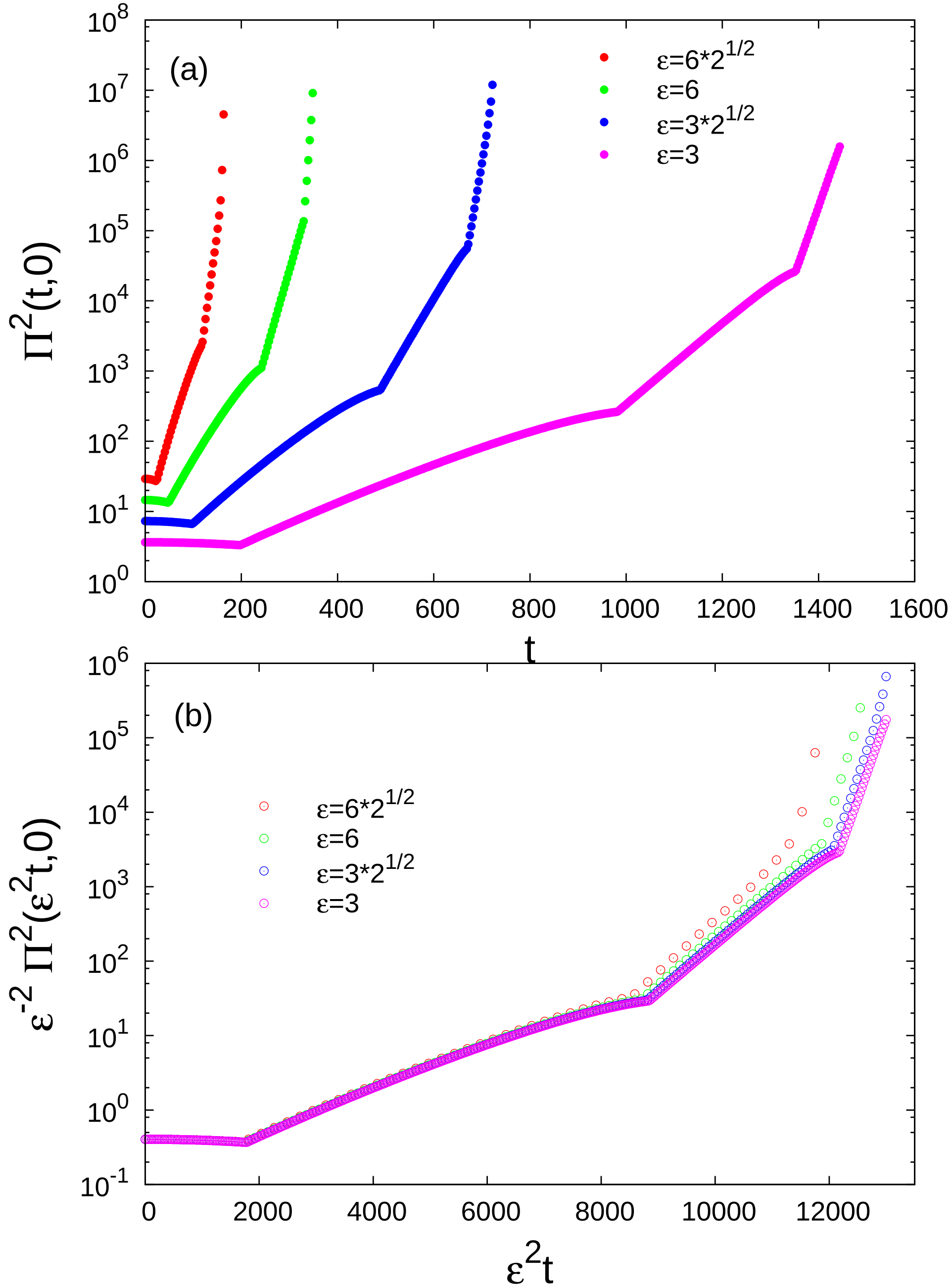
$$\ddot{\phi}_3 + L\phi_3 = S(\phi_1, A_2, \delta_2),$$

- Resonance modes: $w_i = w_k + w_l - w_m$

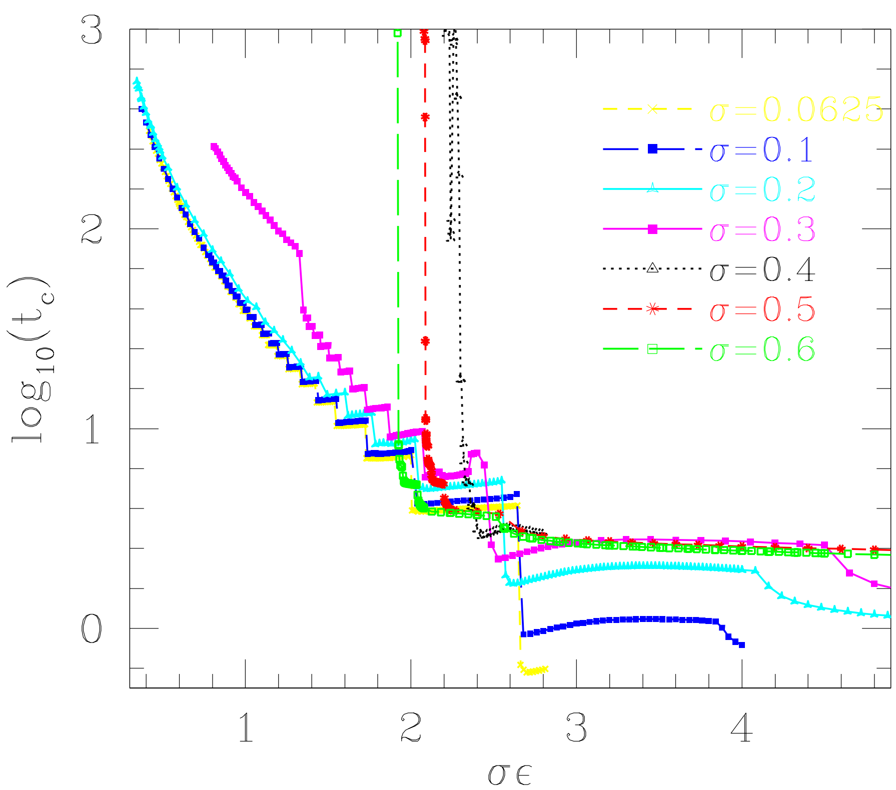
Cf. Dias+ '12, Buchel+ '12

Stability Islands?

Bizoń&Rostworowski('11)



Buchel+('13)



- Collapse time $t_{BH} \propto \epsilon^{-2}$
- Is there any stable solution against the weak turbulence?

Summary of Introduction

- Gravitational collapse occurs in asymptotic flat spacetime when the field initially has larger amplitude than critical value.

- In AdS, the gravitational collapse would always happen after many reflections of waves by the AdS boundary, with some exceptions.

- Stability Islands?
  - Monochromatic mode is stable. (Maliborski&Rostworowski '13)
  - Boson star in AdS? (Buchel+ '13)
  - Secular term cannot be removed. (Craps+ '14)

- Weak turbulence occurs in the asymptotic flat spacetime with artificial mirror. (Maliborski '12)

Methods and Results

Role of confinement

- Massive fields: $V(\Phi) = \frac{1}{2}\mu^2\phi^2$
  (Cf. Brady *et al.* '97)
  - Existence of trapped modes by the potential well.
  - Incomplete confined system

- AdS spacetime
  - Gravitational collapse and Bound states
  - Complete condined system

Massive field potential

Action for scalar cloud collapse

Action

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla^\rho \phi \nabla_\rho \phi - V(\phi) \right],$$

