Tidal deformations of a spinning compact object

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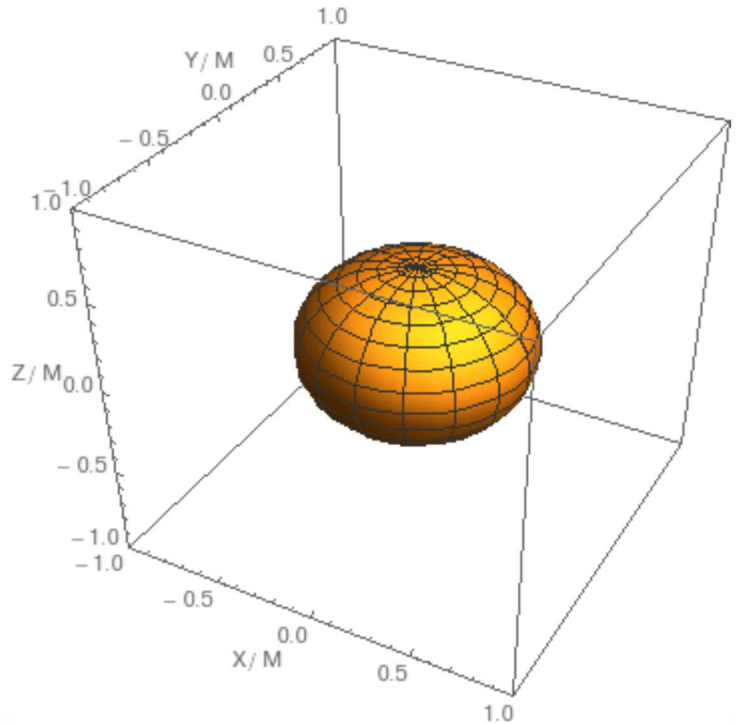
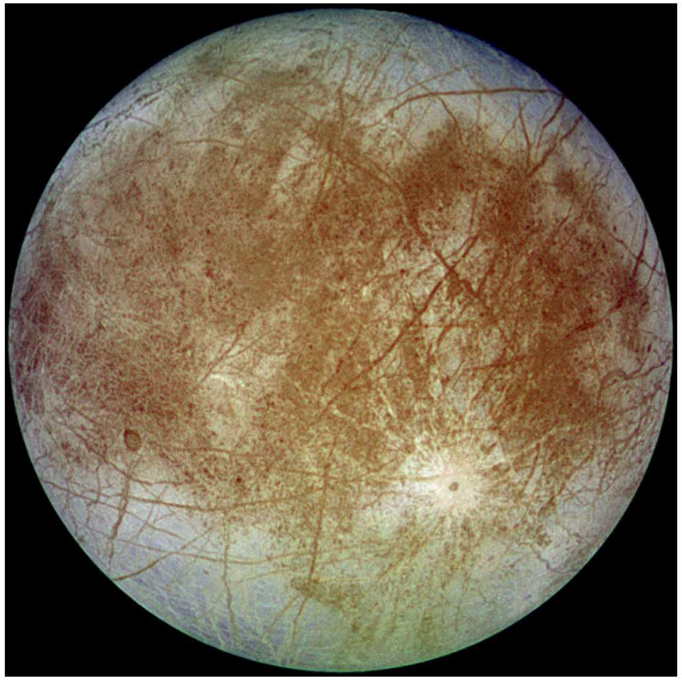


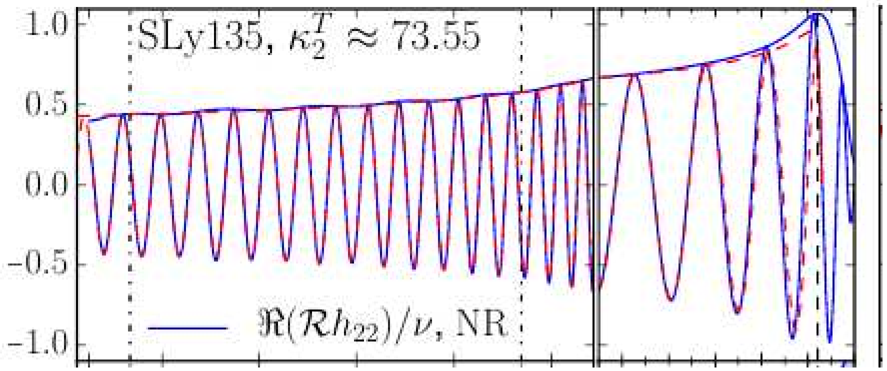
Gravity raises tides

Nazaré, Portugal



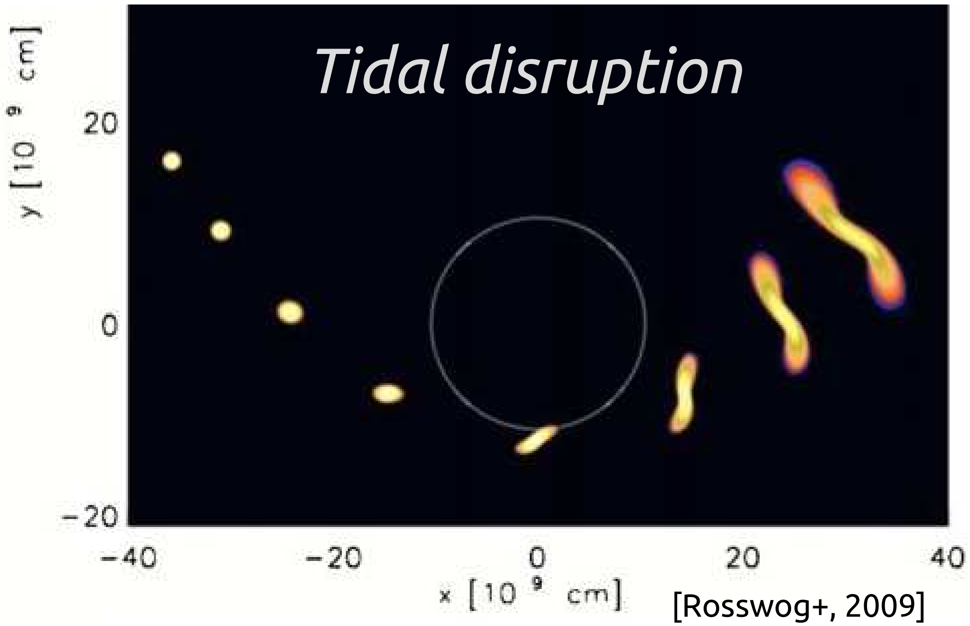
Liquid oceans in Europa?

Tidal effects in compact binaries



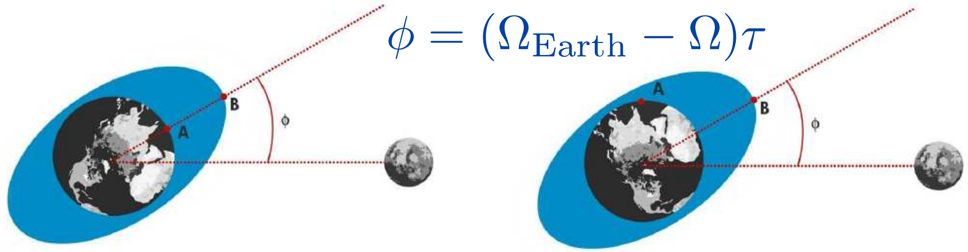
[Bernuzzi+, 2015]

Gravitational waveforms from BNSs



[Rosswog+, 2009]

$\phi = (\Omega_{\rm Earth} - \Omega)\tau$



[Cardoso & Pani, 2013 Class. Quantum Grav. 30 045011]

Tidal acceleration ↔ superradiance

- Ultranuclear matter EoS
- BH no-hair theorems
- BH astrophysics
- EMRIs, …

Tidal Love numbers

- Multipole moments **deformed** by tidal field → $\lambda_\ell^{(M)} \equiv \frac{\partial M_\ell}{\partial \mathcal{E}_0}$
- Tidal Love numbers:
  - depend sensibly on the object's **internal structure**
  - can be measured through **GW detections** [Hinderer & Flanagan, 2009]
  - Mean to constrain the **NS EoS** [Hinderer+, 2010 – Baiotti+, 2010 – Lackey+, 2014....]
- Relativistic theory of tidal Love numbers [Damour & Nagar, 2009] [Binnington & Possion, 2009]
- Tidal Love numbers of Schwarzschild BHs are **zero**
- Most analysis restricted to the **static case** [R. Porto, 2005, 2007, 2008]

Spin-tidal field coupling?

