Rotating AdS black holes and condensed matter physics

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I What is AdS/CFT correspondence?

Original AdS(Anti De Sitter spacetime)/CFT(Conformal field theory):

N=4 SYM ⟺ Type IIB on $AdS_5 \times S^5$

Finite temperature case:

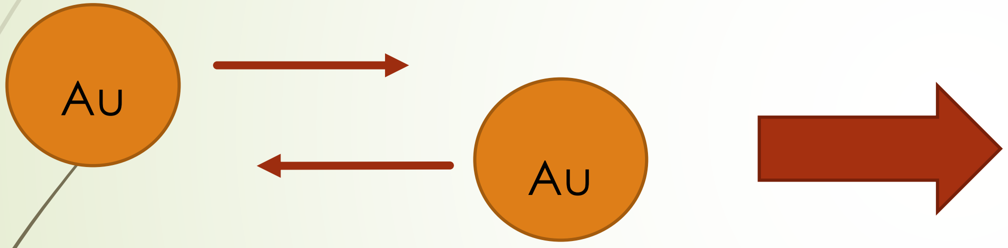
N=4 SYM at finite temp ⟺ Type IIB on AdS black holes

The ***strongly coupled*** gauge theory is described by Classical General Relativity in asymptotically AdS spacetime!

Application to Quark gluon plasma(QGP)

RHIC: Relativistic Heavy Ion Collider at Brookhaven(2005)

- Au-Au collision → Quark-gluon plasma



AdS/CFT correspondence →

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

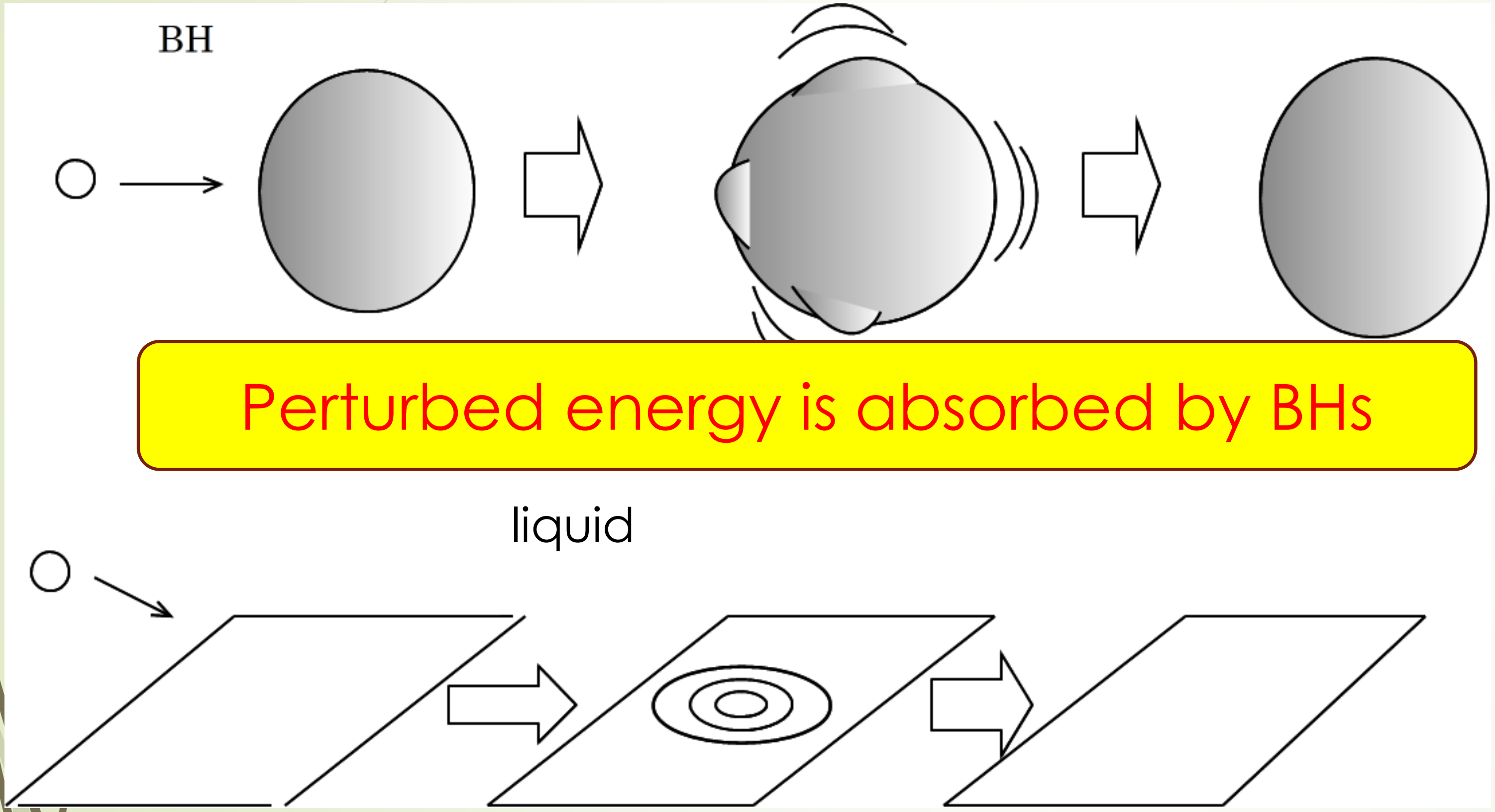
η : shear viscosity

S: entropy density

The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating (2005,, Press release)

Dissipation and AdS black holes

How to describe dissipation in QGP by AdS BHs ?