Einstein-Hilbert part
Matter part: real scalar field

$$\kappa = \frac{8\pi G}{c^4}, \qquad V(\phi) = \frac{1}{2}\mu^2\phi^2.$$

Einstein's equations $\quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$

Numerical Relativity

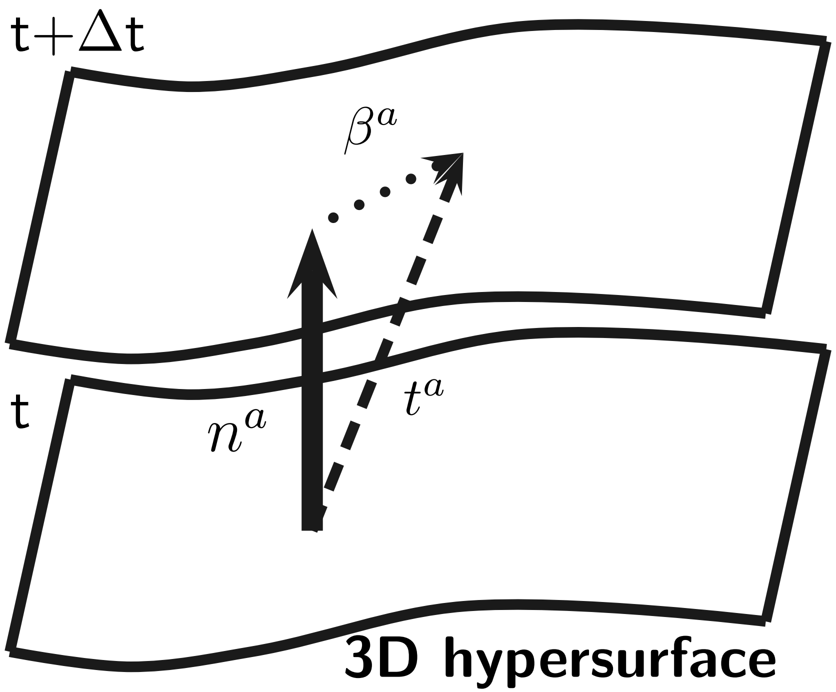
ADM(3+1) decomposition

Energy momentum tensor for a scalar field

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\rho \phi \nabla_\rho \phi - g_{\mu\nu} V(\phi)$$

Klein-Gordon equation $\quad \nabla^\rho \nabla_\rho \phi - V' \phi = 0$

ADM decomposition



n^a is a timelike normal vector
defined as $n^a = (1/\alpha, \beta^i/\alpha), \quad n^a n_a = -1.$

3D spatial metric

$$\gamma_{ab} \equiv g_{ab} + n_a n_b$$

Projection tensor

$$\perp^a_b = \delta^a_b + n^a n_b$$

Extrinsic curvature

$$K_{ab} \equiv -\frac{1}{2\alpha}\left(\partial_t \gamma_{ab} - D_b \beta_a - D_a \beta_b\right)$$

Einstein's equations

$$G_{ab} \equiv \mathcal{R}_{ab} - \frac{1}{2} g_{ab} \mathcal{R} = \kappa T_{ab}$$

Projection of Einstein's equations

$$\left(G_{ab} - \kappa T_{ab}\right) n^a n^b = 0,$$
$$\perp^c_i \left(G_{cb} - \kappa T_{cb}\right) n^b = 0,$$
$$\perp^c_i \perp^d_j \left(G_{cd} - \kappa T_{cd}\right) = 0.$$

Einstein's equations

Hamiltonian Constraint

ρ:energy density, j_i:current density

$$G_{ab}n^a n^b = \kappa T_{ab}n^a n^b \rightarrow {}^{(3)}R + K^2 - K_{ij}K^{ij} = 2\kappa\rho,$$

$$\perp_i^c G_{cb}n^b = \perp_i^c \kappa T_{cb}n^b \rightarrow D_j K_i^j - D_i K = \kappa j_i.$$

Momentum Constraints

All quantities are spatial.
(They must be satisfied on each hypersurface.)

Evolution equations

$$\perp_i^c \perp_j^d G_{cd} = \perp_i^c \perp_j^d \kappa T_{cd} \longrightarrow$$

$$\partial_t K_{ij} = \beta^k D_k K_{ij} + K_{kj} D_i \beta^k + K_{ki} D_j \beta^k - D_j D_i \alpha$$
$$+ \alpha \left(R_{ij} - 2K_{ik}K_j^k + K_{ij}K - \kappa \left[S_{ij} + \frac{\rho - S}{3}\gamma_{ij} \right] \right),$$

$S_{ij} \equiv \perp_i^c \perp_j^d T_{cd}, S \equiv \gamma^{ij} S_{ij}.$

γ_{ij} and K_{ij} are variables to evolve in ADM system.

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_j \beta_i + D_i \beta_j \quad \text{(Definition of } K_{ij}\text{)}$$

Cf. Hamiltonian formalism $\dot{p}_i = -\frac{\partial \mathcal{H}}{\partial q^i}$, $\dot{q}_i = \frac{\partial \mathcal{H}}{\partial p^i}$ $(\mathcal{H} : \mathit{Hamiltonian}, \quad q^i : \mathit{coordinates}, \quad p^i : \mathit{momenta})$

Einstein's equations

Hamiltonian Constraint

ρ:energy density, j_i:current density

$$G_{ab}n^a n^b = \kappa T_{ab}n^a n^b \rightarrow {}^{(3)}R + K^2 - K_{ij}K^{ij} = 2\kappa\rho,$$

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$$\perp_i^c \perp_j^d G_{cd} = \perp_i^c \perp_j^d \kappa T_{cd} \longrightarrow$$

$$\begin{aligned}\partial_t K_{ij} &= \beta^k D_k K_{ij} + K_{kj} D_i \beta^k + K_{ki} D_j \beta^k - D_j D_i \alpha \\ &+ \alpha \left(R_{ij} - 2K_{ik}K_j^k + K_{ij}K - \kappa \left[S_{ij} + \frac{\rho - S}{3}\gamma_{ij} \right] \right),\end{aligned}$$

$S_{ij} \equiv \perp_i^c \perp_j^d T_{cd}, S \equiv \gamma^{ij} S_{ij}.$

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Evolution equations

Ansatz $\quad \mathrm{d}s^2 = -\alpha^2 \mathrm{d}t^2 + \psi^4 \eta_{ij} \mathrm{d}x^i \mathrm{d}x^j \,, \quad K_{ij} = \frac{1}{3}\psi^4 \eta_{ij} K \,.$

Evolution eqs. with spherical symmetry

$$
\begin{aligned}
\partial_t \psi &= -\frac{1}{6}\alpha\psi K \,, \\
\partial_t K &= -\psi^{-4}\alpha_{,rr} - \frac{2\alpha_{,r}}{r\psi^4} - 2\psi^{-5}\psi_{,r}\alpha_{,r} + \frac{1}{3}\alpha K^2 + 4\pi\alpha\left(2\Pi^2 - \mu^2\Phi^2\right) \,, \\
\partial_t \Phi &= -\alpha\Pi \,, \\
\partial_t \Pi &= \alpha\Pi K - \psi^{-4}\alpha_{,r}\Phi_{,r} - \alpha\psi^{-4}\Phi_{,rr} - \frac{2\alpha\Phi_{,r}}{r\psi^4} - 2\alpha\psi^{-5}\psi_{,r}\Phi_{,r} + \alpha\mu^2\Phi \,, \\
\partial_t \alpha &= -\eta\alpha K \,.
\end{aligned}
$$

The metric ansatz permits seeing **BH formation**.

BH in isotropic coord.: $\mathrm{d}s^2 = -\alpha^2 \mathrm{d}t^2 + (1 + \frac{M}{2r})^4 \eta_{ij} \mathrm{d}x^i \mathrm{d}x^j$

Cf.) BH in Schwarzschild coord.: $\mathrm{d}s^2 = -(1 - \frac{2M}{r})\mathrm{d}t^2 + (1 - \frac{2M}{r})^{-1}\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2$

Initial Value Problem

- Initial data must satisfy the constraints.(Cook '02, Gourgoulhon '07)

$$R + K^2 - K_{ij}K^{ij} = 2\kappa\rho, \quad D_j K^j_i - D_i K = \kappa j_i.$$

- We have **14 variables** but only **4 constraints**. $\gamma_{ij}, K_{ij}, \Phi, \Pi.$
- At first, we set **10 variables** in the situation which we want to realize. Then, we solve the constraint equations.
- The constraints can be rewritten to better form by **conformal transformation** ($\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$). $\rightarrow \tilde{\gamma}_{ij}, \psi, K_{ij}, \Phi$ and Π.