Slow-rotation toolkit

"Tidal deformations of a spinning compact object "

PP, Gualtieri, Maselli, Ferrari – arXiv:1503.07365

Landry & Poisson, arXiv:1503.07366

E. Poisson, Phys.Rev. D91 (2015) 044004

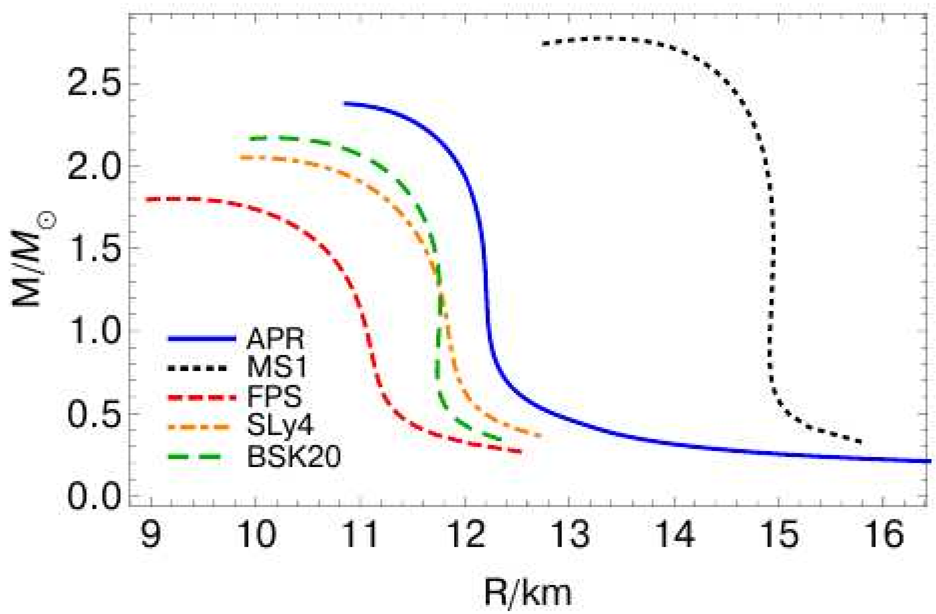
Review: PP, Int.J.Mod.Phys. A28 (2013) 1340018

Isolated central object

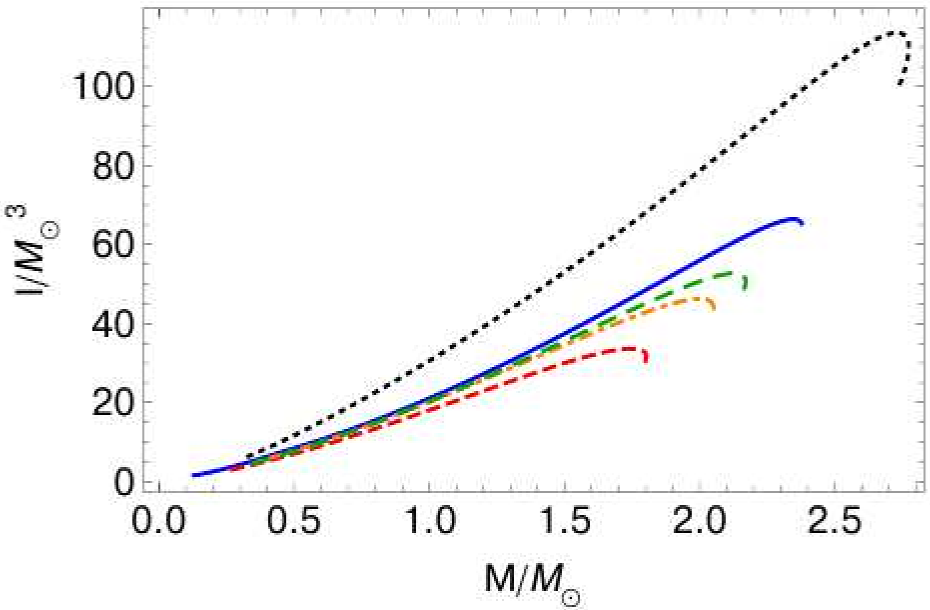
- Hartle-Thorne slow-rotation expansion: [Hartle & Thorne 1967-1968]

$$ds^2 = -e^{\nu}\left[1+2\epsilon_a^2\left(j_0+j_2P_2\right)\right]dt^2 + \frac{1+2\epsilon_a^2(m_0+m_2P_2)/(r-2\mathcal{M})}{1-2\mathcal{M}/r}dr^2$$

$$+r^2\left[1+2\epsilon_a^2(v_2-j_2)P_2\right]\left[d\vartheta^2+\sin^2\vartheta(d\varphi-\epsilon_a\omega dt)^2\right]$$



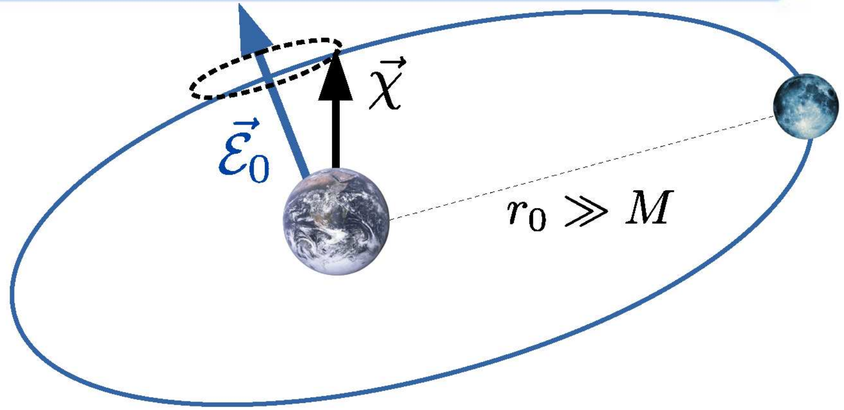
Mass-radius relation



Moment of inertia

Working assumptions

- Central object is **slowly spinning**
- **Quasi-stationary**, **weak** tidal fields
- Tidal sources at large distance

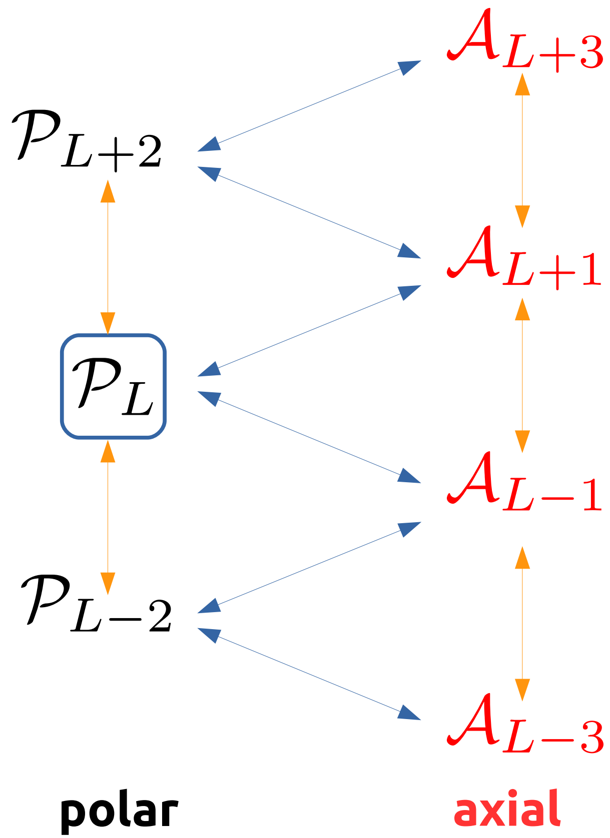


Pani+, gr-qc/1503.07365		**Landry & Poisson, gr-qc/1503.07366**
Kojima's framework	⟷	Irriducible potentials
Generic l (in principle)	⟷	Quadrupolar (l=2) E and B
2nd order in the spin	⟷	1st order in the spin
Stationary tidal fields	⟷	Quasi-stationary fields
Axisymmetry	⟷	Nonaxisymmetry
No matching to PN metric	⟷	Matching to PN metric

Slow-rotation expansion

[Review: P. Pani - Int.J.Mod.Phys. A28 (2013) 1340018]

- No separability besides Kerr
- Slow-rotation expansion [Chandrasekhar & Ferrari, 1991] [Kojima, 1992-1993] [Pani +, 2012-2014]
  - Harmonic decomposition
  - Orthogonality of spherical harmonics
  - Multipole and parity couplings
  - Selection rules (rules of addition of J)
- System of ODEs