Perturbed energy is absorbed by BHs

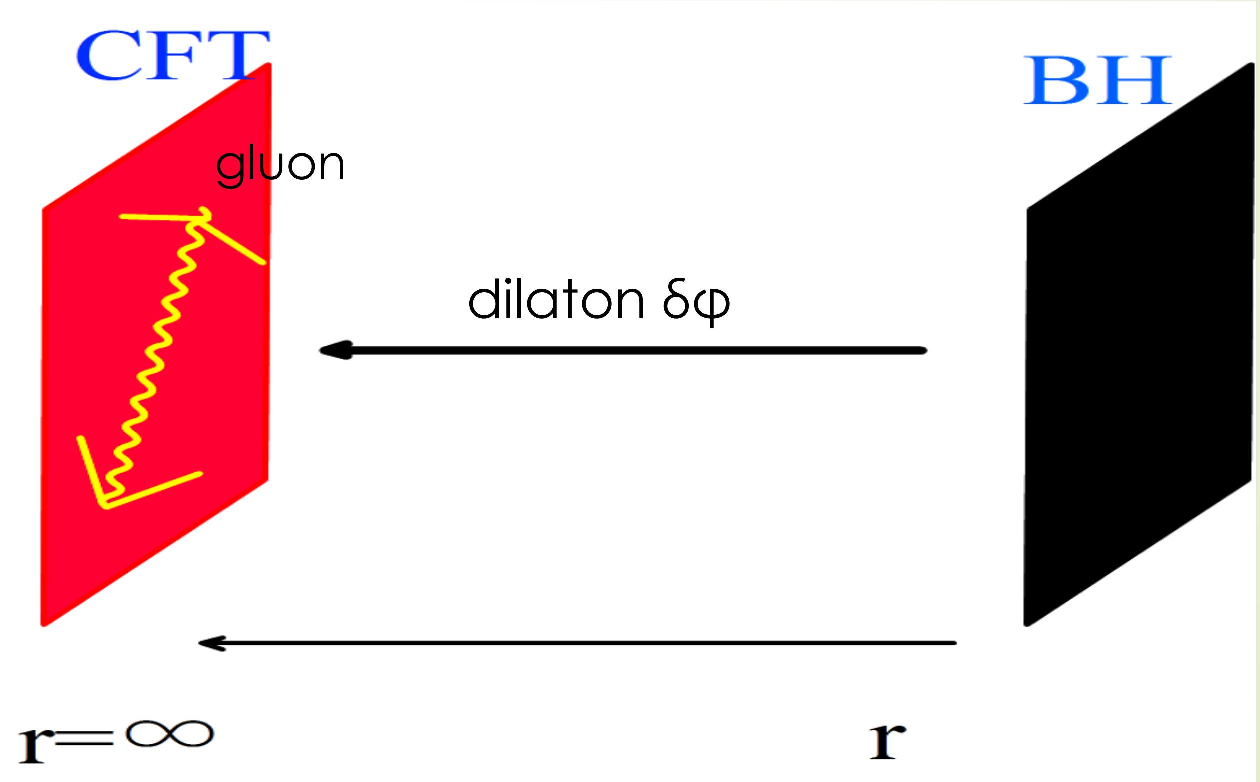
Ripples dissipate by viscosity

The relaxation time is described by Quasi-Normal modes of BHs

Bulk field vs Boundary field

Bulk field fluctuations act as *source term* in Boundary theory

$$S_{int} \sim \int dx^4 (\delta\phi \, F^2_{\mu\nu} + h^{\mu\nu} \, T^{YM}_{\mu\nu}) + \cdots$$



GKP-Witten relation

Source term for the boundary operator $\mathcal{O}(x)$

$$Z[\phi_0] = \exp[-I(\phi)] = \left\langle \exp\left(\int_{\partial M} \phi_0 \mathcal{O} \right) \right\rangle$$

$$\phi \to \phi_0$$

$\mathcal{O}(x)$: BD operator

Ex)

CFT	⟺	Bulk
$(F^{a}{}_{\mu\nu})^2$		Dilaton Φ
EM tensor $T_{\mu\nu}$		Perturbed metric $h_{\mu\nu}$
R-current J_μ		U(1) gauge potential A_μ
Entropy (density) S		BH Entropy (density) S

$h_{\mu\nu}$

. . .

Scalar field case

AdS metric $ds^2 = \frac{r_0^2}{z^2}(\eta_{ij}\, dx^i\, dx^j + dz^2)$

$$\lambda_\pm = \frac{1}{2}(d \pm \sqrt{d^2 + 4m^2})$$

Solution of Eq: $\nabla^2 \phi - m^2 \phi = 0$

$$\phi \cong b_+(x)\phi_+ + b_-(x)\phi_-$$
$$= b_+(x) z^{\lambda_+} + b_-(x) z^{\lambda_-}$$

Boundary operator VEV $\langle \mathcal{O} \rangle$

(Normalizable mode)