Constraints in Initial Value Problem

$$\tilde{\Delta}\psi - \frac{1}{8}\psi\tilde{R} - \frac{1}{8}\psi^5 K^2 + \frac{1}{8}\psi^5 K_{ij}K^{ij} = -\pi\psi^5\left[\Pi^2 + \partial^i\Phi\partial_i\Phi + \mu^2\Phi^2\right],$$

$$D_j K^j_i - D_i K = 8\pi\Pi\partial_i\Phi$$

Initial Data on Minkowski background

Background: Minkowski spacetimes $\quad ds^2 = -dt^2 + \eta_{ij}dx^i dx^j$

Ansatz $\quad \phi = 0\,, \quad K = 0\,, \quad \alpha = 1\,,$ (HO,Witek,Cardoso, '13)

$$\Pi = \frac{A}{2\pi} e^{-\frac{(r-r_0)^2}{w^2}} \psi^{-\frac{5}{2}}\,, \quad \text{and} \quad \psi = \mathbf{1} + \frac{u(r)}{\sqrt{4\pi}r}\,.$$

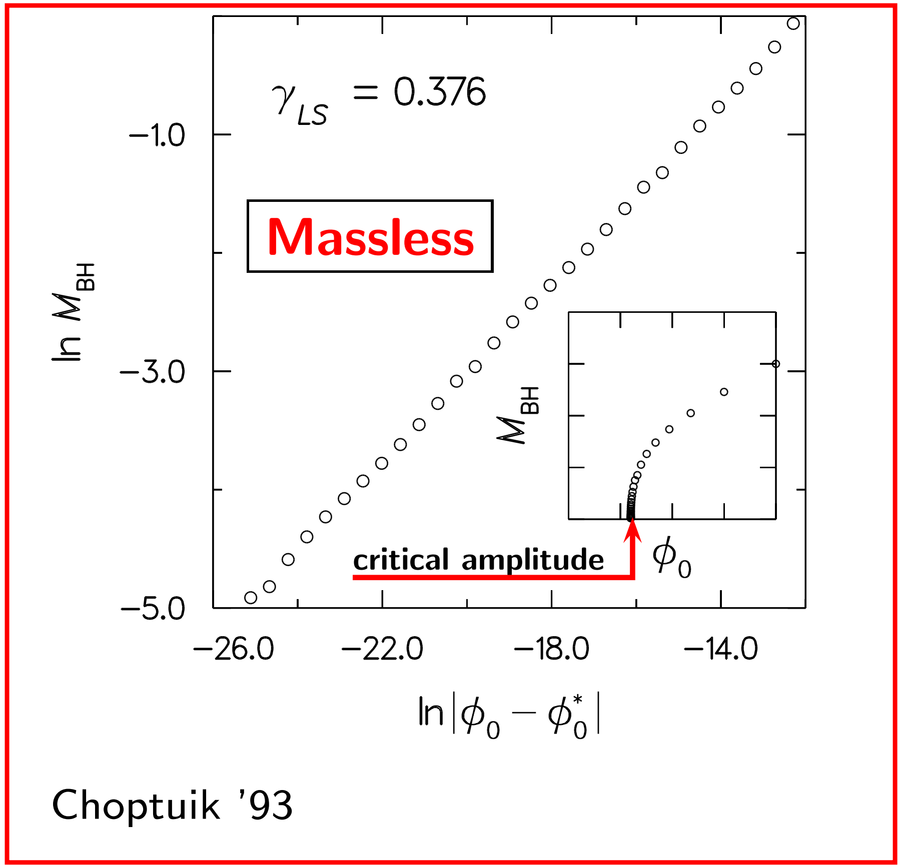
Hamiltonian constraint $\quad u'' + \frac{A^2 r}{\sqrt{4\pi}} e^{-\frac{2(r-r_0)^2}{w^2}} = 0\,.$

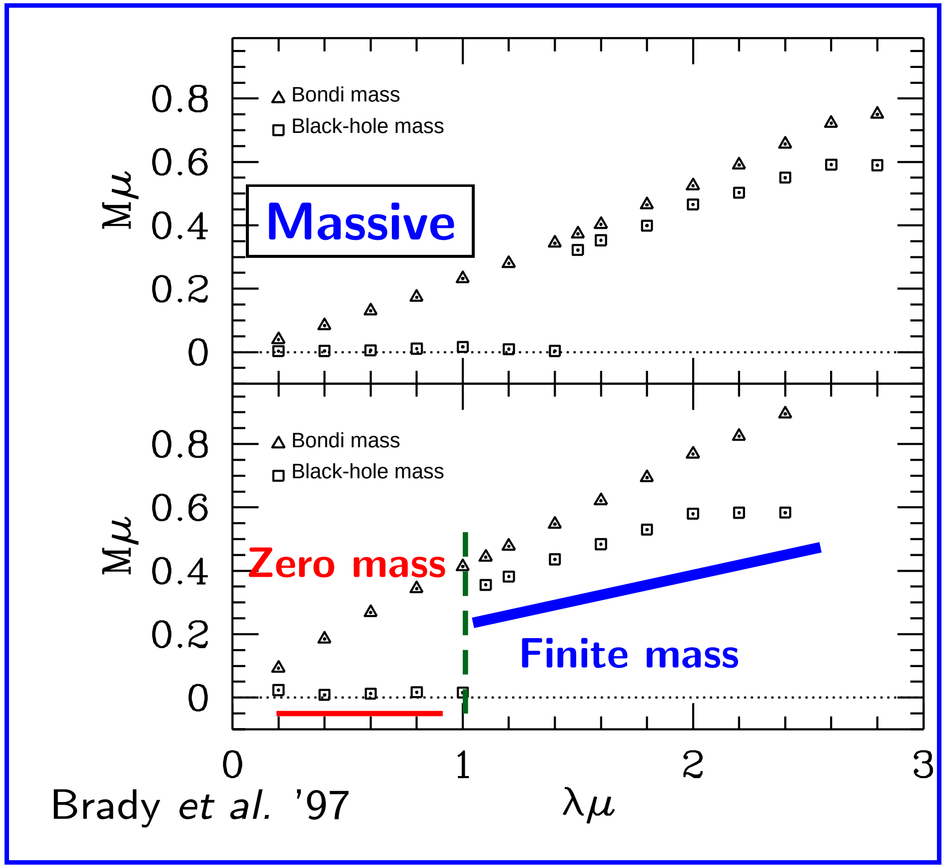
Solution $\quad u \to 0$ as $r \to \infty$,

$$u_0(r) = A^2 \frac{w(w^2 - 4r_0(r-r_0))}{16\sqrt{2}} \left(\mathrm{erf}\left(\frac{\sqrt{2}(\mathrm{r}-\mathrm{r}_0)}{\mathrm{w}} \right) - 1 \right) - A^2 \frac{r_0 w^2}{8\sqrt{\pi}} e^{-\frac{2(r-r_0)^2}{w^2}}\,.$$

We choose $u(r) = u_0(r) - u_0(0)\,.$
ADM energy $M_0 = -u_0(0)/\sqrt{\pi}$.

