Geometry behind the horizon in superposition of Schwarzschild black hole and Bach-Weyl ring.

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INTRODUCTION

- Work group: Oldřich Semerák (supervisor)
  Marek Basovník (PhD. student)
- Submited papers (**G**eneral **R**elativity and **G**ravitation):
  - O.S., M.B; On geometry of deformed black holes: I. Majumdar-Papapetrou binary
  - M.B., O.S; On geometry of deformed black holes: II. Schwarzschild hole surrounded by a Bach–Weyl ring
- Paper in preparation:
  - O.S., M.B; On geometry of deformed black holes: III. Schwarzschild binary held by an Appell ring
- acknowledgement: GAUK 116-10/227329
  (Charles University grant)

SUPERPOSITIONS IN WEYL METRICS

- System for **Static** and **Axisymmetric** spacetimes
- Weyl coordinates: t, ρ, z, ϕ (time-like, radial, axial, equatorial)
- Metric: $\mathrm{d}s^2 = -e^{2\nu}\mathrm{d}t^2 + e^{2\lambda-2\nu}(\mathrm{d}\rho^2 + \mathrm{d}z^2) + \rho^2 e^{-2\nu}\mathrm{d}\theta^2$
- Einstein vacuum eqn:
  $\nu_{,\rho\rho} + \nu_{,zz} + \frac{\nu_{,\rho}}{\rho} = 0,$
  $\lambda_{,\rho} = \rho\left((\nu_{,\rho})^2 - (\nu_{,z})^2\right) \quad , \quad \lambda_{,z} = 2\rho\nu_{\rho}\nu_{,z}$
- Einstein vacuum eqn: (alternative form):
  $\nu(\rho, z) = \frac{1}{\pi}\int_0^{\pi} \nu(0, z + \mathrm{i}\rho\cos\alpha)\mathrm{d}\alpha,$
  $\lambda_{,\rho\rho} + \lambda_{,zz} + \frac{\lambda_{,\rho}}{\rho} = -2\left(\nu_{,z}\right)^2$

SCHWARZSCHILD BH & BACH–WEYL RING

- $\nu = \nu_{\rm schw} + \nu_{\rm ext}$
- $\lambda = \lambda_{\rm schw} + \lambda_{\rm ext}$
- (!) $\lambda_{\rm ext}$ is not the same as λ for a single external source
- $\nu_{\rm schw} = \frac{1}{2} \ln \frac{d_1+d_2-2M}{d_1+d_2+2M}$, $\lambda_{\rm schw} = \frac{1}{2} \ln \frac{(d_1+d_2)^2-4M^2}{4d_1d_2}$
  $d_{1,2} \equiv \sqrt{\rho^2 + (z \mp M)^2}$
- $\nu_{\rm ext} = -\frac{2\mathcal{M}K(k)}{\pi l_2} = -\frac{\mathcal{M}}{\mathrm{agm}(l_1,l_2)}$
  $l_{1,2} \equiv \sqrt{(\rho^2 \mp b^2) + z^2}$, $k^2 \equiv 1 - \frac{(l_1)^2}{(l_2)^2}$

Problem with Weyl coordinates

- $\nu_{\text{schw}} = \frac{1}{2} \ln \frac{d_1 + d_2 - 2M}{d_1 + d_2 + 2M}$
- Coordinate singularity (horizon) is placed at $\rho = 0, z \in (-M, M)$
- (!) Problem: Weyl cordinates do not allow to describe the area behind the horizon
- Solution:
  - A) Trick with pure imaginary ρ
  - B) Transform into Shwarzschild coordinates
  - **C) Transform into our coordinates**

Alternative coordinates

- Weyl coordinates with $\varrho = \mathrm{i}\rho$

  $$\mathrm{d}s^2 = (-e^{2\nu})\mathrm{d}t^2 + e^{2\lambda-2\nu}(-\mathrm{d}\varrho^2 + \mathrm{d}z^2) + \varrho^2(-e^{-2\nu})\mathrm{d}\theta^2,$$

  $$\varrho = \sqrt{r(2M-r)}\sin\theta \quad , \quad z = (r-M)\cos\theta$$

- Schwarzschild coordinates

  $$\mathrm{d}s^2 = (\tfrac{2M}{r} - 1)e^{2\nu_{\mathrm{ext}}}\mathrm{d}t^2 - e^{2\lambda_{\mathrm{ext}}-2\nu_{\mathrm{ext}}}\left(\frac{\mathrm{d}r^2}{\frac{2M}{r}-1} + r^2\mathrm{d}\theta^2\right) + \\ + e^{-2\nu_{\mathrm{ext}}}r^2\sin^2\theta\mathrm{d}\phi^2,$$

  $$r = M\left(1 + \cos\frac{\vartheta_1-\vartheta_2}{2}\right) \quad , \quad \theta = \frac{\vartheta_1+\vartheta_2}{2}$$