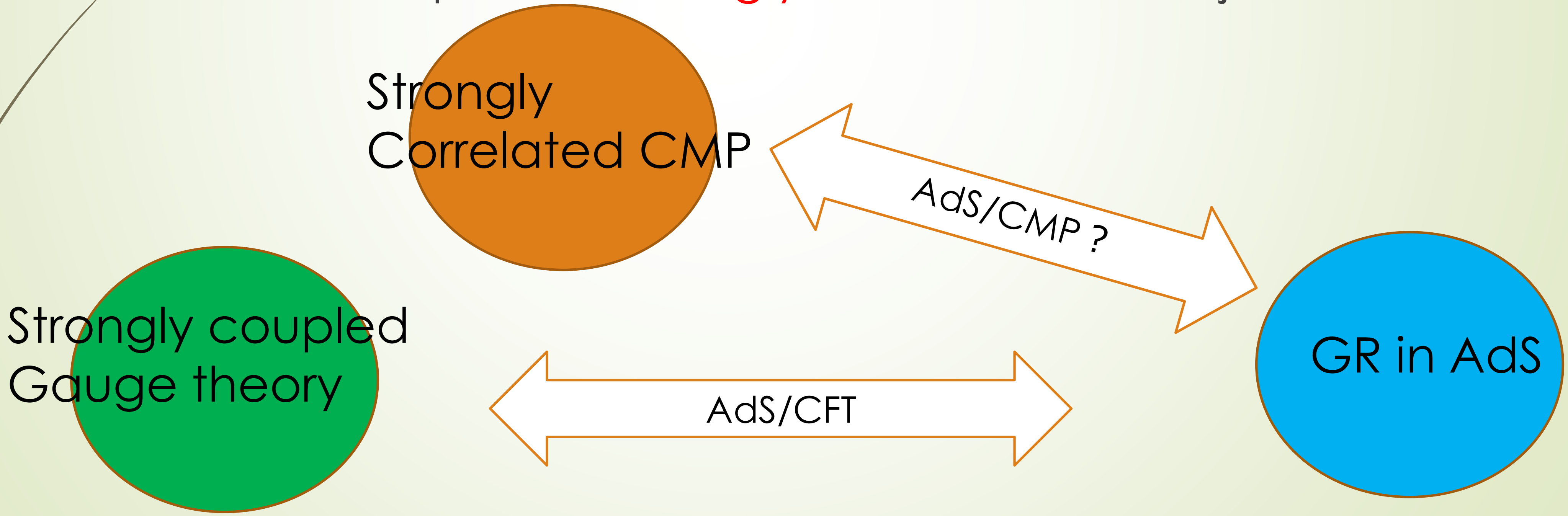
Bulk Source term

(Non-Normalizable mode)

Can we apply AdS/CFT correspondence to Condensed matter physics(CMP) ?

Motivation: Conventional approach to strongly correlated High Tc Superconductor or quantum phase transitions is difficult

Our Hope: Strongly coupled gauge theory or GR in AdS (via AdS/CFT) could describe some aspect of strongly correlated CMP, just as QGP.



Ⅱ Recent progress on AdS/CMP correspondence

Holographic superconductor model (Hartnoll, Herzog, Horowitz (2008))

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

$$D_\mu = \nabla_\mu - iqA_\mu$$

Metric Ansatz: $$ds^2 = -f(r)e^{-\delta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

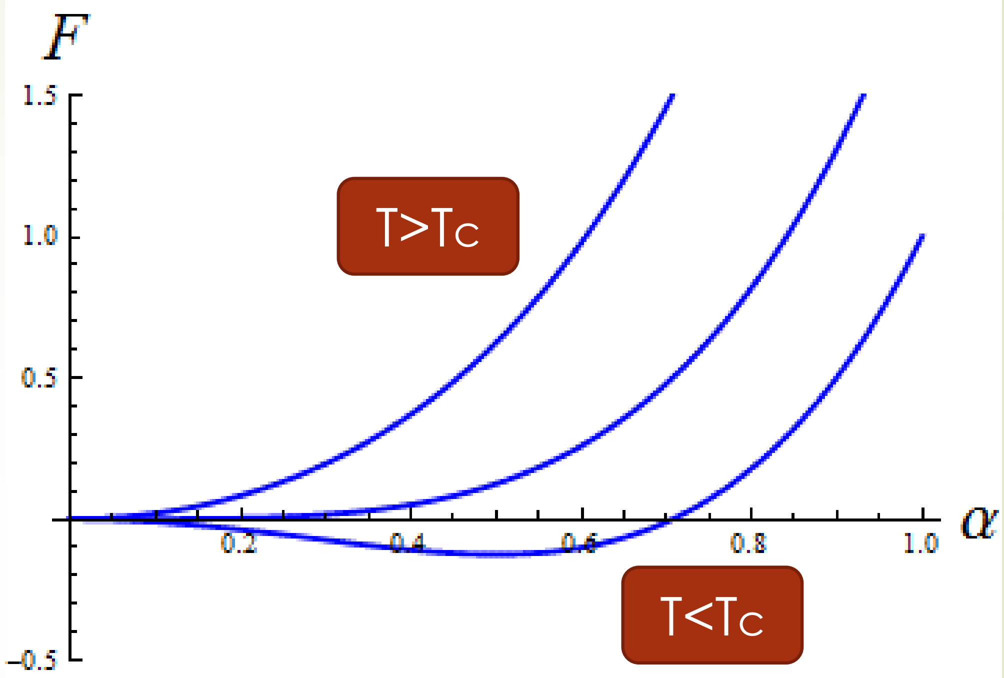
Field Ansatz: $$\Psi = \Psi(r), \qquad A_\mu = \Phi(r)(dt)_\mu$$

Non-Normalizable mode

Asymptotic behavior of ψ:

$$\Psi \simeq \alpha_+ r^{-\lambda_+} + \alpha_- r^{-\lambda_-}, \qquad \lambda_+ > \lambda_-$$

Normalizable mode (Order parameter)



Superconducting state

$$\alpha_- = 0 \qquad \alpha_+ \neq 0$$

Gubser(2008): Near the extremal black hole, such charged hairly BHs could exist in AdS

$$m_{eff}^2 = m^2 - \frac{q^2 A_t^2(r) e^{\delta}}{f(r)} < 0$$

Effective mass is arbitrary negative

Many hairly BHs could exist in AdS spacetime!