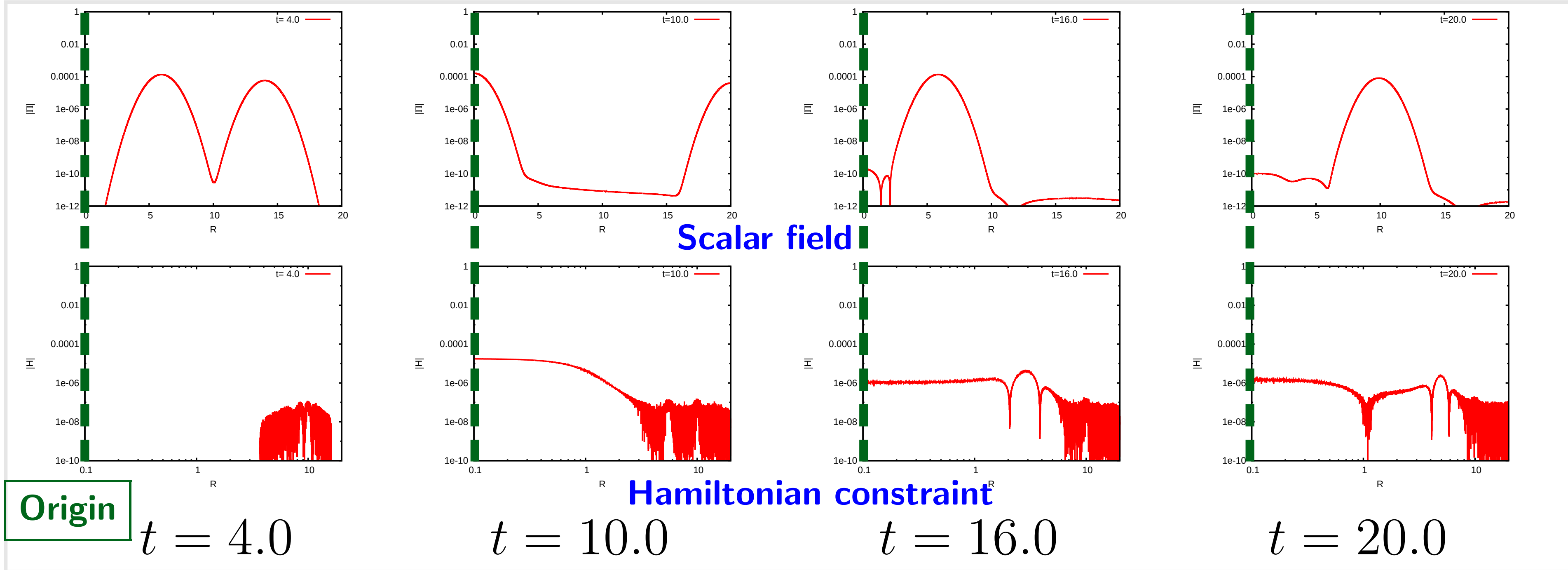
Previous works for critical collapse

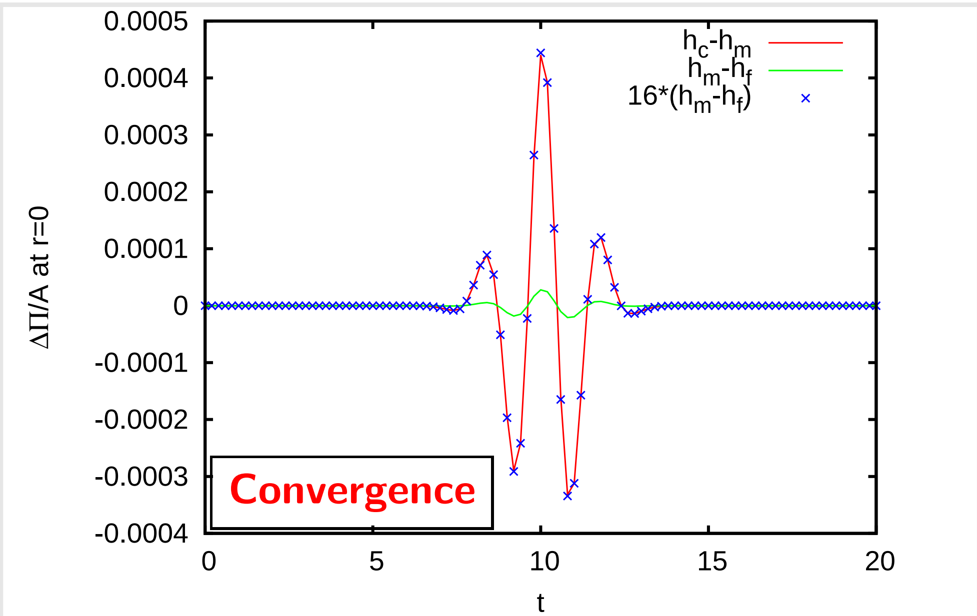


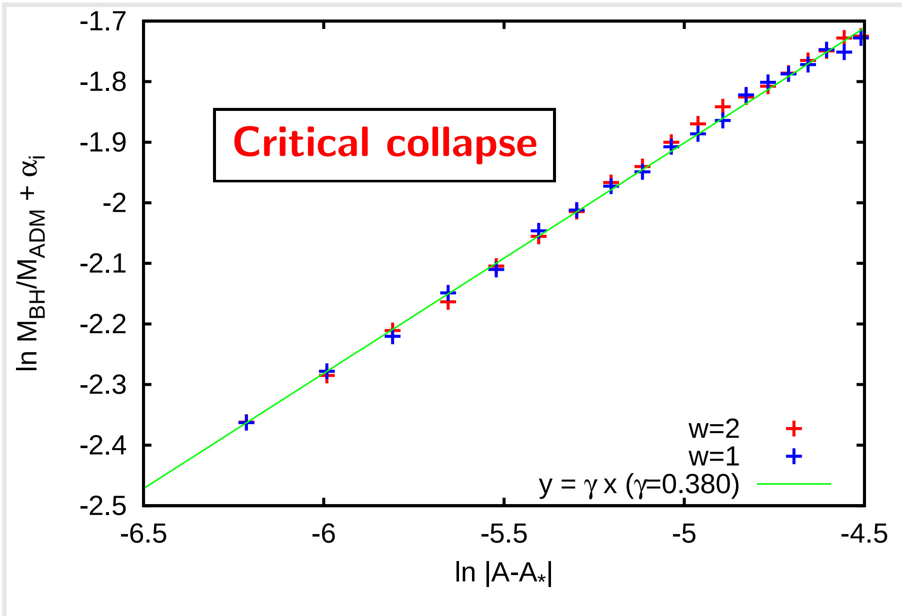


- Critical amplitude produces infinitesimal BH **in massless case**.
- Sufficiently **large mass term** gives finite mass at critical amplitude.

Code tests

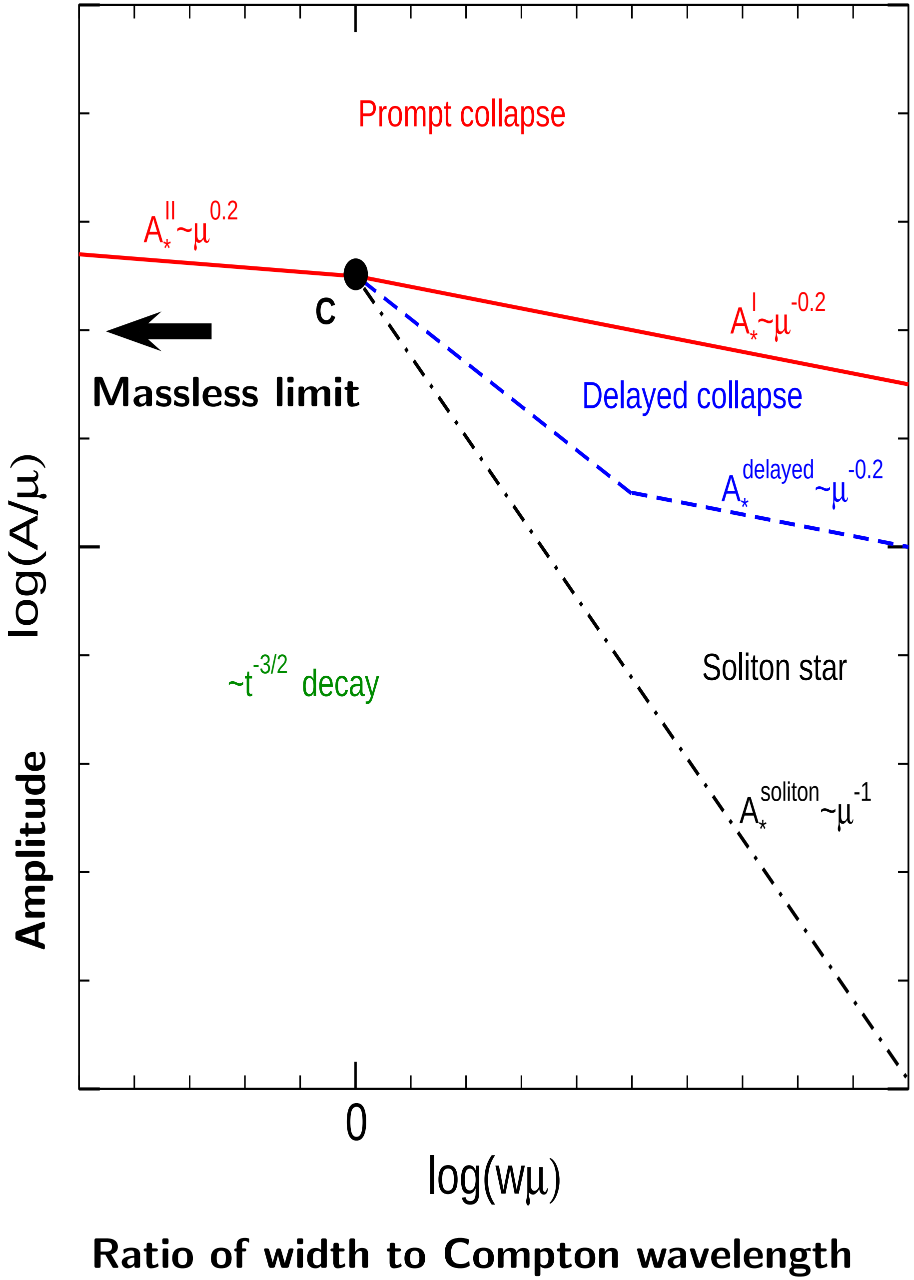

Scalar field
Hamiltonian constraint
Origin
$t = 4.0$
$t = 10.0$
$t = 16.0$
$t = 20.0$


h_c-h_m
h_m-h_f
16*(h_m-h_f)
ΔΠ/A at r=0
t
Convergence


Critical collapse
ln M_{BH}/M_{ADM} + α_i
ln |A-A*|
w=2
w=1
y = γ x (γ=0.380)