Zeroth order → decoupled

First order → polar-axial l±1

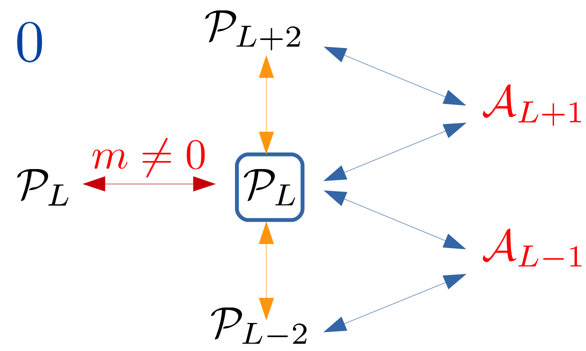
Second order → coupling l±2

Electric- and magnetic-led systems

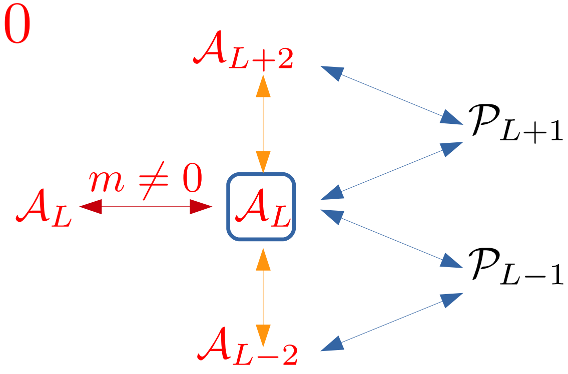
[Review: P. Pani - Int.J.Mod.Phys. A28 (2013) 1340018]

To O(spin^2), various couplings can be truncated:

$$\begin{cases} \mathcal{P}_L + \epsilon_a m \bar{\mathcal{P}}_L + \epsilon_a^2 \hat{\mathcal{P}}_L + \epsilon_a (\mathcal{Q}_L \tilde{\mathcal{A}}_{L-1} + \mathcal{Q}_{L+1} \tilde{\mathcal{A}}_{L+1}) = 0 \\ \mathcal{A}_{L+1} + \epsilon_a m \bar{\mathcal{A}}_{L+1} + \epsilon_a \mathcal{Q}_{L+1} \tilde{\mathcal{P}}_L = 0 \\ \mathcal{A}_{L-1} + \epsilon_a m \bar{\mathcal{A}}_{L-1} + \epsilon_a \mathcal{Q}_L \tilde{\mathcal{P}}_L = 0 \\ \mathcal{P}_{L+2} + \epsilon_a \mathcal{Q}_{L+2} \tilde{\mathcal{A}}_{L+1} + \epsilon_a^2 \mathcal{Q}_{L+1} \mathcal{Q}_{L+2} \breve{\mathcal{P}}_L = 0 \\ \mathcal{P}_{L-2} + \epsilon_a \mathcal{Q}_{L-1} \tilde{\mathcal{A}}_{L-1} + \epsilon_a^2 \mathcal{Q}_L \mathcal{Q}_{L-1} \breve{\mathcal{P}}_L = 0 \end{cases}$$



$$\begin{cases} \mathcal{A}_L + \epsilon_a m \bar{\mathcal{A}}_L + \epsilon_a^2 \hat{\mathcal{A}}_L + \epsilon_a (\mathcal{Q}_L \tilde{\mathcal{P}}_{L-1} + \mathcal{Q}_{L+1} \tilde{\mathcal{P}}_{L+1}) = 0 \\ \mathcal{P}_{L+1} + \epsilon_a m \bar{\mathcal{P}}_{L+1} + \epsilon_a \mathcal{Q}_{L+1} \tilde{\mathcal{A}}_L = 0 \\ \mathcal{P}_{L-1} + \epsilon_a m \bar{\mathcal{P}}_{L-1} + \epsilon_a \mathcal{Q}_L \tilde{\mathcal{A}}_L = 0 \\ \mathcal{A}_{L+2} + \epsilon_a \mathcal{Q}_{L+2} \tilde{\mathcal{P}}_{L+1} + \epsilon_a^2 \mathcal{Q}_{L+1} \mathcal{Q}_{L+2} \breve{\mathcal{A}}_L = 0 \\ \mathcal{A}_{L-2} + \epsilon_a \mathcal{Q}_{L-1} \tilde{\mathcal{P}}_{L-1} + \epsilon_a^2 \mathcal{Q}_L \mathcal{Q}_{L-1} \breve{\mathcal{A}}_L = 0 \end{cases}$$

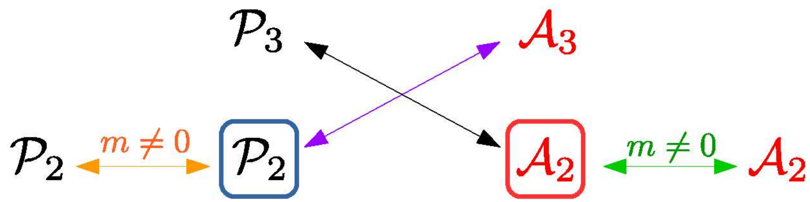


Effective action

[Damour, Soffel, Xu, 1991-1994], [R. Porto, 2005, 2007, 2008]

Work in progress with Jan Steinhoff

- To first order in the spin:



$$S = \int d\tau \sqrt{-u^2}\,\Big[m + \mu_2 \mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} + \sigma_2 \mathcal{B}_{\alpha\beta}\mathcal{B}^{\alpha\beta}$$

$$+\tau_3 S_\mu \nabla_\nu \mathcal{B}^{\mu\lambda}\mathcal{E}^\nu_\lambda + \rho_3 S_\mu \nabla_\nu \mathcal{E}^{\mu\lambda}\mathcal{B}^\nu_\lambda \quad + \lambda_2 S_{\mu\nu}\mathcal{E}^\mu_\alpha \dot{\mathcal{E}}^{\nu\alpha} + \gamma_2 S_{\mu\nu}\mathcal{B}^\mu_\alpha \dot{\mathcal{B}}^{\nu\alpha}\Big]$$

- To second order in the spin (stationary case):

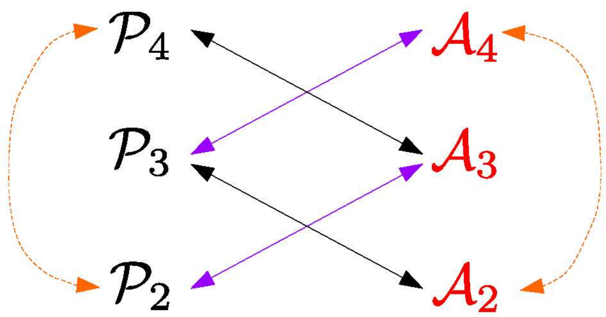
$$S = \int d\tau \sqrt{-u^2}\,\{(m + S^2\hat{m}) + q\mathcal{E}_{\alpha\beta}S^\alpha S^\beta$$

$$+\left[(\mu_2 + \hat{\mu}_2 S^2)\mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} + (\sigma_L + \hat{\sigma}_L S^2)\mathcal{B}_{\alpha\beta}\mathcal{B}^{\alpha\beta}\right.$$

$$+\tau_3 S^\mu H_{\mu\alpha\beta}\mathcal{E}^{\alpha\beta} + \rho_3 S^\mu G_{\mu\alpha\beta}\mathcal{B}^{\alpha\beta}$$

$$+\lambda_4 S^\mu S^\nu G_{\mu\nu\alpha\beta}\mathcal{E}^{\alpha\beta} + \eta_4 S^\mu S^\nu H_{\mu\nu\alpha\beta}\mathcal{B}^{\alpha\beta}$$

$$\left.+\alpha_2 S^\mu S^\nu \mathcal{E}_{\mu\alpha}\mathcal{E}^\alpha_\nu + \beta_2 S^\mu S^\nu \mathcal{B}_{\mu\alpha}\mathcal{B}^\alpha_\nu\right]\}$$



Multipole moments

of a tidally-distorted, slowly-spinning object

"Tidal deformations of a spinning compact object "

PP, Gualtieri, Maselli, Ferrari – arXiv:1503.07365

Exterior solution

- Systems of ODEs can be solved **analytically** in vacuum

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

- Schematically:

$$\delta g_{\mu\nu} = \underbrace{\alpha\delta g^{(\alpha)}_{\mu\nu}}_{\text{divergent}} + \underbrace{\gamma\delta g^{(\gamma)}_{\mu\nu}}_{\text{response}}$$

$$+ \sum_{\ell=0}^{4} \Big[\cancel{\overbrace{\alpha_\ell \delta g^{(\alpha_\ell)}_{\mu\nu}}^{\text{divergent}}} + \overbrace{\gamma_\ell \delta g^{(\gamma_\ell)}_{\mu\nu}}^{\text{response}} \Big]$$