- Ours (under horizon only)

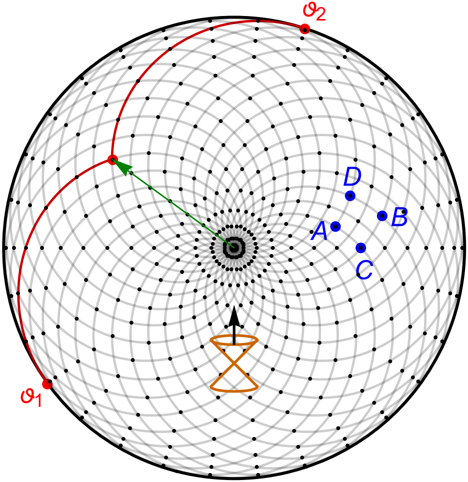
  $$\mathrm{d}s^2 = (\tfrac{2M}{r} - 1)e^{2\nu_{\mathrm{ext}}}\mathrm{d}t^2 + r^2e^{-2\nu_{\mathrm{ext}}}\left(e^{2\lambda_{\mathrm{ext}}}\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

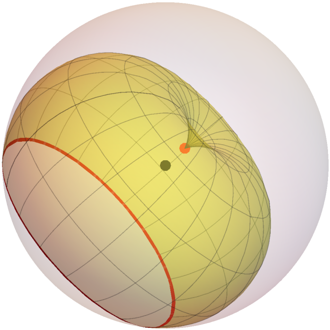
  $$\varrho = \frac{M}{2}(\cos\vartheta_2 - \cos\vartheta_1) \quad , \quad z = \frac{M}{2}(\cos\vartheta_2 + \cos\vartheta_1)$$

EINSTEIN EQUATIONS

- $\nu_{\text{ext}}(\vartheta_1, \vartheta_2) = \frac{1}{\pi} \int_0^\pi \varphi(\sin^2 \alpha \cos \vartheta_1 + \cos^2 \alpha \cos \vartheta_2) \mathrm{d}\alpha$
  $\varphi(\circ) \equiv \nu_{\text{ext}}(r = 2M, \cos\theta = \circ)$
- $\lambda_{\text{ext},12} = -\frac{M}{2} \nu_{\text{ext},r} - \nu_{\text{ext},1} \nu_{\text{ext},2}$

Geometric interpretation of E. eqn





$$\lambda_{\text{ext},12} = -\frac{M}{2}\nu_{\text{ext},r} - \nu_{\text{ext},1}\nu_{\text{ext},2}$$

$$F_{,12} \approx \frac{F_A + F_B - F_C - F_D}{(\pi/n)^2}$$

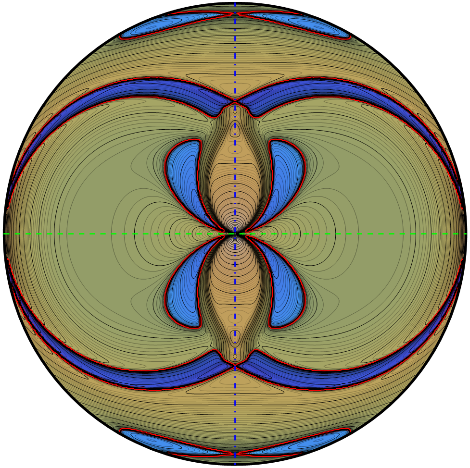
$$r = M\left(1 + \cos\frac{\vartheta_1 - \vartheta_2}{2}\right), \theta = \frac{\vartheta_1 + \vartheta_2}{2}$$

$$\nu_{\text{ext}}(\vartheta_1, \vartheta_2) = \frac{1}{\pi}\int_0^\pi \varphi\left(\sin^2\alpha\cos\vartheta_1 + \cos^2\alpha\cos\vartheta_2\right) d\alpha$$