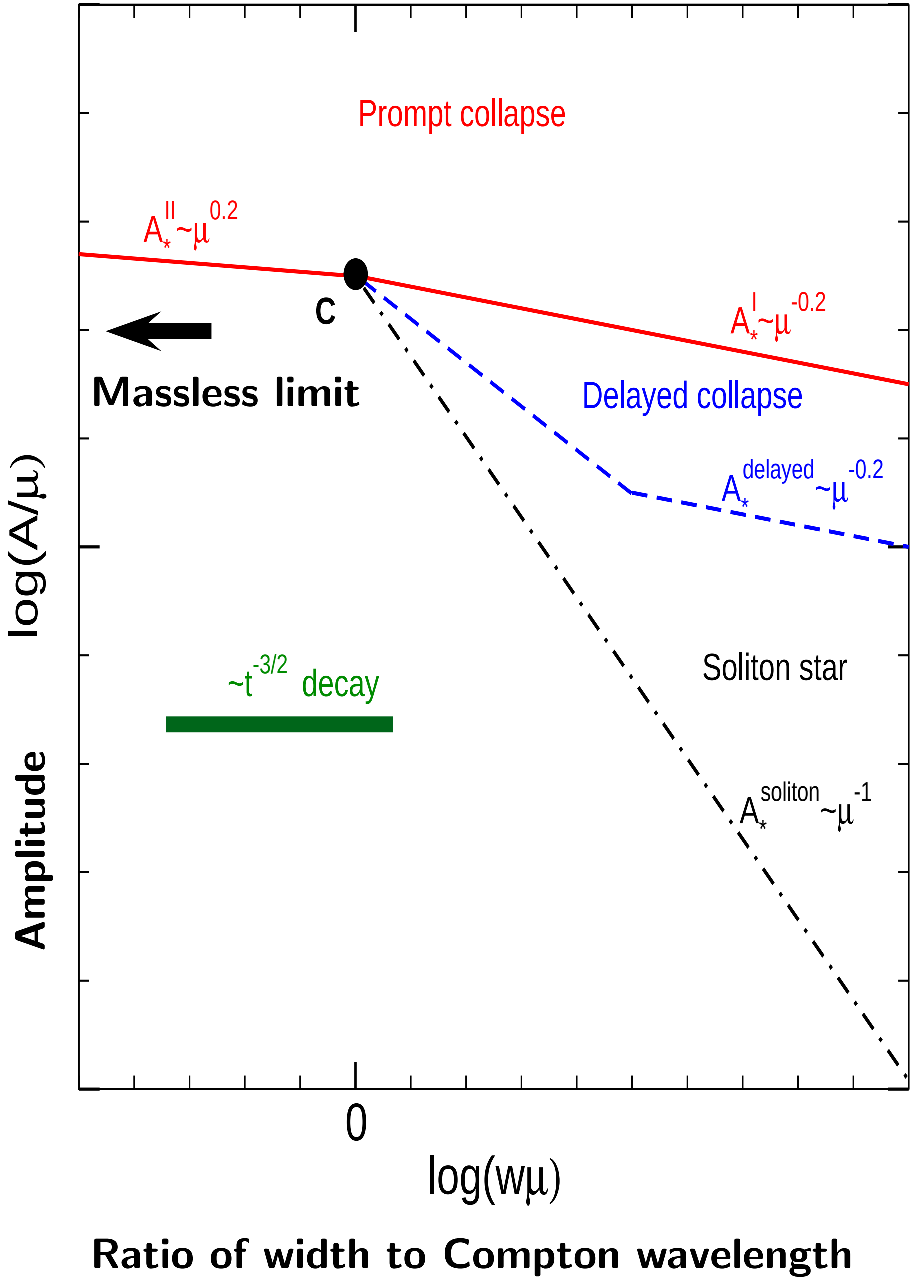
Phase diagram for collapse of massive fields

HO, Cardoso, Pani '13

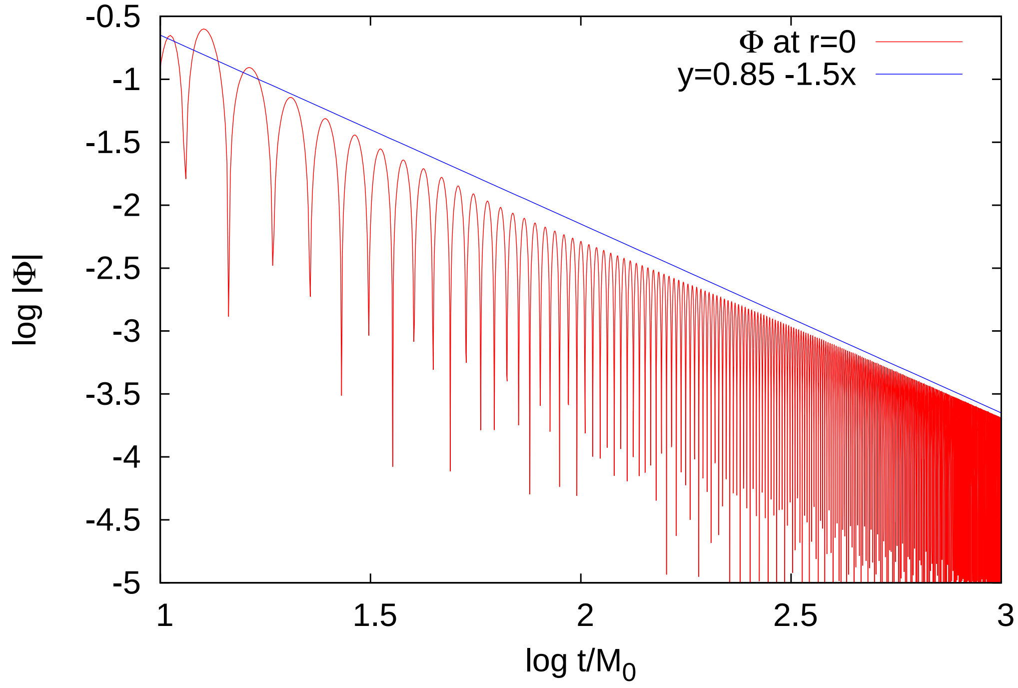


Phase diagram: decay phase

HO, Cardoso, Pani '13



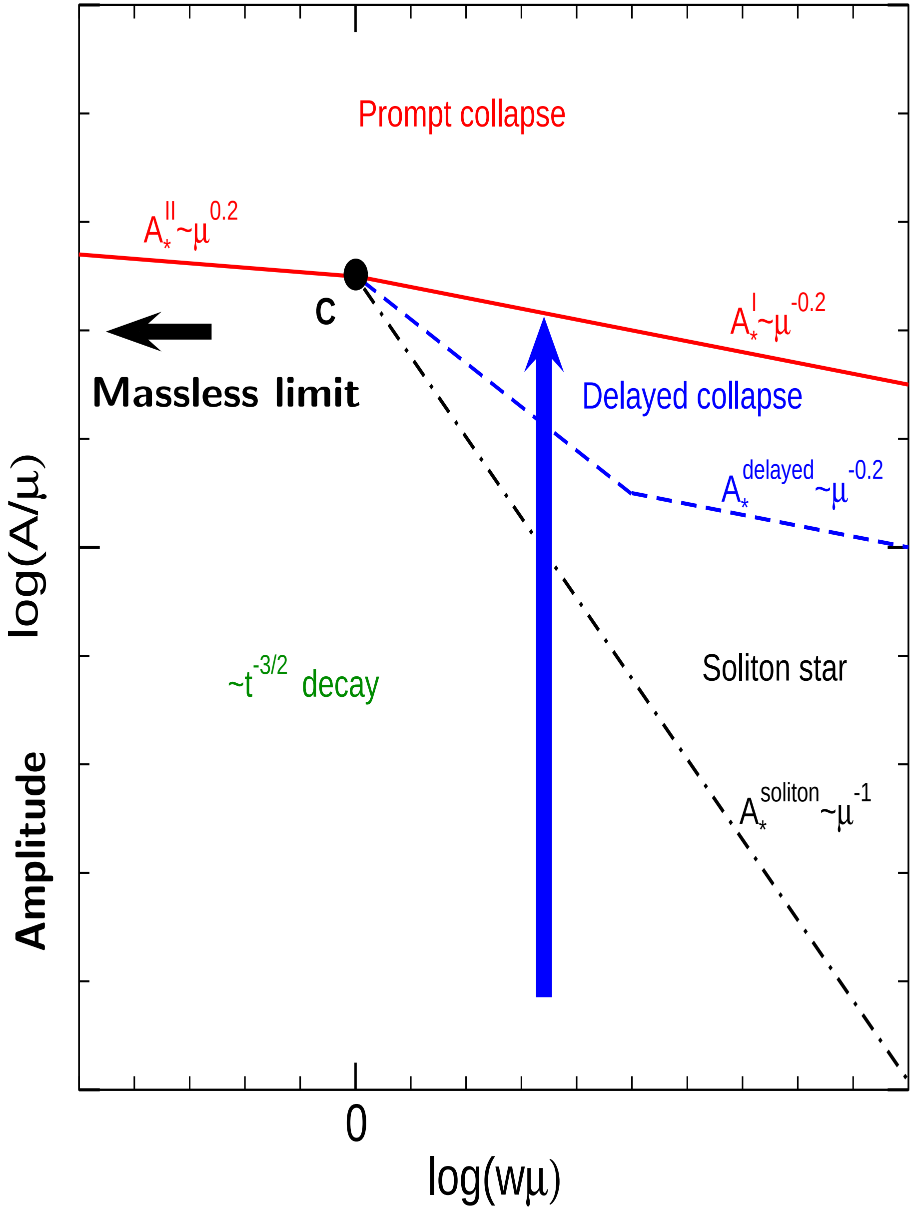
Decay phase



- Scalar fields with enough small amplitude decays by power-law, which is expected by linear analysis.
- Energy can escape from mass term potential well to infinity.