Asymptotics

$\mathcal{O}(\chi,\alpha)$		
$(0,0)$	M	mass
$(1,0)$	χ	spin
$(1,0)$	Ω	angular velocity of the object
$(2,0)$	δm	spin-induced mass shift
$(2,0)$	δq	spin-induced quadrupole-moment shift
$(0,1)$	α	external electric quadrupolar tidal field
$(0,1)$	γ	static response to the external tidal field
$(1,1)$	α_1	external magnetic dipolar tidal field
$(1,1)$	γ_1	tidally-induced spin shift
$(2,1)$	α_0	constant shift of g_{tt} at infinity
$(2,1)$	γ_0	tidally-induced mass shift
$(2,1)$	α_2	external electric quadrupolar tidal field
$(2,1)$	γ_2	tidally-induced quadrupole-moment shift
$(2,1)$	α_3	external magnetic octupolar tidal field
$(2,1)$	γ_3	tidally-induced octupole-moment shift
$(2,1)$	α_4	external electric $L=4$ tidal field
$(2,1)$	γ_4	tidally-induced hexadecapole-moment shift

NOTE: subtleties in separating the linear response from the external tidal field

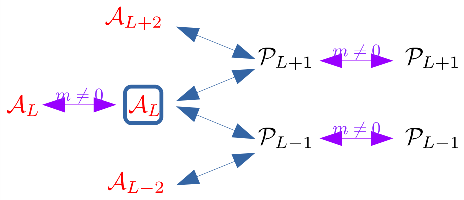
Multipole moments & Love #s

- Removing external tidal contribution:

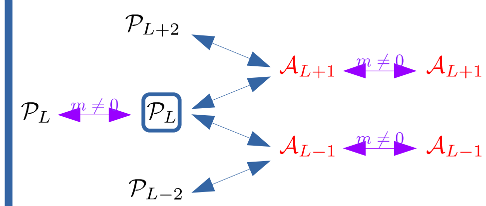
$$\frac{M_2}{M^3} = \left(\frac{4}{5}\delta q - 1\right)\chi^2 - \frac{2\gamma}{\sqrt{5\pi}} - \frac{\chi^2}{280\sqrt{\pi}}\left[195\sqrt{7}\gamma_3 \; +4\sqrt{5}(28\gamma_2 + \gamma(135 + 56\delta m + 76\delta q))\right]$$

$$\frac{S_3}{M^4} = -\frac{3\sqrt{7}\gamma_3 + 44\sqrt{5}\gamma}{28\sqrt{\pi}}\chi \qquad \frac{M_4}{M^5} = \frac{65\sqrt{7}\gamma_3 + 4\left(420\gamma_4 - \sqrt{5}\gamma(221 + 432\delta q)\right)}{2940\sqrt{\pi}}\chi^2$$

- Gamma's computed through matching with the interior $\gamma_i \propto \alpha \propto \mathcal{E}_0$
- **New families of tidal Love numbers:**



$$\boxed{\begin{aligned} \lambda_{E,-}^{(\ell\ell' m)} &\equiv \frac{\partial M_\ell}{\partial \mathcal{B}_m^{(\ell')}}\,, \qquad & \lambda_{M,-}^{(\ell\ell' m)} &\equiv \frac{\partial S_\ell}{\partial \mathcal{E}_m^{(\ell')}}\,, \\ \lambda_{E,+}^{(\ell\ell' m)} &\equiv \frac{\partial M_\ell}{\partial \mathcal{E}_m^{(\ell')}}\,, \qquad & \lambda_{M,+}^{(\ell\ell' m)} &\equiv \frac{\partial S_\ell}{\partial \mathcal{B}_m^{(\ell')}}\,. \end{aligned}}$$



Response VS external field #1

$$\delta g_{\mu\nu} = \overbrace{\alpha \delta g^{(\alpha)}_{\mu\nu}}^{\text{external field}} + \underbrace{\gamma \delta g^{(\gamma)}_{\mu\nu} + \sum_{\ell=0}^{4} \gamma_\ell \delta g^{(\gamma_\ell)}_{\mu\nu}}_{\text{object's response}}$$

Redefinition of integration constants

$$\gamma = \gamma' - \alpha\hat{\gamma}\,, \qquad \gamma_\ell = \gamma'_\ell - \alpha\hat{\gamma}_\ell$$

$$\delta g_{\mu\nu} = \alpha \left[\overbrace{\delta g^{(\alpha)}_{\mu\nu} - \hat{\gamma}\delta g^{(\gamma)}_{\mu\nu} - \sum_{\ell=0}^{4} \hat{\gamma}_\ell \delta g^{(\gamma_\ell)}_{\mu\nu}}^{\text{same external field to leading order}} \right] + \underbrace{\gamma' \delta g^{(\gamma)}_{\mu\nu} + \sum_{\ell=0}^{4} \gamma'_\ell \delta g^{(\gamma_\ell)}_{\mu\nu}}_{\text{different object's response}}$$

How to define the multipole moments uniquely? [Damour & Nagar, 2009]

Response VS external field #2

- In the nonrotating case:
  - 5PN computation
  - Analytic continuation

[Kol & Smolkin, 2012 – Chakrabarti+, 2013]

  - How are multipole moments defined?

$$\delta g_{tt} \sim \overbrace{\alpha r^{\ell}\left(1+\dots\right)}^{\text{external field}} + \underbrace{\gamma r^{-\ell-d+3}\left(1+\dots\right)}_{\text{object's response}}$$

- In the rotating case:
  - Higher PN order
  - No analytical solution for generic multipoles
  - The two solutions overlap

Response VS external field #3

There exists a **unique linear combination** of solutions such that:

- 0^{th} order in the spin:

$$\delta g_{tt} \sim \alpha \underbrace{\left(r^2 + a_1 r + a_0\right)}_{\text{truncated}} + \gamma \overbrace{\left(\frac{1}{r^3} + \dots\right)}^{\text{object's response}}$$

- 1^{st} order in the spin:

$$\delta g_{t\varphi} \sim \alpha \underbrace{\left(r + b_0 + \frac{b_{-1}}{r}\right)}_{\text{truncated}} + \gamma_3 \overbrace{\left(\frac{1}{r^3} + \dots\right)}^{\text{object's response}}$$

- 2^{nd} order in the spin:

$$\delta g_{tt} \sim \alpha \left(\underbrace{r^2 + c_1 r + c_2 \dots \frac{c_5}{r^5}}_{\text{truncated only in the BH case}} + \delta q \left[\frac{c_{-6}}{r^6} + \dots\right]\right) + \overbrace{\gamma_2 \left(\frac{1}{r^3} + \dots\right) + \gamma_4 \left(\frac{1}{r^5} + \dots\right)}^{\text{object's response}}$$

OK in the BH case to 2^{nd} order and in the stellar case to 1^{st} order

Tidally-deformed slowly-spinning BHs

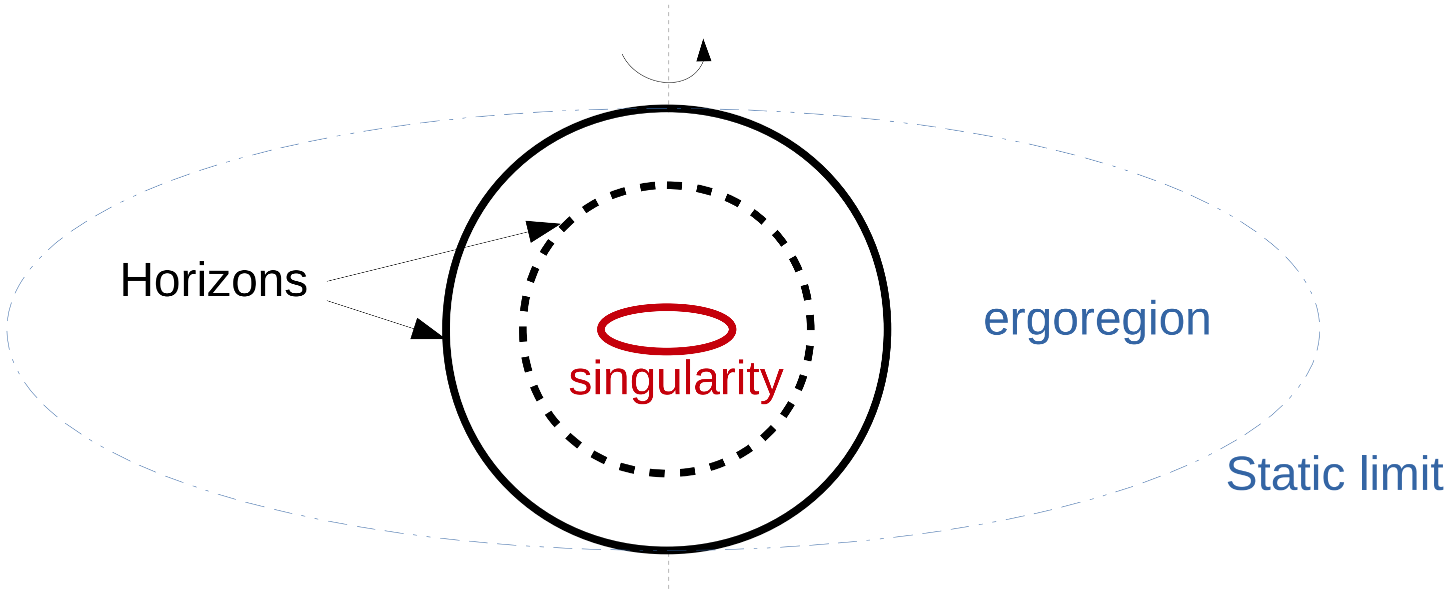
E. Poisson, Phys.Rev. D91 (2015) 044004

PP, Gualtieri, Maselli, Ferrari – arXiv:1503.07365

(Isolated) BHs are simple

$$S = \frac{c^4}{16\pi G}\int dx^4\sqrt{-g}R + S_{\rm m}(\Psi_{\rm SM}, g_{\mu\nu})$$

- **Uniqueness theorem**: Stationary sols. → Kerr family
- **No-hair theorem**: only 3 parameters [Israel, Hawking, Carter, Bekenstein...]