KRETSCHMANN SCALAR

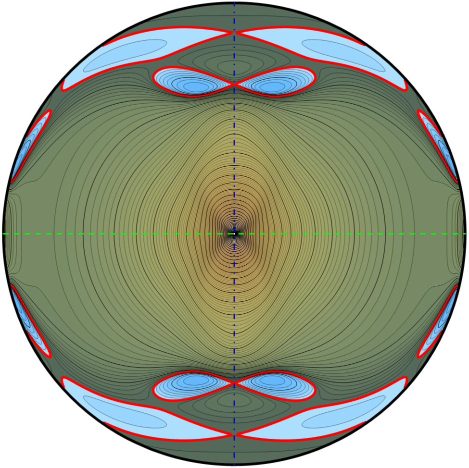
- $K = 12(R^{12}{}_{12})^2 + 16R^{t1}{}_{t2}R^{t2}{}_{t1}$
- $K = 8\left[(R^{tr}{}_{tr})^2 + (R^{t\theta}{}_{t\theta})^2 + (R^{t\phi}{}_{t\phi})^2 - \frac{2(R^{tr}{}_{t\theta})^2}{r(2M-r)}\right]$
- Interpretation of negative scalar
  - Deviation of two particles $u^r, u^t \neq 0$ spread in t
    $\frac{\mathrm{D}^2\delta t}{\mathrm{d}\tau^2} = -R^t{}_{rtr}(u^r)^2\delta t \quad , \quad \frac{\mathrm{D}^2\delta\theta}{\mathrm{d}\tau^2} = R^\theta{}_{trt}u^r u^t \delta t$
  - Deviation of two particles $u^r, u^t \neq 0$ spread in θ
    $\frac{\mathrm{D}^2\delta t}{\mathrm{d}\tau^2} = R^t{}_{rt\theta}u^r u^t \delta\theta \quad , \quad \frac{\mathrm{D}^2\delta\theta}{\mathrm{d}\tau^2} = -\left[R^\theta{}_{t\theta t}(u^t)^2 + R^\theta{}_{r\theta r}(u^r)^2\right]\delta\theta$

KRETSCHMANN SCALAR



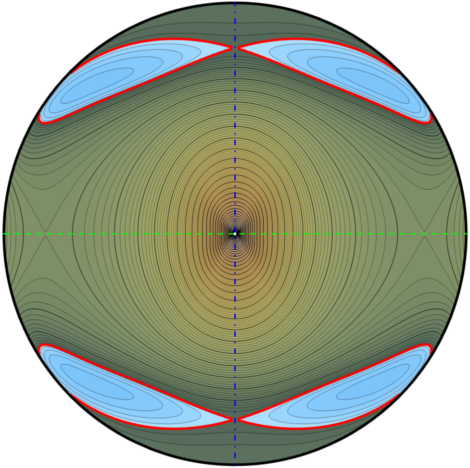
$\mathcal{M} = 0.2M, b = 0.08M$

KRETSCHMANN SCALAR



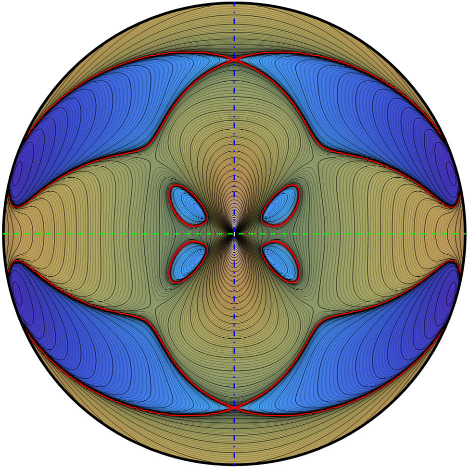
$\mathcal{M} = 0.2M, b = 0.4M$

Kretschmann scalar



$$\mathcal{M} = M, b = M$$

KRETSCHMANN SCALAR



$$\mathcal{M} = 5M, b = M$$

CONCLUSIONS

- Well known methods (using Weyl coordinates) are not very fit for under–horizon computation
- We found a different type of coordinates (describing under–horizon only) with well–aranged geometric interpretation
- We recognize a negative Kretschmann scalar under the horizon even the spacetime is **static** and axially symmetric (however the *t*-symmetry become space-like, so negative values are not so suprisingly)

Thank you

Thank you for your attention!