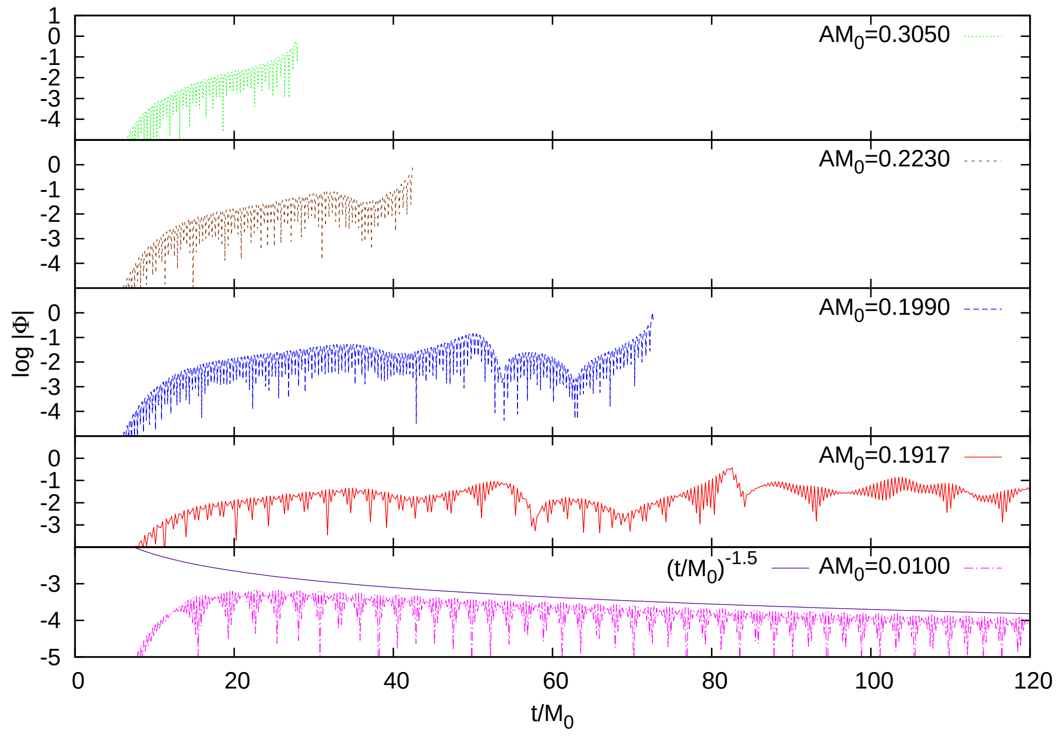
Phase diagram: collapse phase

HO, Cardoso, Pani '13



Ratio of width to Compton wavelength

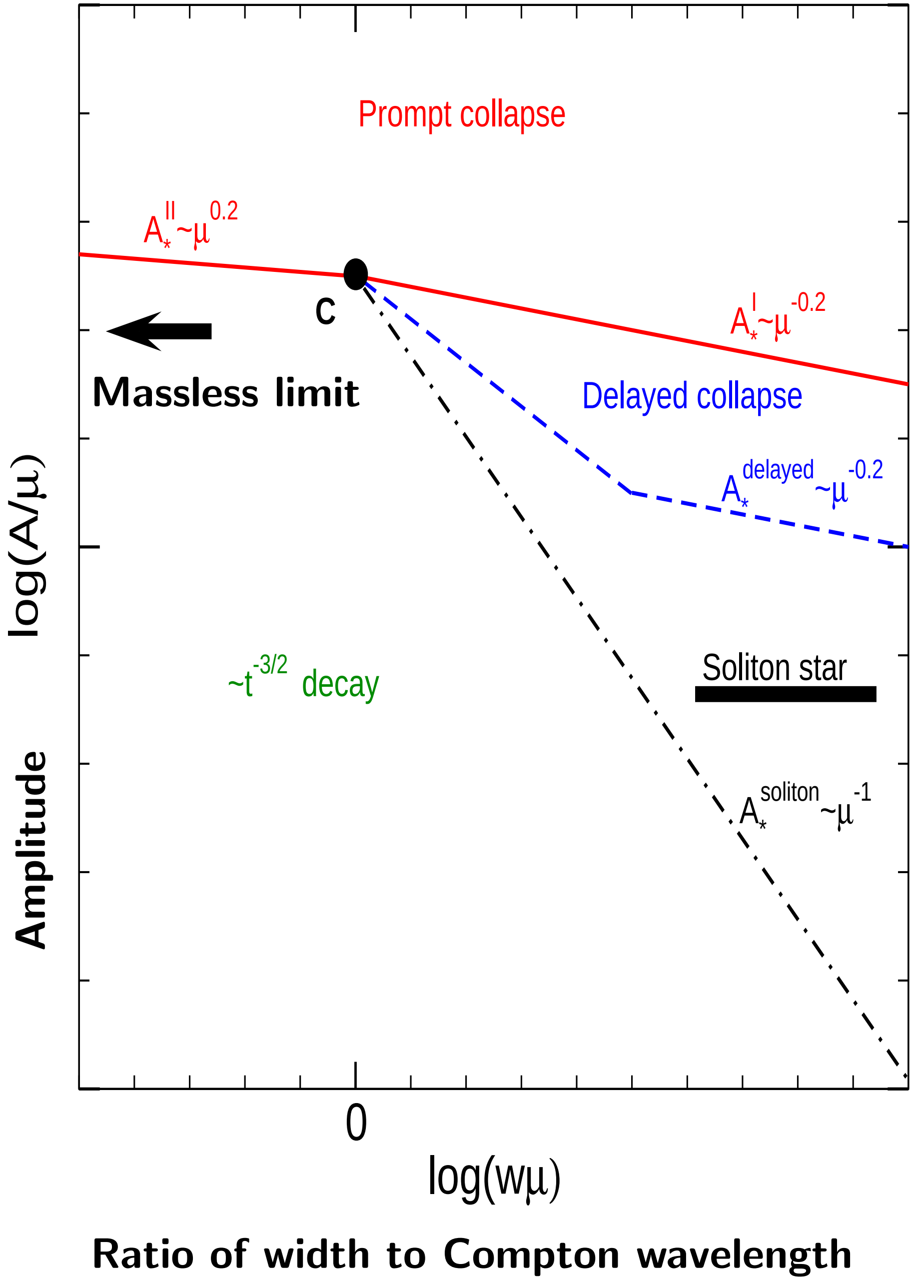
Collapse phase

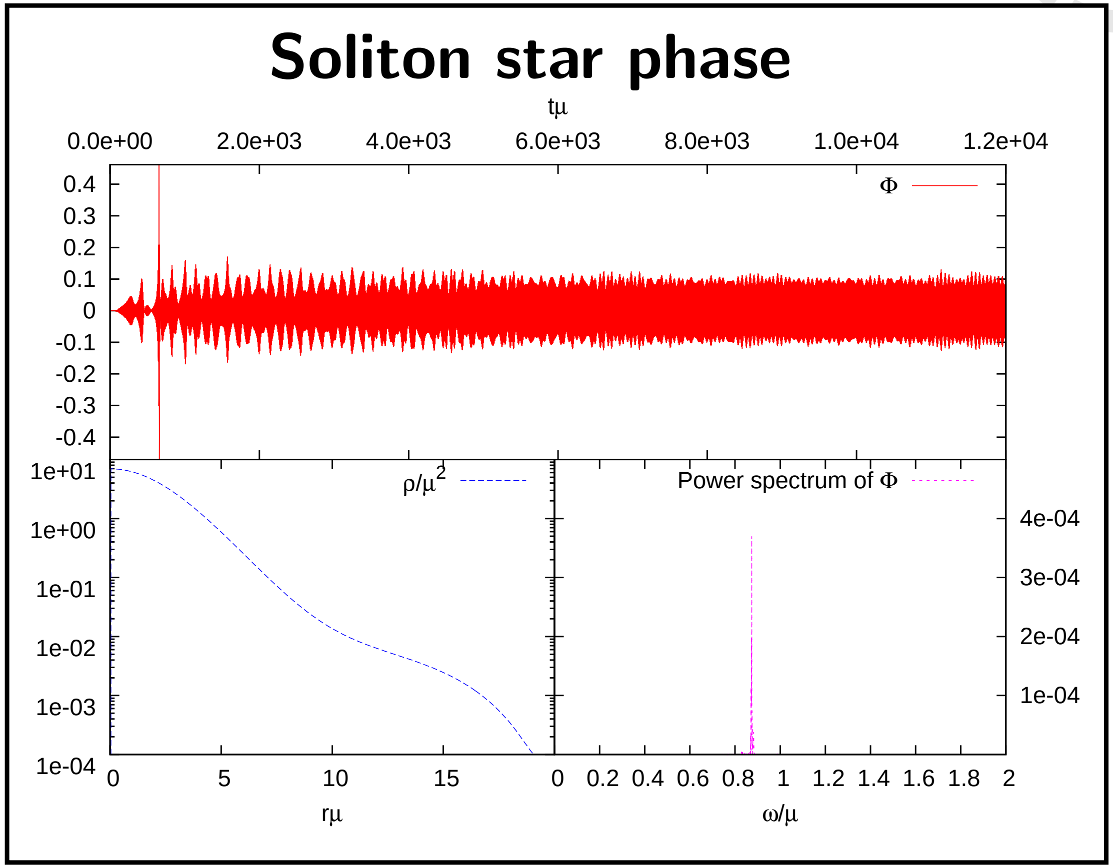


- We monitored scalar field at the origin with fixed mass parameter.
- BHs are formed after many reflections by mass term.