Are realistic BHs so simple?

$$M_\ell + iS_\ell = M^{\ell+1}\,(i\chi)^\ell$$

- **Astrophysical BHs are never isolated**
  - Halos, binaries, DM, …
- **Do no-hair theorems generalize to tidally-deformed BHs?**
  - Weak tides, *static* case → **YES** [Binnington & Possion, 2009, Damour & Nagar, 2009]
  - Strong tides, *static* case → **YES** [Gurlebeck, 2015]
  - Spinning case → **?**

BH rotational Love numbers = zero

Regularity at the horizon requires → $\delta q = 0$ and $\gamma_\ell = 0 = \gamma$

$$\frac{M_2}{M^3} = \left(\cancel{\frac{4}{5}\delta q} - 1\right)\chi^2 - \frac{2\gamma}{\sqrt{5\pi}} - \cancel{\frac{\chi^2}{280\sqrt{\pi}}\left[195\sqrt{7}\gamma_3 + 4\sqrt{5}(28\gamma_2 + \gamma(135 + 56\delta m + 76\delta q))\right]}$$

$$\frac{S_3}{M^4} = -\cancel{\frac{3\sqrt{7}\gamma_3 + 44\sqrt{5}\gamma}{28\sqrt{\pi}}}\chi \qquad \frac{M_4}{M^5} = \cancel{\frac{65\sqrt{7}\gamma_3 + 4\left(420\gamma_4 - \sqrt{5}\gamma(221 + 432\delta q)\right)}{2940\sqrt{\pi}}}\chi^2$$

$$\frac{\partial M_2}{\partial \mathcal{E}_0} = \frac{\partial S_3}{\partial \mathcal{E}_0} = \frac{\partial M_4}{\partial \mathcal{E}_0} = 0$$

Multipole moments of a **slowly-rotating** BH are **NOT** deformed by an external, **weak**, **axisymmetric** tidal field

Tidally-distorted spinning BHs

1st order, generic m

$\mathcal{E}_{ab} \neq 0 \quad \mathcal{B}_{ab} \neq 0$

$$g_{vv} = -f - r^2 e_1^{\mathsf{q}} \mathcal{E}^{\mathsf{q}} - r^2 \hat{e}_1^{\mathsf{q}} \chi \partial_\phi \mathcal{E}^{\mathsf{q}} + r^2 k_1^{\mathsf{d}} \mathcal{K}^{\mathsf{d}} - r^2 k_1^{\mathsf{o}} \mathcal{K}^{\mathsf{o}}$$

$$\begin{aligned} g_{vA} = {} & \frac{2M^2}{r} \chi_A^{\mathsf{d}} - \frac{2}{3} r^3 (e_4^{\mathsf{q}} \mathcal{E}_A^{\mathsf{q}} - b_4^{\mathsf{q}} \mathcal{B}_A^{\mathsf{q}}) \\ & - r^3 \chi \partial_\phi (\hat{e}_4^{\mathsf{q}} \mathcal{E}_A^{\mathsf{q}} - \hat{b}_4^{\mathsf{q}} \mathcal{B}_A^{\mathsf{q}}) - r^3 (f_4^{\mathsf{d}} \mathcal{F}_A^{\mathsf{d}} - k_4^{\mathsf{d}} \mathcal{K}_A^{\mathsf{d}}) \\ & + r^3 (f_4^{\mathsf{o}} \mathcal{F}_A^{\mathsf{o}} + k_4^{\mathsf{o}} \mathcal{K}_A^{\mathsf{o}}), \end{aligned}$$

$$\begin{aligned} g_{AB} = {} & r^2 \Omega_{AB} - \frac{1}{3} r^4 (e_7^{\mathsf{q}} \mathcal{E}_{AB}^{\mathsf{q}} - b_7^{\mathsf{q}} \mathcal{B}_{AB}^{\mathsf{q}}) \\ & - r^4 \chi \partial_\phi (\hat{e}_7^{\mathsf{q}} \mathcal{E}_{AB}^{\mathsf{q}} - \hat{b}_7^{\mathsf{q}} \mathcal{B}_{AB}^{\mathsf{q}}) \\ & - r^4 (f_7^{\mathsf{o}} \mathcal{F}_{AB}^{\mathsf{o}} - k_7^{\mathsf{o}} \mathcal{K}_{AB}^{\mathsf{o}}), \end{aligned}$$

[E. Poisson, Phys.Rev. D91 (2015) 044004]

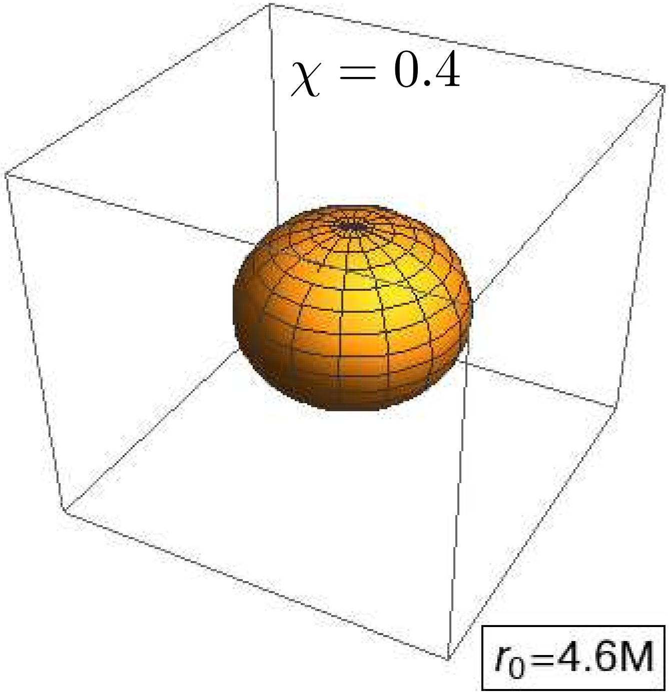
2nd order, m=0, $\mathcal{E}_{ab} \neq 0 \quad \mathcal{B}_{ab} = 0$