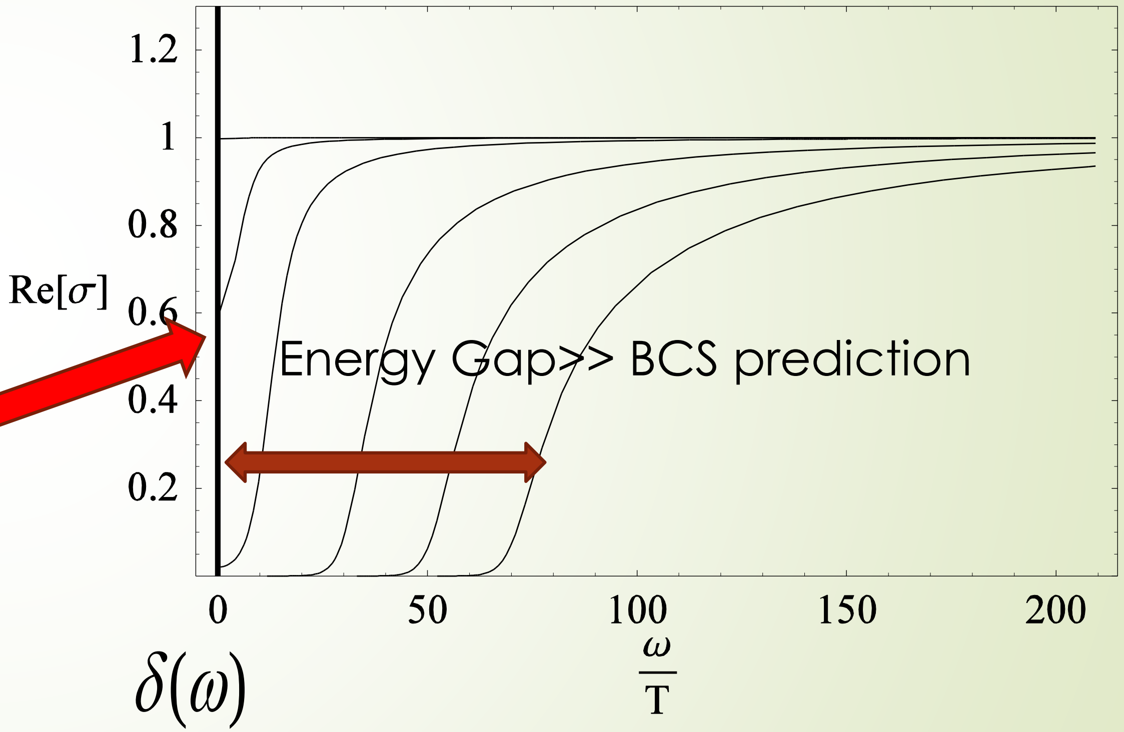
Maxwell perturbations and the conductivity

Perturbation of $A_x \sim e^{-i\omega t}$ ⟷ current J_μ

$$A_x \simeq \alpha_x + \frac{J_x}{r}$$

$$\sigma(\omega) = \frac{J_x}{E_x} = -\frac{J_x}{\dot{\alpha}_x} = -\frac{iJ_x}{\omega \alpha_x}$$

- Delta function appears instead of Drude peak due to no dissipation
- Large energy gap suggests ***strongly coupled effect*** of boundary theory



PRL101.031601 Hartnoll, Herzog, Horowitz

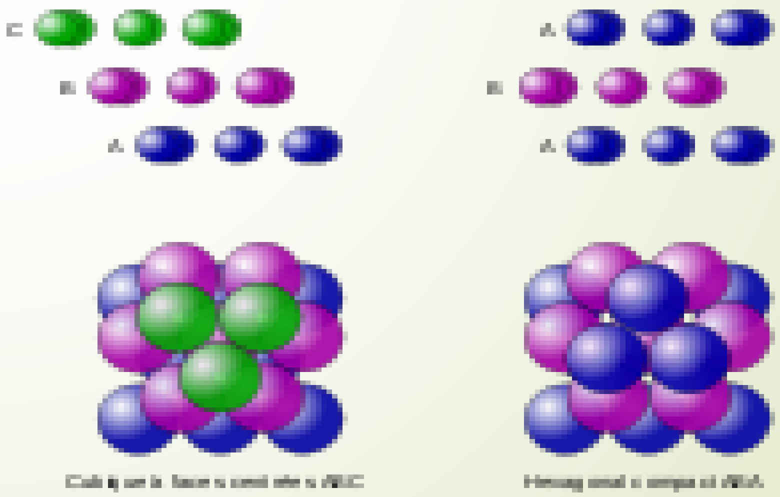
However…

Delta function $\delta(\omega)$ always appears even in RN-AdS BH!

Momentum conservation is always satisfied in translationally invariant (planar) BHs!

We need to explore holographic superconductor model in *some broken translationally system*

Ex) Lattice structure
in condensed matter system



Perturbative model:

- K. M., T. Okamura, J. Koga (2011) AdS BH solutions with ***spatially modulated chemical potential*** In Einstein Maxwell System
- N. Iizuka, K. M. (2012) Confirmation of Delta function $\delta(\omega)$ of conductivity in a superconducting state in a massive U(1) gauge toy model with lattices

Non-perturbative model:

- G. T. Horowitz, J. E. Santos, D. Tong (2012, 2013) The construction of spatially modulated charged AdS BHs and confirmation of Delta function $\delta(\omega)$ of conductivity in a superconducting state

Need to solve Nonlinearly PD Eqs!

Within linear response theory*, strongly correlated superconductor model is well described by the holographic model*

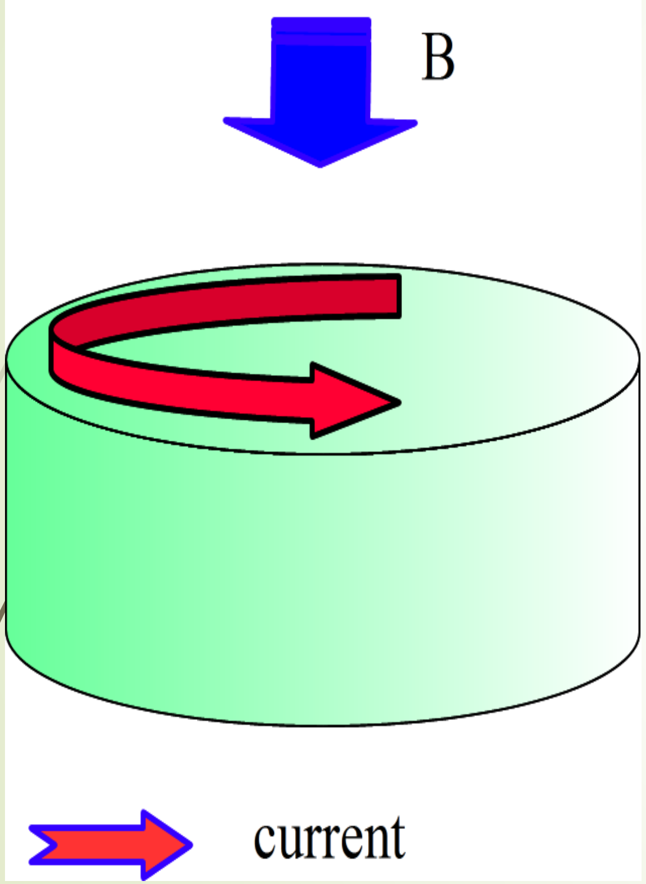
Question

Another simple model ? !