Phase diagram: soliton star phase

HO, Cardoso, Pani '13



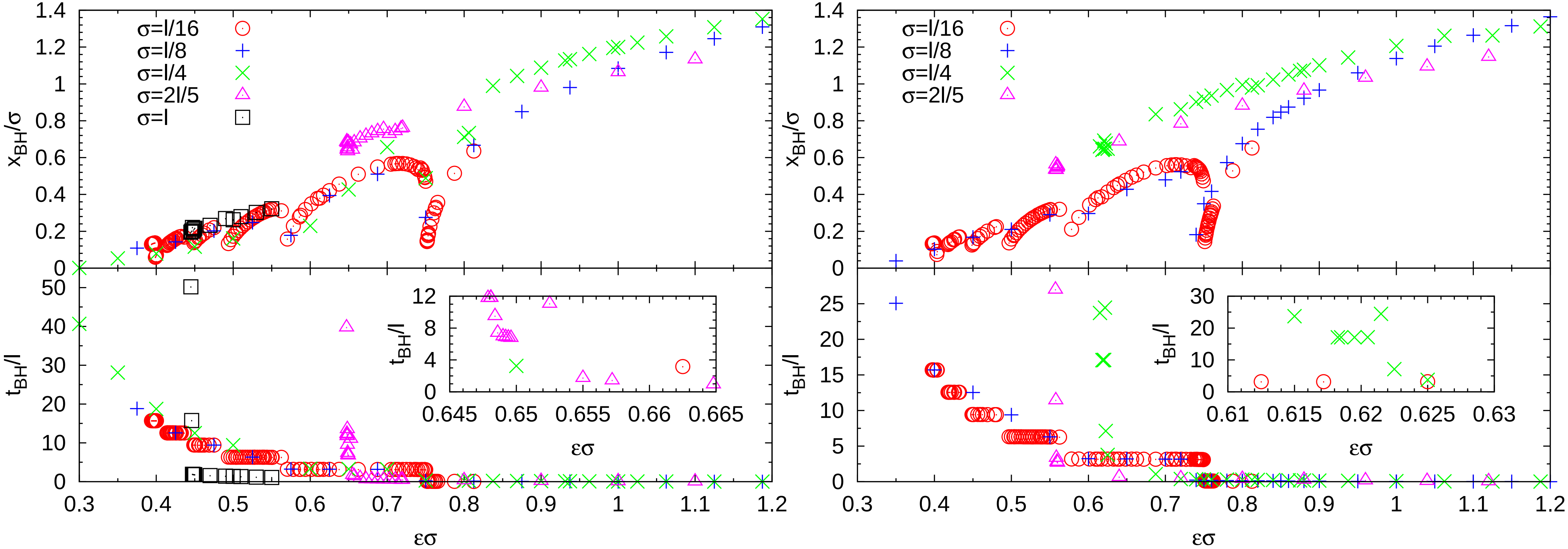


- Quasi-stable soliton star forms in the sense of "realistic time."
- Nonlinear quasi-stable solutions can prevent from collapsing to a BH.

AdS spacetime

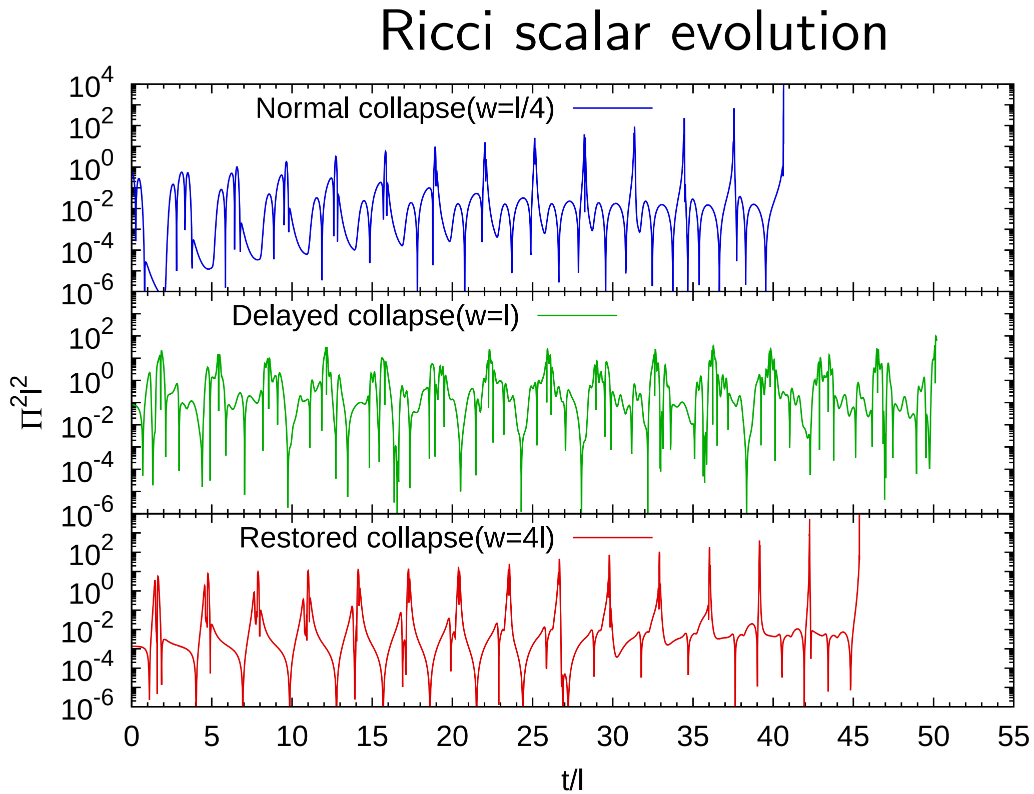
Collapse of massive fields in AdS(BH size, time)