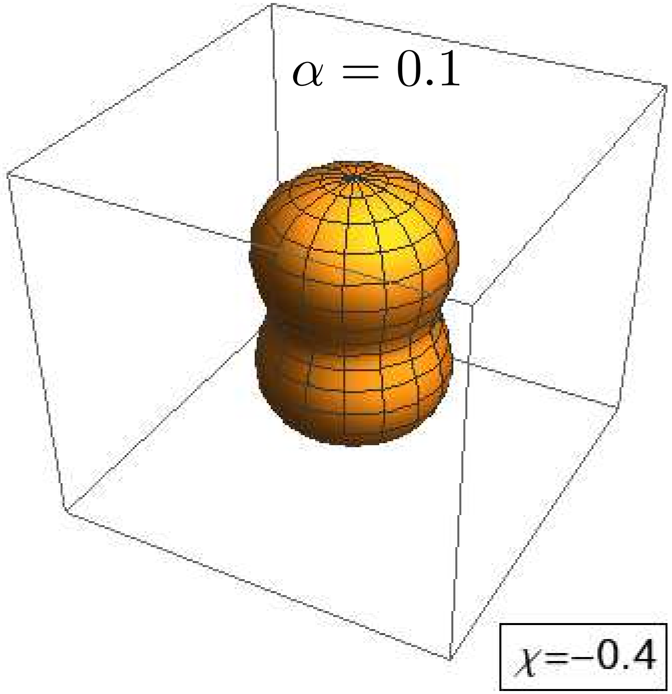
$$\begin{aligned}
g_{tt} &= -1 + \frac{2}{y} + \frac{\chi^2}{y^5}\left(4 - 2y^2 + \left(-6 + y + 3y^2\right)\sin^2\vartheta\right) + \frac{3}{8}\sqrt{\frac{5}{\pi}}(y-2)^2\alpha[1 + 3\cos(2\vartheta)] \\
&\quad + \frac{3\alpha\chi^2}{64\sqrt{5\pi}y^5}\left[104 + 112y + 344y^2 - 773y^3 + 558y^4\right. \\
&\quad + 4\left(-104 + 104y + 152y^2 - 305y^3 + 30y^4\right)\cos(2\vartheta) \\
&\quad + 5\left(-40 + 48y + 40y^2 - 75y^3 + 18y^4\right]\cos(4\vartheta)\Big) \,, \\
g_{t\varphi} &= -\frac{2M\chi\sin\vartheta^2}{y} - \frac{3\alpha M\chi}{4\sqrt{5\pi}y}\left(-12 + 15y + 20y^2 + 5(-4 + 5y)\cos(2\vartheta)\right)\sin\vartheta^2 \,, \\
g_{rr} &= \frac{y}{y-2} - \frac{\chi^2\left(10 - 3y + y^2 + 3\left(10 - 7y + y^2\right)\cos(2\vartheta)\right)}{2(y-2)^2y^3} + \frac{3}{8}\sqrt{\frac{5}{\pi}}y^2\alpha[1 + 3\cos(2\vartheta)] \\
&\quad + \frac{3\alpha\chi^2}{448\sqrt{5\pi}(y-2)^2y^3}\left[-896\left(10 - 15y + 7y^2\right)\right. \\
&\quad + 32(y-2)\left(-55 - 4y + 20y^2 + 10y^3\right)[1 + 3\cos(2\vartheta)] \\
&\quad \left. -(y-2)\left(100 + 62y - 93y^2 + 6y^3\right)(9 + 20\cos(2\vartheta) + 35\cos(4\vartheta))\right] \,, \\
g_{\vartheta\vartheta} &= y^2M^2 - \frac{(2+y)M^2\chi^2(1 + 3\cos(2\vartheta))}{2y^2} + \frac{3}{8}\sqrt{\frac{5}{\pi}}y^2(y^2 - 2)\alpha M^2[1 + 3\cos(2\vartheta)] \\
&\quad - \frac{\alpha M^2\chi^2}{64\sqrt{5\pi}y^2}\left[-1008 + 36y + 804y^2 + 321y^3 + 4\left(-672 + 44y + 540y^2 + 225y^3\right)\cos(2\vartheta)\right. \\
&\quad \left. + 5\left(-336 - 68y + 252y^2 + 63y^3\right)\cos(4\vartheta)\right] \,, \\
g_{\varphi\varphi} &= g_{\vartheta\vartheta}\sin^2\vartheta \,,
\end{aligned}$$

[PP, Gualtieri, Maselli, Ferrari – arXiv:1503.07365]

Tidally-distorted spinning BHs

$$M^2 R_{\rm intr} = \frac{1}{2} - \frac{3}{8}\chi^2 c(2\vartheta) + \frac{3}{4}\sqrt{\frac{5}{\pi}}\alpha\left[1+3c(2\vartheta)\right] - \frac{9\alpha\chi^2}{128\sqrt{5\pi}}\left[1+172c(2\vartheta)+115c(4\vartheta)\right]$$





NOTE: tidal effect enormously exaggerated → $\alpha \sim \dfrac{m_c M^2}{r_0^3}$

Tidally-distorted spinning BHs

- Geodetic properties can be computed analytically
  - ISCO, binding energy, epicyclic frequencies, ...
- Tidal VS spin effects

$$\frac{\Omega^{\rm ISCO}}{\Omega_0^{\rm ISCO}} \approx 1+0.150\chi_{0.2}+0.022\chi_{0.2}^2+0.017r_{30}^{-1}-0.004r_{30}^{-1}\chi_{0.2}+0.00008r_{30}^{-1}\chi_{0.2}^2$$

$$\frac{\Omega_\vartheta^{\rm ISCO}}{\Omega_{\vartheta,0}^{\rm ISCO}} \approx 1+0.123\chi_{0.2}+0.015\chi_{0.2}^2+0.010r_{30}^{-1}-0.003r_{30}^{-1}\chi_{0.2}-0.0002r_{30}^{-1}\chi_{0.2}^2$$

- Small effects but degenerate with, e.g., modified gravity
  - EH Telescope, GRAVITY, BH Cam, eLISA...

[Barausse, Cardoso, PP, Phys.Rev. D89 (2014) 104059]

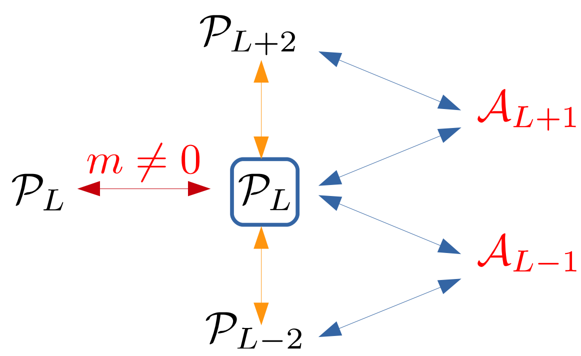
Tidal Love numbers of slowly-spinning NSs

In preparation (2015)

Are spin-tidal couplings important?

Tidal l=2 → $\mathcal{E}_{ab} \sim \frac{M}{r_0^3}\,, \quad \mathcal{B}_{ab} \sim \frac{M}{r_0^3}\sqrt{\frac{M}{r_0}}$

Tidal l=3 → $\mathcal{E}_{abc} \sim \frac{M}{r_0^4}\,, \quad \mathcal{B}_{abc} \sim \frac{M}{r_0^4}\sqrt{\frac{M}{r_0}}$



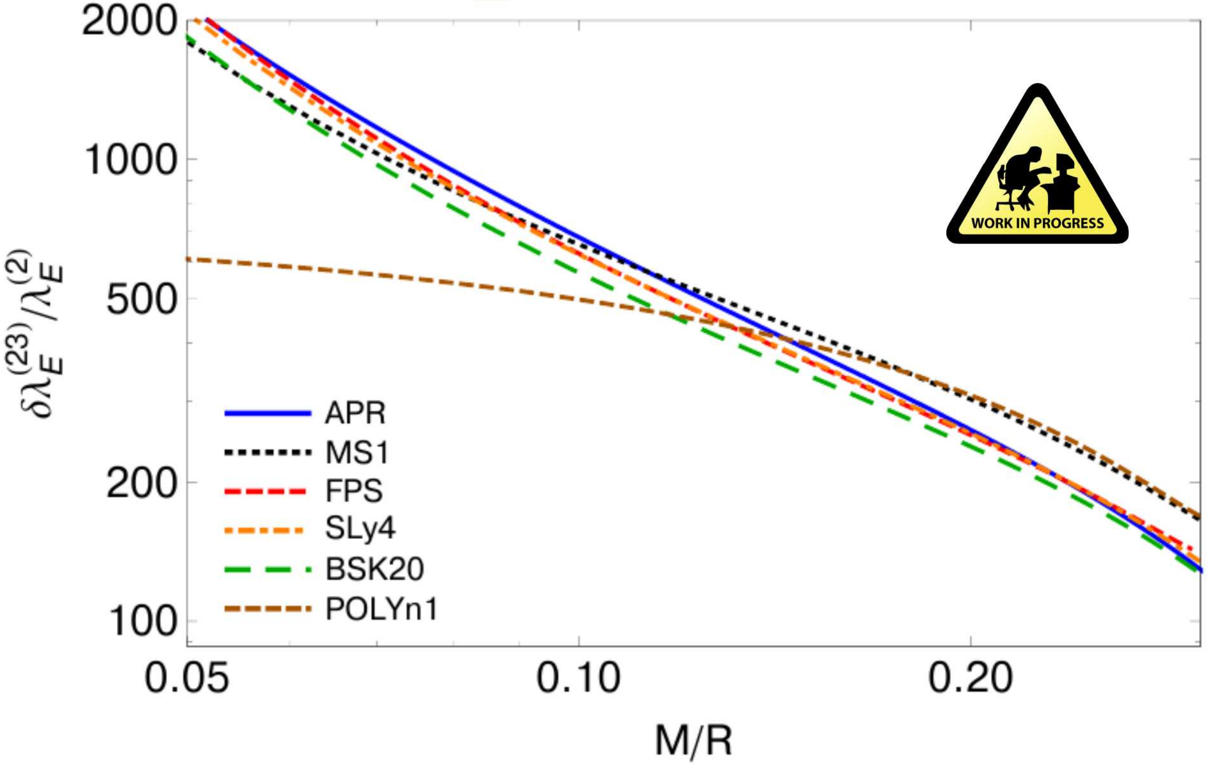
$$\frac{M_2}{M^3} \sim \frac{M^3}{r_0^3}\left[\bar{\lambda}_E^{(2)} + \underbrace{\chi\left(\frac{M}{r_0}\right)^{3/2}\delta\bar{\lambda}_E^{(23)} + \chi^2\delta\bar{\lambda}_E^{(22)}}_{\text{spin-tidal coupling}}\right] + \chi^2\left[\frac{4}{5}\delta q - 1\right]$$

$$\delta\lambda_E^{\ell\ell'} \equiv \frac{\partial M_\ell}{\partial \mathcal{B}_{\ell'}}\,, \qquad \delta\lambda_M^{\ell\ell'} \equiv \frac{\partial S_\ell}{\partial \mathcal{E}_{\ell'}}$$