What happens beyond linear response theory ?

III. Superfluid flow and rotating AdS Black Holes

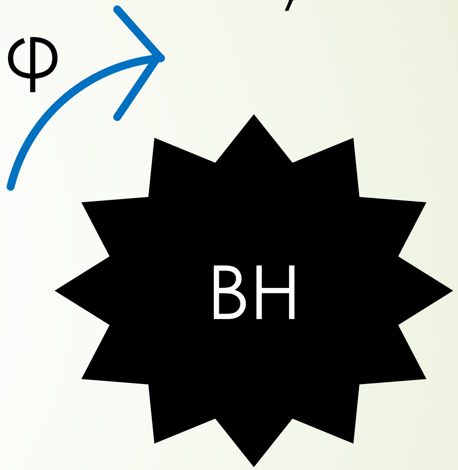
CMP: Persistent current



AdS/CMP ?

Hairly rotating lumpy BH ?

$\partial_\phi \neq$ Killing vector



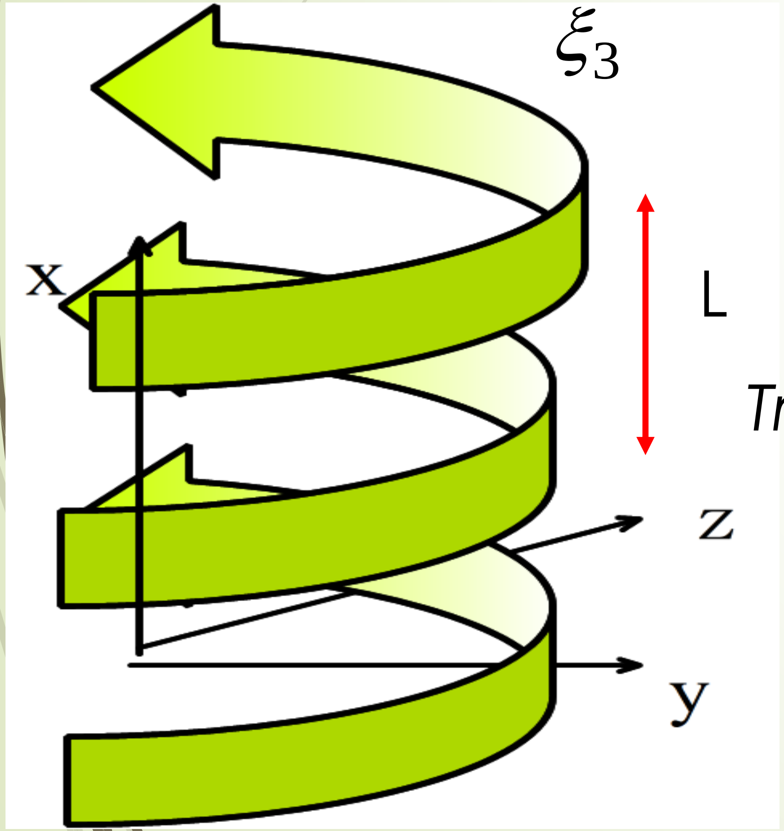
BH rigidity theorem is violated ?

To answer this question, we apply Bianchi type anisotropic model!

Bianchi type VII$_0$ space –Helical structure-

Three Killing vectors:

$$\xi_1 = \partial_y, \qquad \xi_2 = \partial_z, \qquad \xi_3 = \partial_x - z\partial_y + y\partial_z$$



Invariant one-form:

$$\omega^1 = \cos(x)\,dy + \sin(x)\,dz, \qquad \omega^2 = -\sin(x)\,dy + \cos(x)\,dz,$$

$$\omega^3 = dx$$

Translationally invariance is violated along x direction!

Helical structure is naturally incorporated in the Bianchi VII$_0$ space!

Holographic superconductor model:

$$\mathcal{L} = \left(R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - |D\psi|^2 - m^2|\psi|^2 \right),$$

$F = dA, \quad W = dB, \quad A, B$: U(1) gauge field

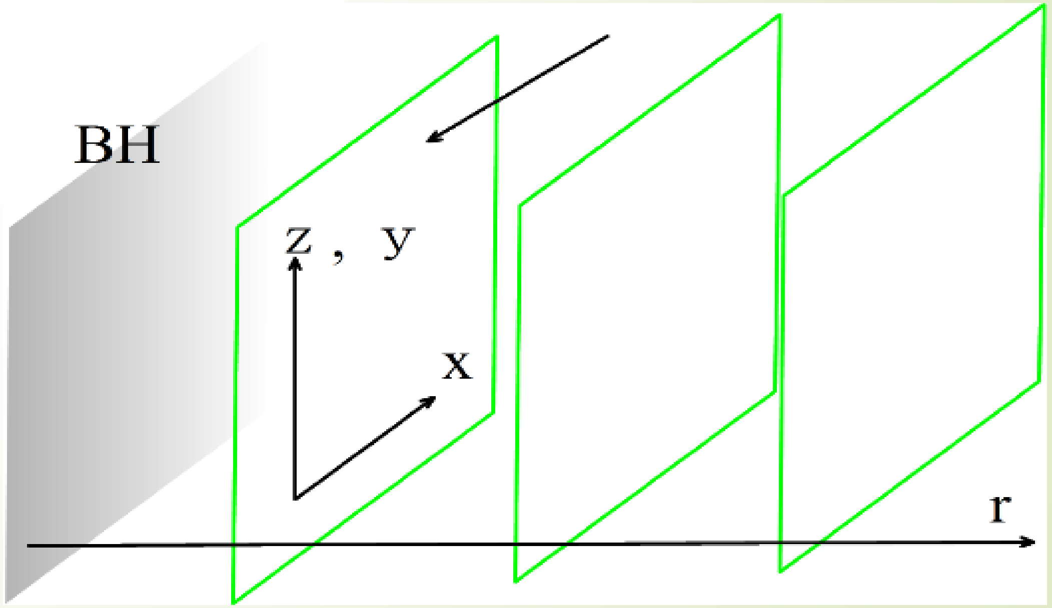
Metric ansatz:

$$ds_5^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + e^{2v_3(r)}(\omega^3 - \Omega(r)dt)^2 + e^{2v_1(r)}(\omega^1)^2 + e^{2v_2(r)}(\omega^2)^2$$

5-dim AdS Bianchi VII0 Black brane

Einstein Eqs.