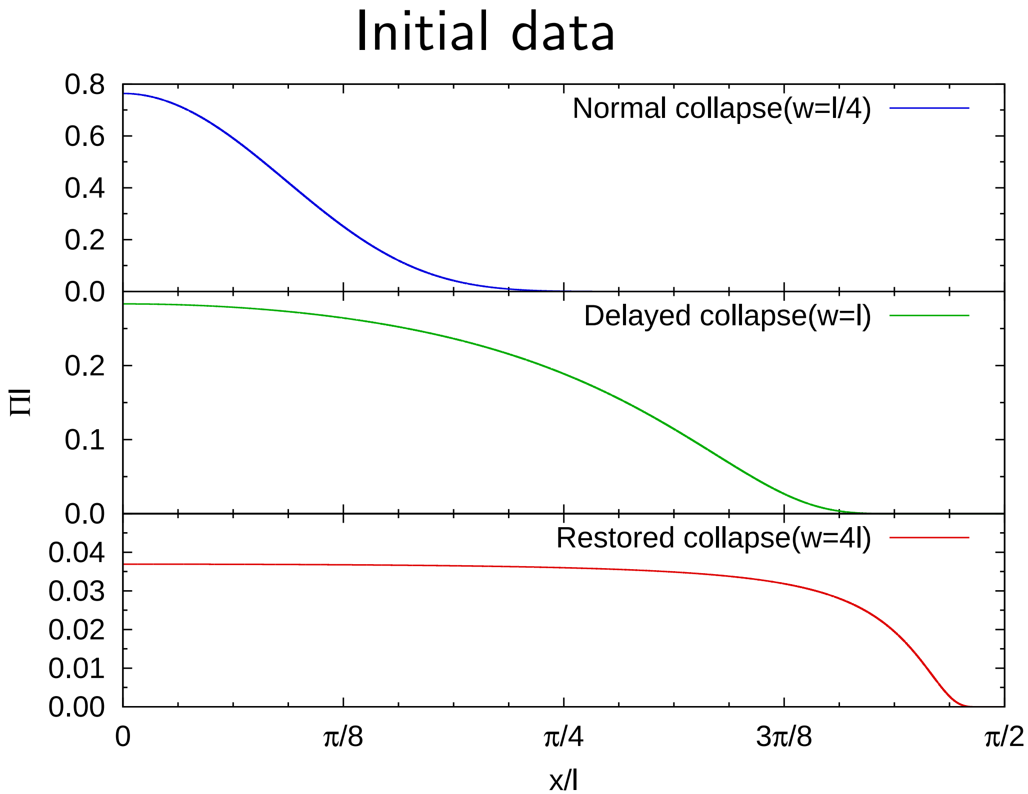
HO, Lopes, Cardoso '15



- Left: BH radius as a function of amplitude(**Massless**)
- Right: (**Massive fields**)
- Different width of initial pulse might give a quisi-stable solution.

Massless collapse in AdS(Width)



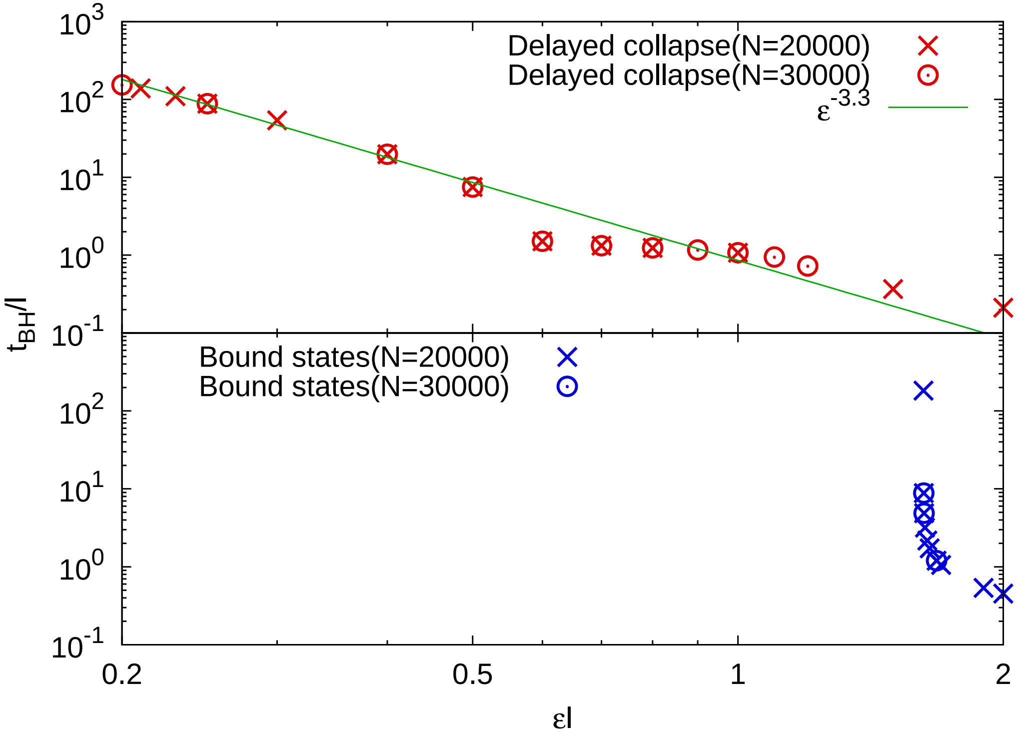


$$\Pi = \frac{2\epsilon}{\pi} \exp\left[-\frac{4\tan^2 x}{\sigma^2\pi^2}\right]$$

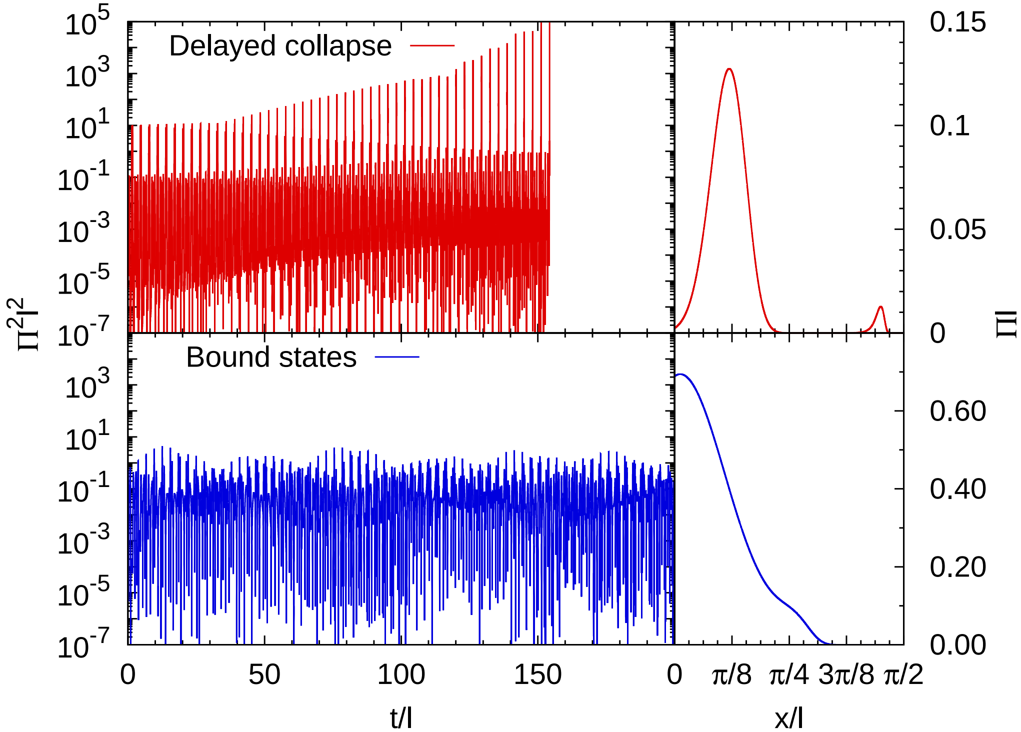
- Energy transfer from lower modes to higher arises for small width(Bizoń&Rostworowski '11).
- Moderately large width of initial pulse makes bound-states(Buchel+ '13).
- Much larger width restores the similar energy transfer to small width and collapses to a BH.(Maliborski&Rostworowski '13)

Collapse in AdS(several wavepackets)

collapse time



Ricci evolution and Initial profile



- $$\Pi(0,x) = \epsilon \sum_{i=1}^{N} \frac{2a_i}{\pi} \exp\left[\frac{-4(\tan x - r_i)^2}{\pi^2 \sigma_i^2}\right].$$
- Two wavepackets initial data delays the collapse time.
- Three wavepackets can make bound-states with relatively narrower width.

Summary

Summary

- We investigated the collapse in two different confined system.
- **Mass term**:
  - Equations are rewritten by the coordinates to find horizon.
  - We use **analytic initial data** and draw **the phase diagram** between amplitude and width of the scalar field.
  - The energy can escape from the mass term potential.
- **AdS spacetime**:
  - We reconfirm that **bound-states** can exist in the parameter range of single wavepacket initial data. Mass term can broaden the parameter region.
  - The second wavepacket would delay the collapse.
  - Many wavepackets can make bound-states easier.