Rotational tidal Love numbers

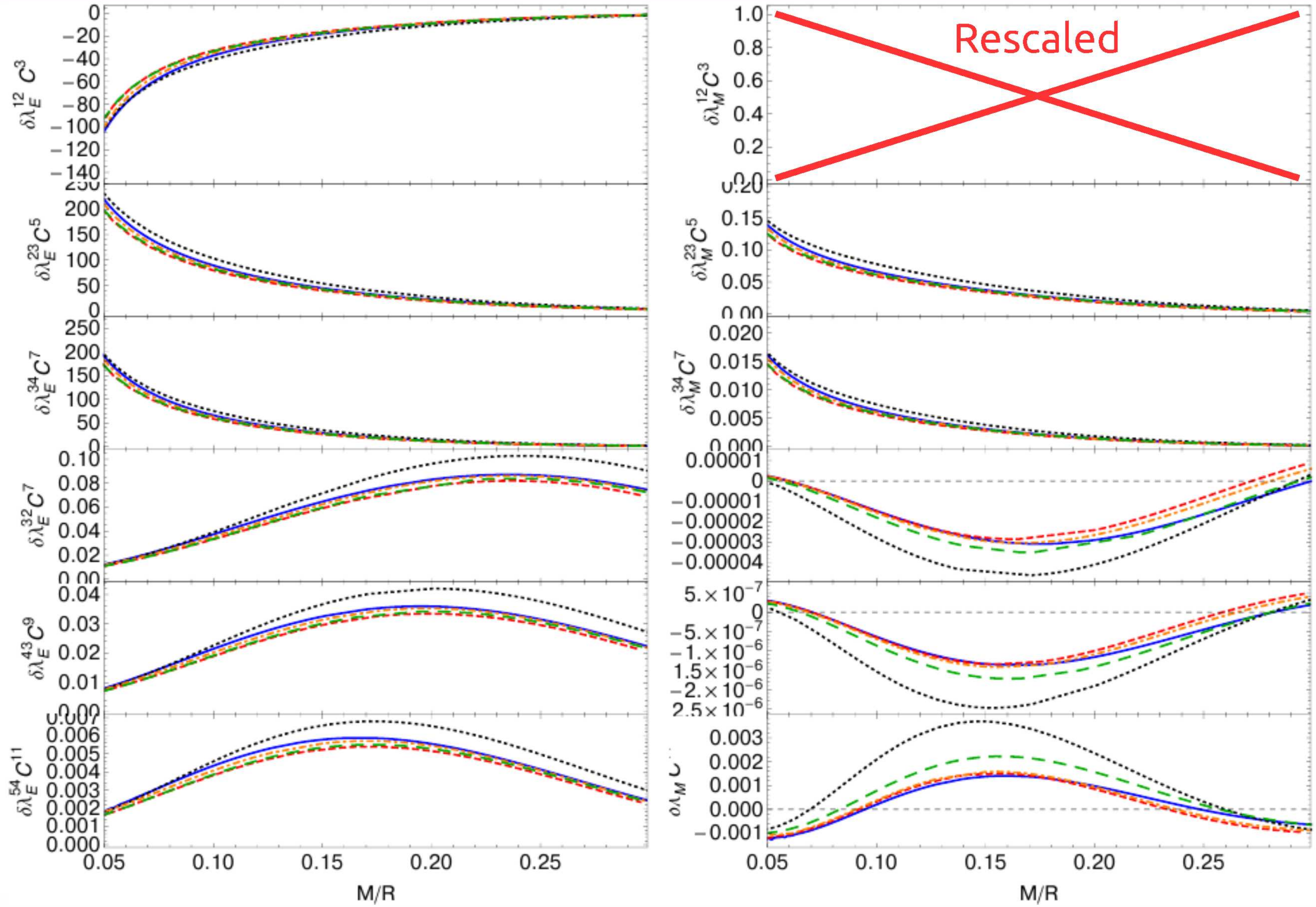
Is spin-tidal coupling important?

$$\frac{\delta M_2^{\mathcal{O}(\chi)}}{\delta M_2^{\mathcal{O}(\chi^0)}} \sim 10^{-2}\frac{\delta\bar{\lambda}_E^{(23)}}{\bar{\lambda}_E^{(2)}}\left(\frac{\chi}{0.1}\right)\left(\frac{M}{R}\right)^{3/2}\left(\frac{5R}{r_0}\right)^{3/2}$$



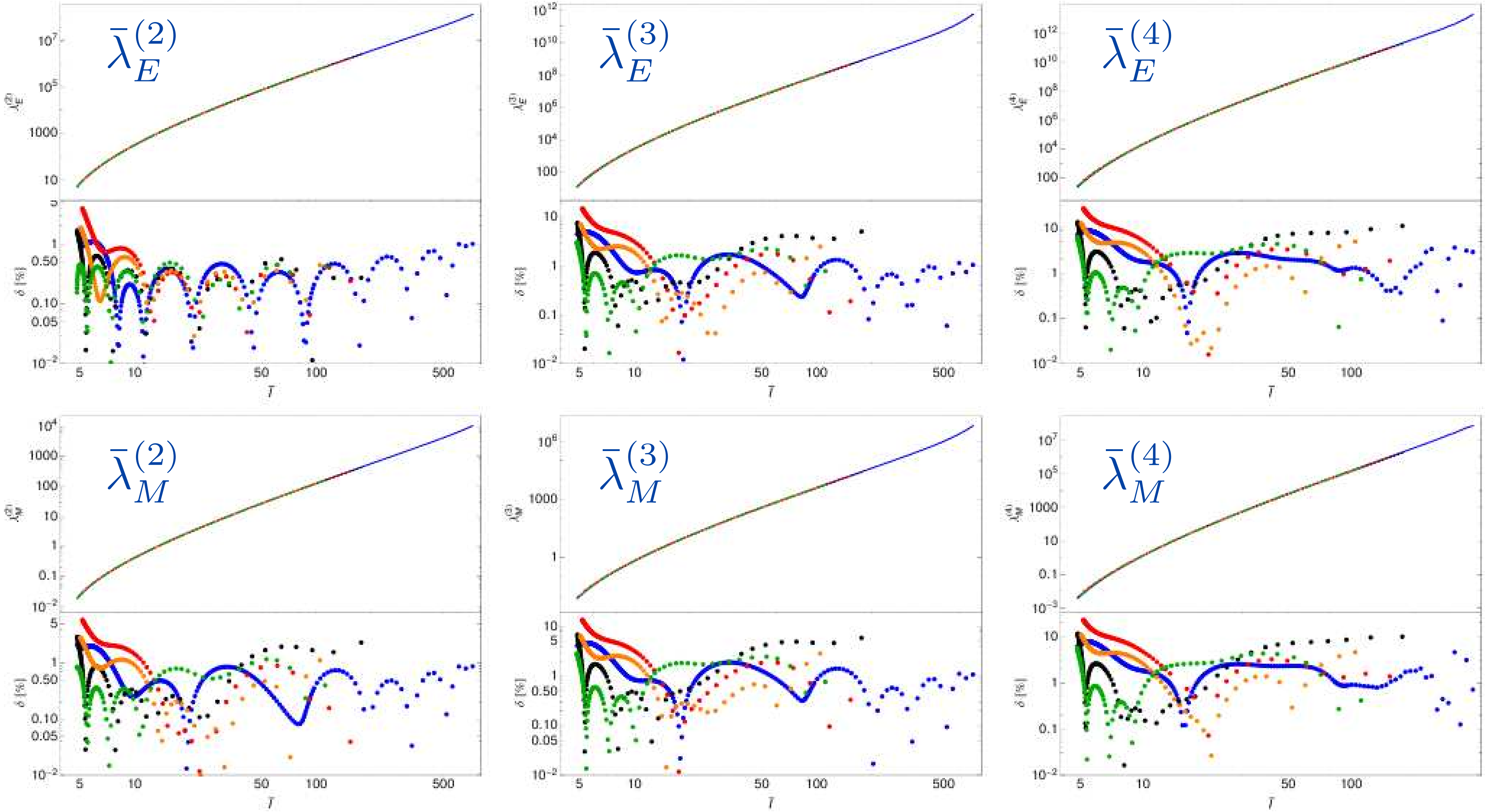
1st order spin-tidal corrections can be ~20% near merger