Ordinary differential equations



Asymptotic behavior of Ω

$$\Omega \cong \frac{P}{r^4}$$

← Momentum of superconducting persistent current

Q. ***P is carried by BH or matters outside of BH ?***

A. P cannot be carried by BH according to regularity condition at the horizon

→ Complex scalar field ψ carry the whole angular momentum P

No contradiction with BH rigidity theorem

Non-trivial relation:

$$P = -\mu \times J$$

μ: chemical potential

J : Supercurrent

N. Iizuka, A. Ishibashi, K. M. PRL(2014)

Landau and Tisza's two fluid model

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P\eta_{\mu\nu} + \mu\rho_s v_\mu v_\nu ,$$
$$j_\mu = \rho_n u_\mu + \rho_s v_\mu ,$$

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu j^\mu = 0$$

u_μ : Velocity of normal component

v_μ : Velocity of superfluid component

Josephson Eq.

$$v_\mu u^\mu = -1$$

When $u_x = 0$

We find $\frac{T_{tx}}{\mu j_x} = v_t = -(u^t)^{-1} = -1$

Precise agreement with numerical calculation

IV. Critical phenomena on the superfluid flow

Rotating AdS black hole is unstable ?

- Some AdS BHs is ***unstable*** against ***superradiant instability***

  V. Cardoso, O. J. C. Dias, J. P. S. Lemos, S.Yoshida (2004)

- Possible final state should be less symmetric BHs with ***only one Killing vector***

  H. S. Reall (2003)

- Many ***less symmetric*** AdS BH soluions:

O. J. C. Dias, G. T. Horowitz, and J. E. Santos(2011); C. A. R. Herdeiro and E. Radu (2014)

N. Iizuka, A. Ishibashi, K. M.(2015); O. J. C. Dias, J. E. Santos, B. Way (2015)

Metric possesses only one Killing vector

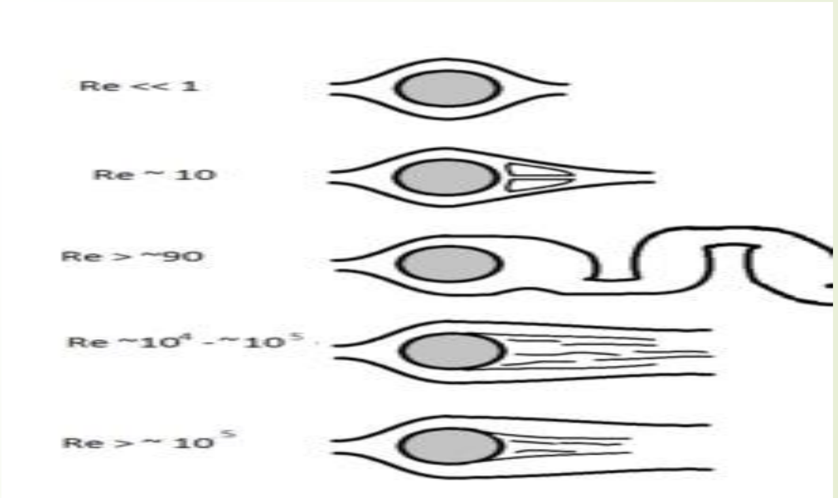
High frequency modes are continued to be generated (Cascade behavior)

BH Turbulence!

Q. Under what conditions BH turbulence occurs?

Maybe, AdS/CMP correspondence predicts the conditions and help to understand the nature of turbulence based on the knowledge of condensed matter physics

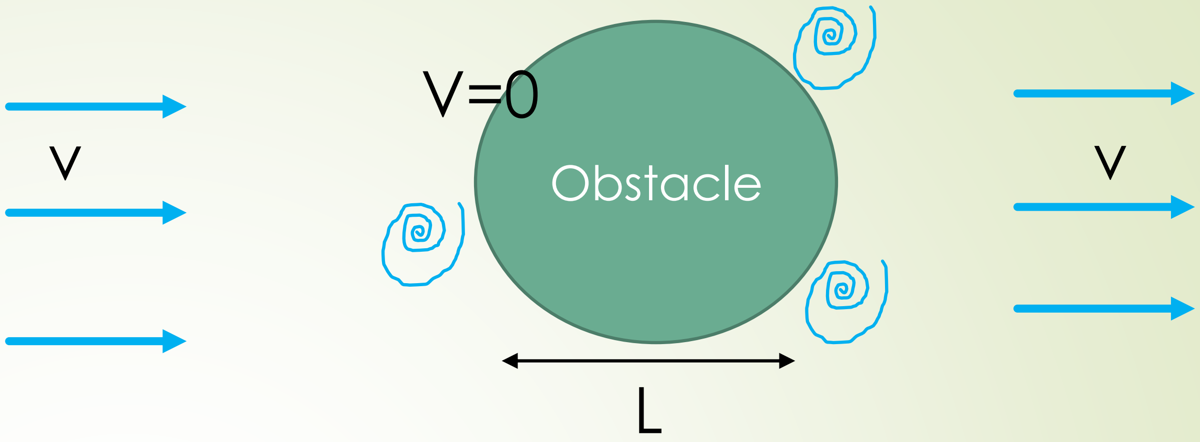
AdS/CMP



Normal fluid case

$$\boldsymbol{Re} \approx \frac{\rho v L}{\eta}$$

η : shear viscosity



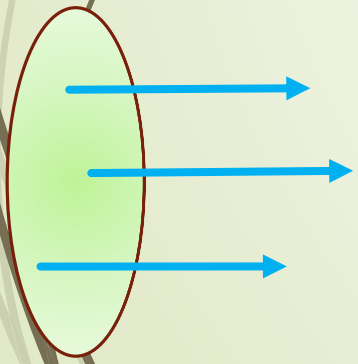
High ***Re*** → turbulence

- S. R. Green, F. Carrasco, L. Lehner (2014) *2-dim*. Relativistic hydrodynamics shows turbulence