Rotational tidal Love numbers

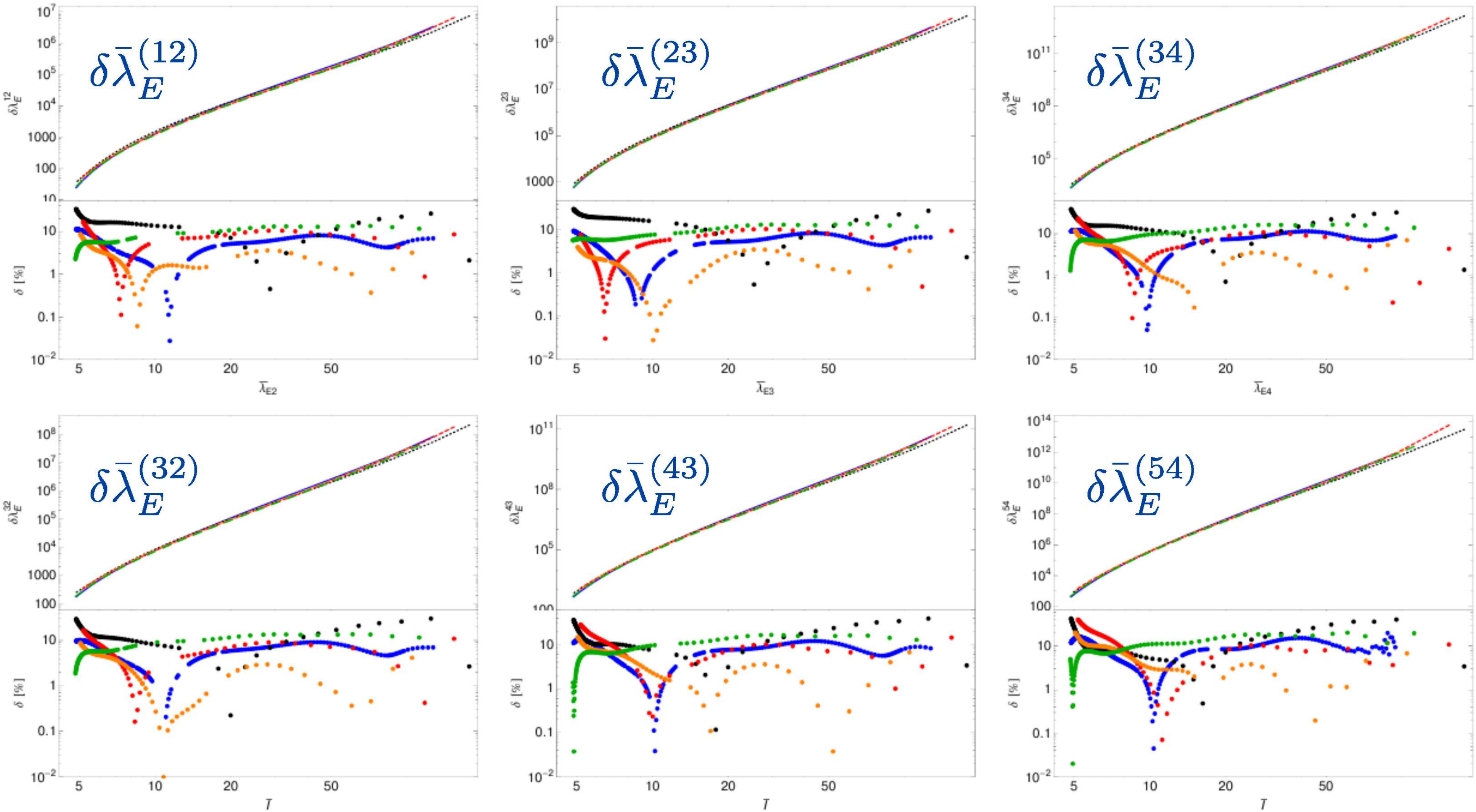


Universality #2: static case

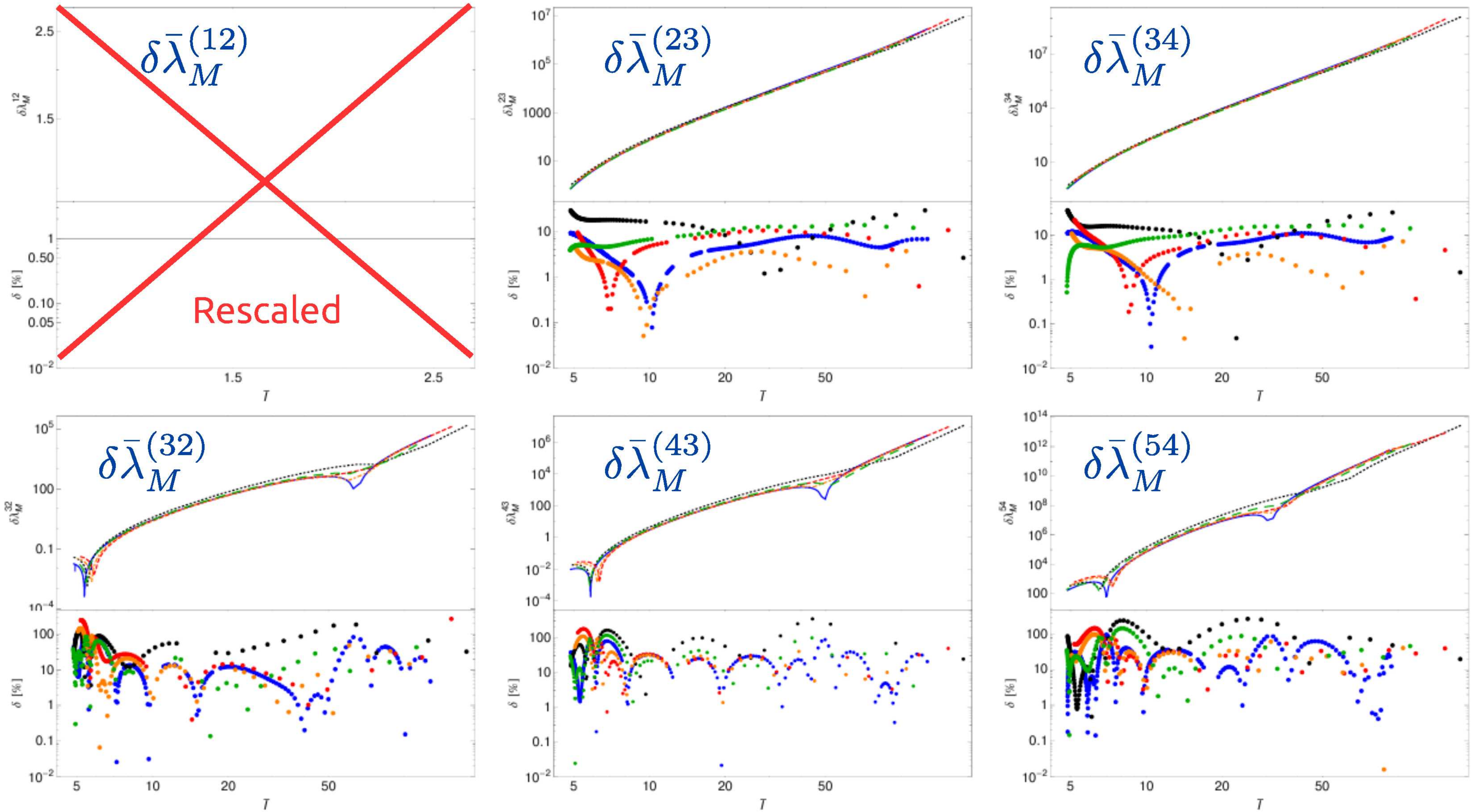
[Yagi & Yunes, 2013]
[Yagi, Phys.Rev. D89 (2014) 4, 043011]



Universality: spin electric



Universality: spin magnetic



Conclusions & Outlook

- Tidal deformations of a spinning compact object: fascinating & challenging
- **Tidally-deformed spinning BHs:**
  - (axisymmetric?) tidal Love numbers of spinning BHs are zero
  - Metric known analytically to second order in the spin
- **Rotational tidal Love numbers of spinning NSs:**
  - Important effects? Spin-tidal couplings in the waveforms?
  - Universality breaking?

- Proper definition of the **multipole moments, effective action, waveforms, ...**
- **Time-dependence**, **nonaxisymmetric** perturbations
- **Large spin** BH case: [Yunes & Gonzales, 2009 – O'Sullivan & Hughes, 2014-2015]
- **Matching to PN metric** [Poisson, 2014 – Yunes 2006]
- **Differential rotation**, **higher-order** spin corrections, **irrotational** fluids, ...

[Laundry & Poisson, 2015 – Delsate, 2015]

Thanks!

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