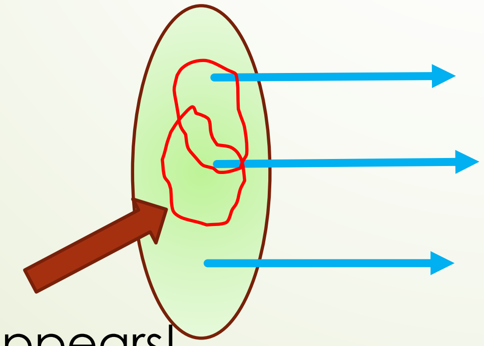
Superfluid case

Superfluid velocity v> V_c (Critical velocity) → turbulence

$V < V_c$

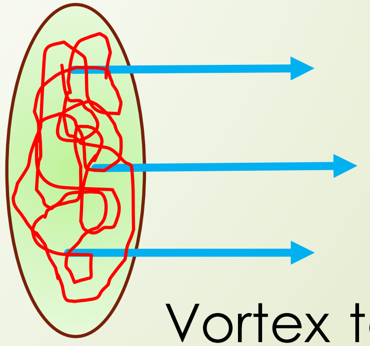


$V \cong V_c$



Vortex appears!

$V > V_c$



Vortex tangle

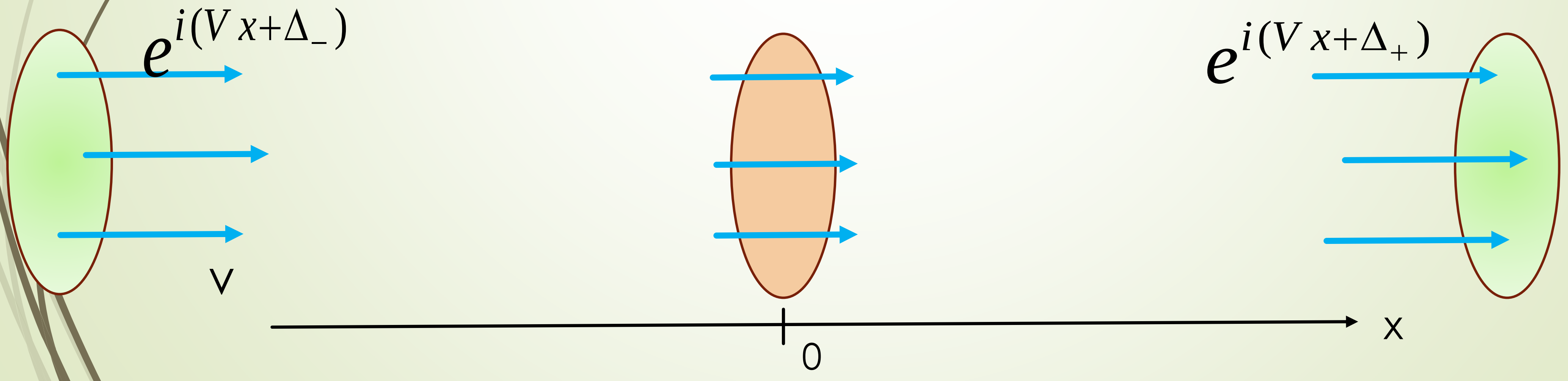
As a starting point, it may be interesting to know how superfluid steady flow state is broken

The simplest model: 1-dim. ***Non-linear Schrodinger model***

$$i\partial_t\varphi - iv\partial_x\varphi = -\partial_{xx}\varphi - \varphi + |\varphi|^2\varphi + U(x)\varphi$$

Weakly interacting Bose-Einstein superfluid model

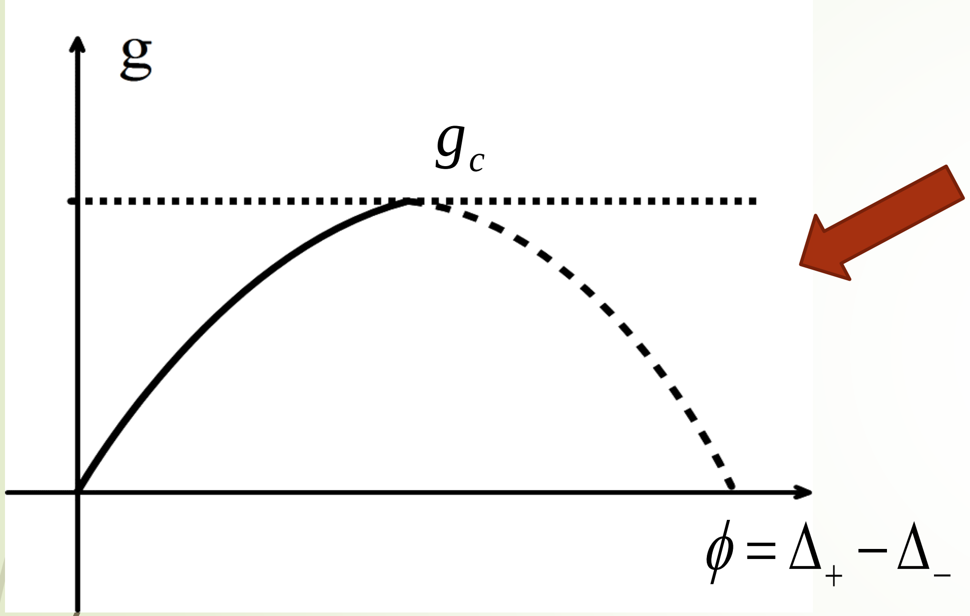
Hakim (1997) → $U(x) = g\,\delta(x), \quad g > 0 \quad \textit{repulsive potential}$



Hakim (1997)

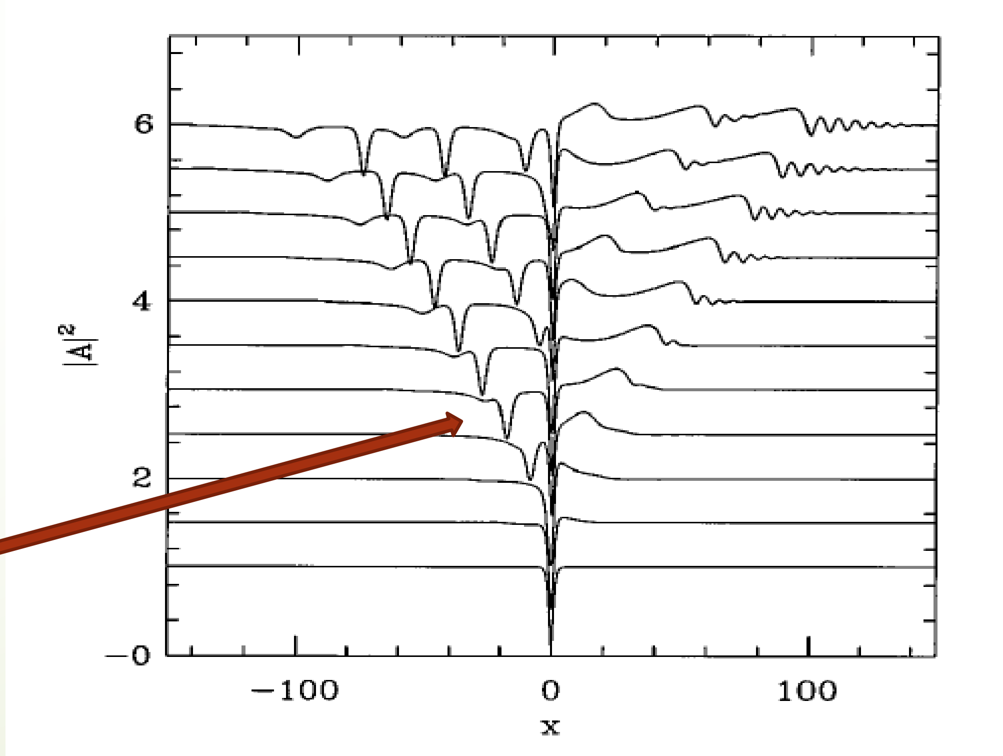
When g< g_c two steady flow solutions appear

When g= g_c the two solutions coalesce and disappear.



Saddle node bifurcation

Hakim PRE55



g> g_c ***the gray soliton*** solutions are created by the obstacle.

Gray Soliton

What happens in holographic model ?

A. Ishibashi, K. M., T. Okamura(work in progress)

$$\mathcal{L} = -|\nabla\psi - iA\psi|^2 - m^2|\psi|^2 - V(x,u)|\psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

We analytically solve the equations and confirmed that saddle node bifurcation always occurs.

In detail, we give a talk in VIII BH workshop!

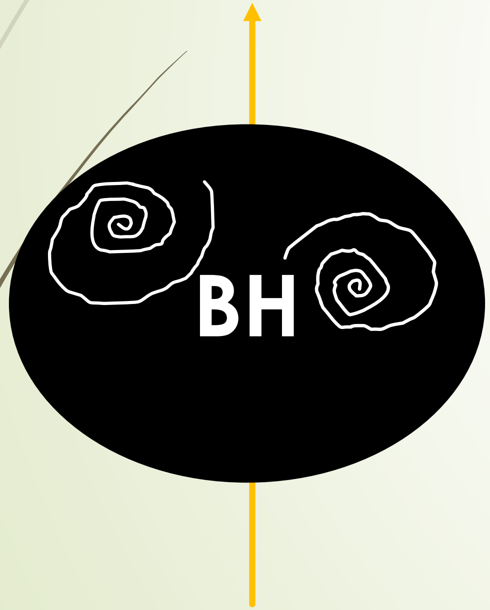
g> g_c

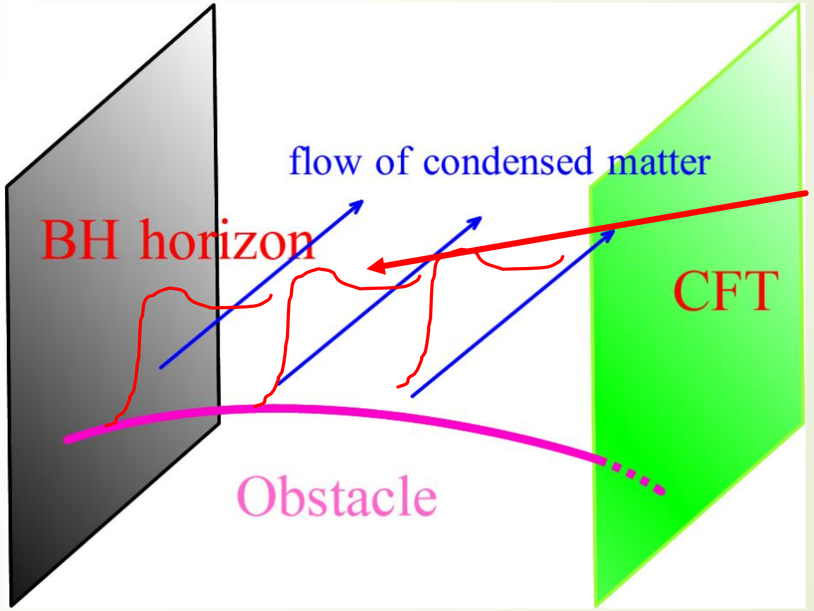
Onset of turbulence!

Rotating AdS BHs with momentum

Superradiance instability

Obstacle instability





Gray Soliton

Rotating BH turbulence!

Thank you!